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Some *KV* Indices of Certain Dendrimers

V. R. Kulli

Department of Mathematics, Gulbarga University, Gulbarga, 585106, India; e-mail: vrkulli@gmail.com

Abstract

In this paper, we define the modified first and second *KV* indices, *F-KV* and F_1-*KV* indices, hyper *F-KV* index and augmented *KV* index of a graph and compute exact formulas for POPAM and tetrathiafulvalene dendrimers. Furthermore, we determine the *F-KV*, hyper *F-KV* and augmented *KV* polynomials of POPAM dendrimers and tetrathiafulvalene dendrimers.

1. Introduction

A molecular graph is a simple graph such that its vertices correspond to the atoms and edges to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Numerous topological indices have been considered in Theoretical Chemistry, especially in QSPR/QSAR study, see [1, 2].

Let $V(G)$, $E(G)$ be a vertex set and an edge set of a finite simple connected graph G respectively. The degree $d(v)$ of a vertex v is the number of edges incident to v. Let $M_G(v)$ denote the product of the degrees of all vertices adjacent to a vertex v. We refer to [3] for undefined term and notation.

In [4], Kulli introduced the first and second *KV* indices, defined as

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$$KV_1(G) = \sum_{uv \in E(G)} [M_G(u) + M_G(v)], \qquad KV_2(G) = \sum_{uv \in E(G)} M_G(u) M_G(v).$$

We introduce the modified first and second *KV* indices of a graph, defined as

$$^{m}KV_1(G) = \sum_{uv \in E(G)} \frac{1}{M_G(u) + M_G(v)}, \tag{1}$$

$$^{m}KV_2(G) = \sum_{uv \in E(G)} \frac{1}{M_G(u) M_G(v)}. \tag{2}$$

In [5], Furtula and Gutman proposed the *F*-index of a graph *G*, defined as

$$F(G) = \sum_{u \in V(G)} d(u)^3 = \sum_{uv \in E(G)} [d(u)^2 + d(v)^2].$$

The *F*-index was studied, for example, in [6, 7, 8, 9, 10, 11].

We introduce the F_1-*KV* index of a graph *G*, defined as

$$F_1KV(G) = \sum_{uv \in E(G)} [M_G(u)^2 + M_G(v)^2]. \tag{3}$$

We define the F_1-*KV* polynomial of a graph G as

$$F_1KV(G, x) = \sum_{uv \in E(G)} x^{[M_G(u)^2 + M_G(v)^2]}. \tag{4}$$

We define the harmonic *KV* index of a graph *G* as

$$HKV(G) = \sum_{uv \in E(G)} \frac{2}{M_G(u) + M_G(v)}.$$

We propose the general harmonic *KV* index of a graph *G* and it is defined as

$$HKV^a(G) = \sum_{uv \in E(G)} \left(\frac{2}{M_G(u) + M_G(v)} \right)^a. \tag{5}$$

The harmonic index was studied in [12, 13, 14].

We introduce the augmented *KV* index of a graph as follows:

The augmented *KV* index of a graph *G* is defined as

$$AKVI(G) = \sum_{uv\in E(G)} \left(\frac{M_G(u)M_g(v)}{M_G(u)+M_G(v)-2}\right)^3. \tag{6}$$

The augmented index was studied in [15, 16, 17].

Considering the augmented *KV* index, we introduce the augmented *KV* polynomial of a graph *G* as

$$AKVI(G, x) = \sum_{uv\in E(G)} x^{\left(\frac{M_G(u)M_G(v)}{M_G(u)+M_G(v)-2}\right)^3}. \tag{7}$$

We propose the hyper *F-KV* index and hyper *F-KV* polynomial of a graph as follows:

The hyper *F-KV* index of a graph *G* is defined as

$$HFKV(G) = \sum_{uv\in E(G)} [M_G(u)^2 + M_G(v)^2]^2. \tag{8}$$

The hyper *F-KV* polynomial of a graph *G* is defined as

$$HFKV(G, x) = \sum_{uv\in E(G)} x^{[M_G(u)^2+M_G(v)^2]^2}. \tag{9}$$

Very recently, some new *KV* indices have been introduced and studied such as hyper *KV* and square *KV* indices [13], connectivity *KV* indices [19], multiplicative connectivity *KV* indices [20], multiplicative *KV* indices and multiplicative hyper *KV* indices [21]. In this paper, we compute the modified first and second *KV* indices, *F-KV* and hyper *F-KV* indices, general harmonic *KV* index, augmented *KV* index of POPAM and tetrathiafulvalene dendrimers. Also the *F-KV* polynomial, F_1-*KV* polynomial, augmented *KV* polynomial of POPAM and tetrathiafulvalene dendrimers are determined. For dendrimers see [22].

2. Results for POPAM Dendrimers

The family of POPAM dendrimers is symbolized by $POD_2[n]$, where *n* is the steps of growth in this type of dendrimers. The graph of $POD_2[2]$ is shown in Figure 1.

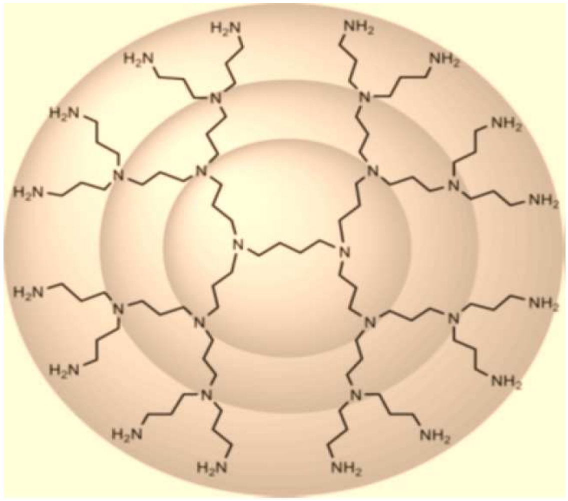

Figure 1. The graph of $POD_2[2]$.

Let G be the graph of a POPAM dendrimer $POD_2[n]$. By algebraic method, we obtain that G has $2^{n+5}-10$ vertices and $2^{n+5}-11$ edges. The edge partition of $POD_2[n]$ based on the degree product of neighbors of end vertices of each edge is given in Table 1.

Table 1. Edge partition of $POD_2[n]$.

$M_G(u), M_G(v)\backslash uv \in E(G)$	(2, 2)	(2, 4)	(4, 4)	(4, 6)	(6, 8)
Number of edges	2^{n+2}	2^{n+2}	1	$3\times 2^{n+2}-6$	$3\times 2^{n+2}-6$

Theorem 1. *The modified first and second KV indices of a POPAM dendrimer* $POD_2[n]$ *are given by*

(i) $^{m}KV_1(POD_2[n]) = \frac{391}{420}2^{n+2} - \frac{253}{280}.$

(ii) $^{m}KV_2(POD_2[n]) = \frac{9}{16}2^{n+2} - \frac{5}{16}.$

Proof. Let G be the graph of $POD_2[n]$.

(i) From equation (1) and by using Table 1, we deduce

$$ {}^{m}KV_1(POD_2[n]) = \sum_{uv \in E(G)} \frac{1}{M_G(u) + M_G(v)} $$

$$ = \left(\frac{1}{2+2}\right)2^{n+2} + \left(\frac{1}{2+4}\right)2^{n+2} + \left(\frac{1}{4+4}\right) + \left(\frac{1}{4+6}\right)(3 \times 2^{n+2} - 6) $$

$$ + \left(\frac{1}{6+8}\right)(3 \times 2^{n+2} - 6) $$

$$ = \frac{391}{420}2^{n+2} - \frac{253}{280}. $$

(ii) By using equation (2) and Table 2, we obtain

$$ {}^{m}KV_2(POD_2[n]) = \sum_{uv \in E(G)} \frac{1}{M_G(u)M_G(v)} $$

$$ = \left(\frac{1}{2 \times 2}\right)2^{n+2} + \left(\frac{1}{2 \times 4}\right)2^{n+2} + \left(\frac{1}{4 \times 4}\right) + \left(\frac{1}{4 \times 6}\right)(3 \times 2^{n+2} - 6) $$

$$ + \left(\frac{1}{6 \times 8}\right)(3 \times 2^{n+2} - 6) $$

$$ = \frac{9}{16}2^{n+2} - \frac{5}{16}. $$

Theorem 2. *The F_1-KV index and its polynomial of a POPAM dendrimer $POD_2[n]$ are given by*

(i) $F_1KV(POD_2[n]) = 484 \times 2^{n+2} - 880.$

(ii) $F_1KV(POD_2[n], x) = 2^{n+2}x^8 + 2^{n+2}x^{20} + x^{32}$

$$ + (3 \times 2^{n+2} - 6)x^{52} + (3 \times 2^{n+2} - 6)x^{100}. $$

Proof. Let G be the graph of a POPAM dendrimer $POD_2[n]$.

(i) From equation (3) and using Table 1, we derive

$$ F_1KV(POD_2[n]) = \sum_{uv \in E(G)} [M_G(u)^2 + M_G(v)^2] $$

$$= (2^2 + 2^2)2^{n+2} + (2^2 + 4^2)2^{n+2} + (4^2 + 4^2)$$

$$+ (4^2 + 6^2)(3\times 2^{n+2} - 6) + (6^2 + 8^2)(3\times 2^{n+2} - 6)$$

$$= 484\times 2^{n+2} - 880.$$

(ii) By using equation (4) and Table 1, we have

$$F_1KV(POD_2[n],\ x) = \sum_{uv\in E(G)} x^{[M_G(u)^2 + M_G(v)^2]}$$

$$= 2^{n+2}x^{2^2+2^2} + 2^{n+2}x^{2^2+4^2} + x^{4^2+4^2}$$

$$+ (3\times 2^{n+2} - 6)x^{4^2+6^2} + (3\times 2^{n+2} - 6)x^{6^2+8^2}$$

$$= 2^{n+2}x^8 + 2^{n+2}x^{20} + x^{32}$$

$$+ (3\times 2^{n+2} - 6)x^{52} + (3\times 2^{n+2} - 6)x^{100}.$$

Theorem 3. *The general harmonic KV index of* $POD_2[n]$ *is*

$$HKV^a(POD_2[n]) = \left[\left(\frac{1}{2}\right)^a + \left(\frac{1}{3}\right)^a\right]2^{n+2} + \left[\left(\frac{1}{5}\right)^a + \left(\frac{1}{7}\right)^a\right](3\times 2^{n+2} - 6) + \left(\frac{1}{4}\right)^a. \quad (10)$$

Proof. Let $G = POD_2[n]$. By using equation (5) and Table 1, we deduce

$$HKV^a(POD_2[n]) = \sum_{uv\in E(G)} \left(\frac{2}{M_G(u) + M_G(v)}\right)^a$$

$$= \left(\frac{2}{2+2}\right)^a 2^{n+2} + \left(\frac{2}{2+4}\right)^a 2^{n+2} + \left(\frac{2}{4+4}\right)^a$$

$$+ \left(\frac{2}{4+6}\right)^a (3\times 2^{n+2} - 6) + \left(\frac{2}{6+8}\right)^a (3\times 2^{n+2} - 6)$$

$$= \left[\left(\frac{1}{2}\right)^a + \left(\frac{1}{3}\right)^a\right]2^{n+2} + \left[\left(\frac{1}{5}\right)^a + \left(\frac{1}{7}\right)^a\right](3\times 2^{n+2} - 6) + \left(\frac{1}{4}\right)^a.$$

Corollary 3.1. *The harmonic KV index of* $POD_2[n]$ *is*

$$\frac{391}{210}2^{n+2} - \frac{253}{140}.$$

Proof. Put $a = 1$ in equation (10), we get the desired result.

Theorem 4. *The augmented KV index and its polynomial of a POPAM dendrimer* $POD_2[n]$ *are given by*

(i) $AKVI(POD_2[n]) = 289 \times 2^{n+2} - \frac{14230}{27}.$

(ii) $AKVI(POD_2[n], x) = 2 \times 2^{n+2}x^8 + (3 \times 2^{n+2} - 6)x^{27}$

$$+ (3 \times 2^{n+2} - 6)x^{64} + x^{\frac{512}{27}}.$$

Proof. Let $G = POD_2[n]$.

(i) From equation (6) and by using Table 1, we deduce

$$AKVI(POD_2[n]) = \sum_{uv \in E(G)} \left(\frac{M_G(u)M_G(v)}{M_G(u) + M_G(v) - 2}\right)^3$$

$$= \left(\frac{2 \times 2}{2 + 2 - 2}\right)^3 2^{n+2} + \left(\frac{2 \times 4}{2 + 4 - 2}\right)^3 2^{n+2} + \left(\frac{4 \times 4}{4 + 4 - 2}\right)^3$$

$$+ \left(\frac{4 \times 6}{4 + 6 - 2}\right)^3 (3 \times 2^{n+2} - 6) + \left(\frac{6 \times 8}{6 + 8 - 2}\right)^3 (3 \times 2^{n+2} - 6)$$

$$= 289 \times 2^{n+2} - \frac{14230}{27}.$$

(ii) By using equation (7) and Table 1, we derive

$$AKVI(POD_2[n], x) = \sum_{uv \in E(G)} x^{\left(\frac{M_G(u)M_G(v)}{M_G(u)+M_G(v)-2}\right)^3}$$

$$= 2^{n+2}x^{\left(\frac{2\times2}{2+2-2}\right)^3} + 2^{n+2}x^{\left(\frac{2\times4}{2+4-2}\right)^3} + x^{\left(\frac{4\times4}{4+4-2}\right)^3}$$

$$+(3\times 2^{n+2}-6)x^{\left(\frac{4\times 6}{4+6-2}\right)^3}+(3\times 2^{n+2}-6)x^{\left(\frac{6\times 8}{6+8-2}\right)^3}$$

$$=2\times 2^{n+2}x^{8}+(3\times 2^{n+2}-6)x^{27}+(3\times 2^{n+2}-6)x^{64}+x^{\frac{512}{27}}.$$

Theorem 5. *The hyper F-KV index and its polynomial of a POPAM dendrimer* $POD_2[n]$ *are given by*

(i) $HFKV(POD_2[n])=38576\times 2^{n+2}-75200.$

(ii) $HFKV(POD_2[n],\ x)=2^{n+2}x^{64}+2^{n+2}x^{400}+x^{1024}$

$$+(3\times 2^{n+2}-6)x^{8112}+(3\times 2^{n+2}-6)x^{30000}.$$

Proof. Let $G=POD_2[n]$.

(i) From equation (8) and using Table 1, we deduce

$$HFKV(POD_2[n])=\sum_{uv\in E(G)}[M_G(u)^2+M_G(v)^2]^2$$

$$=(2^2+2^2)^2 2^{n+2}+(2^2+4^2)^2 2^{n+2}+(4^2+4^2)^2$$

$$+(4^2+6^2)^2(3\times 2^{n+2}-6)+(6^2+8^2)^2(3\times 2^{n+2}-6)$$

$$=38576\times 2^{n+2}-75200.$$

(ii) From equation (9) and Table 1, we obtain

$$HFKV(POD_2[n],\ x)=\sum_{uv\in E(G)}x^{[M_G(u)^2+M_G(v)^2]^2}$$

$$=2^{n+2}x^{(2^2+2^2)^2}+2^{n+2}x^{(2^2+4^2)^2}+x^{(4^2+4^2)^2}$$

$$+(3\times 2^{n+2}-6)x^{(4^2+6^2)^2}+(3\times 2^{n+2}-6)x^{(6^2+8^2)^2}$$

$$=2^{n+2}x^{64}+2^{n+2}x^{400}+x^{1024}+(3\times 2^{n+2}-6)x^{8112}$$

$$+(3\times 2^{n+2}-6)x^{30000}.$$

3. Results for Tetrathiafulvalene Dendrimers

We consider the family of tetrathiafulvalene dendrimers. This family of dendrimers is symbolized by $TD_2[n]$, where n is the steps of growth in this type of dendrimers. The graph of $TD_2[2]$ is presented in Figure 2.

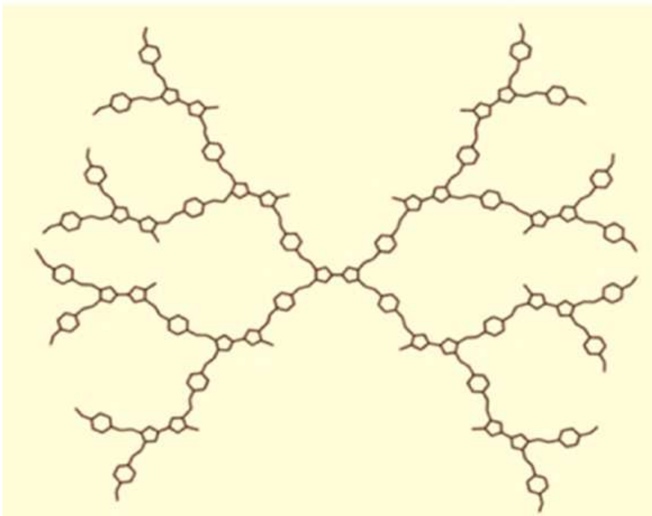

Figure 2. The graph of $TD_2[2]$.

Let G be the graph of a tetrathiafulvalene dendrimer $TD_2[n]$. By calculation, we obtain that G has $31 \times 2^{n+2} - 24$ vertices and $32 \times 2^{n+2} - 85$ edges. The edge partition of G based on the degree product of neighbors of end vertices of each edge is given in Table 2.

Table 2. Edge partition of $TD_2[n]$.

$M_G(u), M_G(v) \backslash uv \in E(G)$	Number of edges
(2, 3)	2^{n+2}
(3, 6)	$2^{n+2} - 4$
(3, 8)	2^{n+2}
(6, 6)	$7 \times 2^{n+2} - 16$

(6, 8)	$11 \times 2^{n+2} - 24$
(6, 9)	$2^{n+2} - 4$
(6, 12)	$3 \times 2^{n+2} - 8$
(9, 12)	$8 \times 2^{n+2} - 24$
(12, 12)	$2 \times 2^{n+2} - 5$

Theorem 6. *The modified first and second KV indices of $TD_2[n]$ are given by*

(i) $^{m}KV_1(TD_2[n]) = \frac{562}{165}2^{n+2} - \frac{13997}{2520}$.

(ii) $^{m}KV_2(TD_2[n]) = \frac{53}{72}2^{n+2} - \frac{85}{54}$.

Proof. Let $G = TD_2[n]$.

(i) By using equation (1) and Table 2, we obtain

$$^{m}KV_1(TD_2[n]) = \sum_{uv \in E(G)} \frac{1}{M_G(u) + M_G(v)}$$

$$= \left(\frac{1}{2+3}\right)2^{n+2} + \left(\frac{1}{3+6}\right)(2^{n+2} - 4) + \left(\frac{1}{3+8}\right)2^{n+2}$$

$$+ \left(\frac{1}{6+6}\right)(7 \times 2^{n+2} - 16) + \left(\frac{1}{6+8}\right)(11 \times 2^{n+2} - 24)$$

$$+ \left(\frac{1}{6+9}\right)(2^{n+2} - 4) + \left(\frac{1}{6+12}\right)(3 \times 2^{n+2} - 8)$$

$$+ \left(\frac{1}{9+12}\right)(8 \times 2^{n+2} - 24) + \left(\frac{1}{12+12}\right)(2 \times 2^{n+2} - 5)$$

$$= \frac{562}{162}2^{n+2} - \frac{13997}{2520}.$$

(ii) By using equation (2) and Table 2, we deduce

$$^{m}KV_2(TD_2[n]) = \sum_{uv \in E(G)} \frac{1}{M_G(u)M_G(v)}$$

$$= \left(\frac{1}{2\times 3}\right)2^{n+2} + \left(\frac{1}{3\times 6}\right)(2^{n+2} - 4) + \left(\frac{1}{3\times 8}\right)2^{n+2}$$

$$+ \left(\frac{1}{6\times 6}\right)(7\times 2^{n+2} - 16) + \left(\frac{1}{6\times 8}\right)(11\times 2^{n+2} - 24)$$

$$+ \left(\frac{1}{6\times 9}\right)(2^{n+2} - 4) + \left(\frac{1}{6\times 12}\right)(3\times 2^{n+2} - 8)$$

$$+ \left(\frac{1}{9\times 12}\right)(8\times 2^{n+2} - 24) + \left(\frac{1}{12\times 12}\right)(2\times 2^{n+2} - 5)$$

$$= \frac{53}{72}2^{n+2} - \frac{85}{54}.$$

Theorem 7. *The F_1-KV index and its polynomial of $TD_2[n]$ are given by*

(i) $F_1KV(TD_2[n]) = 4768\times 2^{n+2} - 12480.$

(ii) $F_1KV(TD_2[n], x) = 2^{n+2}x^{13} + (2^{n+2} - 4)x^{45} + 2^{n+2}x^{73}$

$$+ (7\times 2^{n+2} - 16)x^{72} + (11\times 2^{n+2} - 24)x^{225}$$

$$+ (2^{n+2} - 4)x^{117} + (3\times 2^{n+2} - 8)x^{180}$$

$$+ (8\times 2^{n+2} - 24)x^{225} + (2\times 2^{n+2} - 5)x^{288}.$$

Proof. Let $G = TD_2[n]$.

(i) From equation (3) and using Table 2, we derive

$$F_1KV(TD_2[n]) = \sum_{uv \in E(G)} [M_G(u)^2 + M_G(v)^2]$$

$$= (2^2 + 3^2)2^{n+2} + (3^2 + 6^2)(2^{n+2} - 4) + (3^2 + 8^2)2^{n+2}$$

$$+ (6^2 + 6^2)(7\times 2^{n+2} - 16) + (6^2 + 8^2)(11\times 2^{n+2} - 24)$$

$$+(6^2+9^2)(2^{n+2}-4)+(6^2+12^2)(3\times 2^{n+2}-8)$$

$$+(9^2+12^2)(8\times 2^{n+2}-24)+(12^2+12^2)(2\times 2^{n+2}-5)$$

$$=4768\times 2^{n+2}-12480.$$

(ii) From equation (4) and using Table 2, we deduce

$$F_1KV(TD_2[n],\ x)=\sum_{uv\in E(G)} x^{[M_G(u)^2+M_G(v)^2]}$$

$$=2^{n+2}x^{(2^2+3^2)}+(2^{n+2}-4)x^{(3^2+6^2)}+2^{n+2}x^{(3^2+8^2)}$$

$$+(7\times 2^{n+2}-16)x^{(4^2+6^2)}+(11\times 2^{n+2}-24)x^{(6^2+8^2)}$$

$$+(2^{n+2}-4)x^{(6^2+9^2)}+(3\times 2^{n+2}-8)x^{(6^2+12^2)}$$

$$+(8\times 2^{n+2}-24)x^{(9^2+12^2)}+(2\times 2^{n+2}-5)x^{(12^2+12^2)}$$

$$=2^{n+2}x^{13}+(2^{n+2}-4)x^{45}+2^{n+2}x^{73}+(7\times 2^{n+2}-16)x^{72}$$

$$+(11\times 2^{n+2}-24)x^{225}+(2^{n+2}-4)x^{117}+(3\times 2^{n+2}-8)x^{180}$$

$$+(8\times 2^{n+2}-24)x^{225}+(2\times 2^{n+2}-5)x^{288}.$$

Theorem 8. *The general harmonic KV index of $TD_2[n]$ is given by*

$$HKV^a(TD_2[n])=\left[\left(\frac{2}{5}\right)^a+\left(\frac{2}{9}\right)^a+\left(\frac{2}{11}\right)^a+7\left(\frac{1}{6}\right)^a+11\left(\frac{1}{7}\right)^a+\left(\frac{2}{15}\right)^a\right.$$

$$\left.+3\left(\frac{1}{9}\right)^a+8\left(\frac{2}{11}\right)^a+2\left(\frac{1}{12}\right)^a\right]2^{n+2}$$

$$-\left[4\left(\frac{2}{9}\right)^a+16\left(\frac{1}{6}\right)^a+24\left(\frac{1}{7}\right)^a+4\left(\frac{2}{15}\right)^a+8\left(\frac{1}{9}\right)^a\right.$$

$$\left.+24\left(\frac{2}{11}\right)^a+5\left(\frac{1}{12}\right)^a\right]. \tag{11}$$

Proof. Let $G = TD_2[n]$. From equation (5) and Table 2, we derive

$$HKV^a(TD_2[n]) = \sum_{uv \in E(G)} \left(\frac{2}{M_G(u) + M_G(v)}\right)^a$$

$$= \left(\frac{2}{2+3}\right)^a 2^{n+2} + \left(\frac{2}{3+6}\right)^a (2^{n+2} - 4) + \left(\frac{2}{3+8}\right)^a 2^{n+2}$$

$$+ \left(\frac{2}{6+6}\right)^a (7 \times 2^{n+2} - 16) + \left(\frac{2}{6+8}\right)^a (11 \times 2^{n+2} - 24)$$

$$+ \left(\frac{2}{6+9}\right)^a (2^{n+2} - 4) + \left(\frac{2}{6+12}\right)^a (3 \times 2^{n+2} - 8)$$

$$+ \left(\frac{2}{9+12}\right)^a (8 \times 2^{n+2} - 24) + \left(\frac{2}{12+12}\right)^a (2 \times 2^{n+2} - 5)$$

$$= \left[\left(\frac{2}{5}\right)^a + \left(\frac{2}{9}\right)^a + \left(\frac{2}{11}\right)^a + 7\left(\frac{1}{6}\right)^a + 11\left(\frac{1}{7}\right)^a\right.$$

$$\left. + \left(\frac{2}{15}\right)^a + 3\left(\frac{1}{9}\right)^a + 8\left(\frac{2}{11}\right)^a + 2\left(\frac{1}{12}\right)^a\right] 2^{n+2}$$

$$- \left[4\left(\frac{2}{9}\right)^a + 16\left(\frac{1}{6}\right)^a + 24\left(\frac{1}{7}\right)^a + 4\left(\frac{2}{15}\right)^a + 8\left(\frac{1}{9}\right)^a\right.$$

$$\left. + 24\left(\frac{2}{11}\right)^a + 5\left(\frac{1}{12}\right)^a\right].$$

Corollary 8.1. *The harmonic KV index of $TD_2[n]$ is*

$$HKV(TD_2[n]) = \left(\frac{11}{7} + \frac{5}{9} + \frac{18}{11} + \frac{28}{15}\right) 2^{n+2} + \left(\frac{25}{7} + \frac{40}{9} + \frac{48}{11} + \frac{5}{12} + \frac{8}{15}\right).$$

Proof. Put $a = 1$ in equation (11), we obtain the desired result.

Theorem 9. *The augmented KV index and its polynomial of $TD_2[n]$ are given by*

(i) $AKVI(TD_2[n]) = \left[\left(\frac{18}{7}\right)^3 + \left(\frac{8}{3}\right)^3 + 7\left(\frac{18}{5}\right)^3 + \left(\frac{54}{13}\right)^3 + 8\left(\frac{108}{19}\right)^3\right.$

$$+ 2\left(\frac{72}{11}\right)^3 + 2899\Bigg]2^{n+2} - \left[4\left(\frac{18}{7}\right)^3 + 16\left(\frac{18}{5}\right)^3 + 4\left(\frac{54}{13}\right)^3\right.$$

$$\left. + 24\left(\frac{108}{19}\right)^3 + 5\left(\frac{72}{11}\right)^3 + 7368\right].$$

(ii) $AKVI(TD_2[n], x) = 2^{n+2}x^8 + (2^{n+2} - 4)x^{\left(\frac{18}{7}\right)^3} + 2^{n+2}x^{\left(\frac{8}{3}\right)^3}$

$$+ (7 \times 2^{n+2} - 16)x^{\left(\frac{18}{5}\right)^3} + (11 \times 2^{n+2} - 24)x^{64}$$

$$+ (2^{n+2} - 4)x^{\left(\frac{54}{13}\right)^3} + (3 \times 2^{n+2} - 8)x^{\left(\frac{9}{2}\right)^3}$$

$$+ (8 \times 2^{n+2} - 24)x^{\left(\frac{108}{19}\right)^3} + (2 \times 2^{n+2} - 5)x^{\left(\frac{72}{11}\right)^3}.$$

Proof. Let G be the graph of $TD_2[n]$.

(i) By using equation (6) and Table 2, we obtain

$$AKVI(TD_2[n]) = \sum_{uv \in E(G)} \left(\frac{M_G(u)M_G(v)}{M_G(u) + M_G(v) - 2}\right)^3$$

$$= \left(\frac{2 \times 3}{2 + 3 - 2}\right)^3 2^{n+2} + \left(\frac{3 \times 6}{3 + 6 - 2}\right)^3 (2^{n+2} - 4) + \left(\frac{3 \times 8}{3 + 8 - 2}\right)^3 2^{n+2}$$

$$+ \left(\frac{6 \times 6}{6 + 6 - 2}\right)^3 (7 \times 2^{n+2} - 16) + \left(\frac{6 \times 8}{6 + 8 - 2}\right)^3 (11 \times 2^{n+2} - 24)$$

$$+ \left(\frac{6 \times 9}{6 + 9 - 2}\right)^3 (2^{n+2} - 4) + \left(\frac{6 \times 12}{6 + 12 - 2}\right)^3 (3 \times 2^{n+2} - 8)$$

$$+ \left(\frac{9 \times 12}{9 + 12 - 2}\right)^3 (8 \times 2^{n+2} - 24) + \left(\frac{12 \times 12}{12 + 12 - 2}\right)^3 (2 \times 2^{n+2} - 5)$$

$$= \left[\left(\frac{18}{7}\right)^3 + \left(\frac{8}{3}\right)^3 + 7\left(\frac{18}{5}\right)^3 + \left(\frac{54}{13}\right)^3 + 8\left(\frac{108}{19}\right)^3 + 2\left(\frac{72}{11}\right)^3 + 2899\right]2^{n+2}$$

$$-\left[4\left(\frac{18}{7}\right)^3+16\left(\frac{18}{5}\right)^3+4\left(\frac{54}{13}\right)^3+24\left(\frac{108}{19}\right)^3+5\left(\frac{72}{11}\right)^3+7368\right].$$

(ii) From equation (7) and by using Table 2, we deduce

$$AKVI(TD_2[n],\ x)=\sum_{uv\in E(G)} x^{\left(\frac{M_G(u)M_G(v)}{M_G(u)+M_G(v)-2}\right)^3}$$

$$= 2^{n+2}x^{\left(\frac{2\times 3}{2+3-2}\right)^3}+(2^{n+2}-4)x^{\left(\frac{3\times 6}{3+6-2}\right)^3}+2^{n+2}x^{\left(\frac{3\times 8}{3+8-2}\right)^3}$$

$$+(7\times 2^{n+2}-16)x^{\left(\frac{6\times 6}{6+6-2}\right)^3}+(11\times 2^{n+2}-24)x^{\left(\frac{6\times 8}{6+8-2}\right)^3}$$

$$+(2^{n+2}-4)x^{\left(\frac{6\times 9}{6+9-2}\right)^3}+(3\times 2^{n+2}-8)x^{\left(\frac{6\times 12}{6+12-2}\right)^3}$$

$$+(8\times 2^{n+2}-24)x^{\left(\frac{9\times 12}{9+12-2}\right)^3}+(2\times 2^{n+2}-5)x^{\left(\frac{12\times 12}{12+12-2}\right)^3}.$$

$$= 2^{n+2}x^{8}+(2^{n+2}-4)x^{\left(\frac{18}{7}\right)^3}+2^{n+2}x^{\left(\frac{8}{3}\right)^3}+(7\times 2^{n+2}-16)x^{\left(\frac{18}{5}\right)^3}$$

$$+(11\times 2^{n+2}-24)x^{64}+(2^{n+2}-4)x^{\left(\frac{54}{13}\right)^3}+(3\times 2^{n+2}-8)x^{\left(\frac{9}{2}\right)^3}$$

$$+(8\times 2^{n+2}-24)x^{\left(\frac{108}{19}\right)^3}+(2\times 2^{n+2}-5)x^{\left(\frac{72}{11}\right)^3}.$$

Theorem 10. *The hyper F-KV index and its polynomial of $TD_2[n]$ are given by*

(i) $HFKV(TD_2[n]) = 835588\times 2^{n+2}-2274720.$

(ii) $$HFKV(TD_2[n],\ x)=2^{n+2}x^{169}+(2^{n+2}-4)x^{2025}+2^{n+2}x^{5329}$$

$$+(7\times 2^{n+2}-16)x^{5184}+(11\times 2^{n+2}-24)x^{10000}$$

$$+(2^{n+2}-4)x^{13689}+(3\times 2^{n+2}-8)x^{32400}$$

$$+(8\times 2^{n+2}-24)x^{50625}+(2\times 2^{n+2}-5)x^{82944}.$$

Proof. Let $G = TD_2[n]$.

(i) By using equation (8) and Table 2, we deduce

$$HFKV(TD_2[n]) = \sum_{uv \in E(G)} [M_G(u)^2 + M_G(v)^2]^2$$

$$= (2^2 + 3^2)^2 2^{n+2} + (3^2 + 6^2)^2 (2^{n+2} - 4) + (3^2 + 8^2)^2 2^{n+2}$$
$$+ (6^2 + 6^2)^2 (7 \times 2^{n+2} - 16) + (6^2 + 8^2)^2 (11 \times 2^{n+2} - 24)$$
$$+ (6^2 + 9^2)^2 (2^{n+2} - 4) + (6^2 + 12^2)^2 (3 \times 2^{n+2} - 8)$$
$$+ (9^2 + 12^2)^2 (8 \times 2^{n+2} - 24) + (12^2 + 12^2)^2 (2 \times 2^{n+2} - 5)$$

$$= 835588 \times 2^{n+2} - 2274720.$$

(ii) From equation (9), we have

$$HFKV(TD_2[n], x) = \sum_{uv \in E(G)} x^{[M_G(u)^2 + M_G(v)^2]^2}.$$

Then by using Table 2, we obtain

$$HFKV(TD_2[n], x) = 2^{n+2} x^{(2^2+3^2)^2} + (2^{n+2} - 4) x^{(3^2+6^2)^2}$$
$$+ 2^{n+2} x^{(3^2+8^2)^2} + (7 \times 2^{n+2} - 16) x^{(6^2+6^2)^2}$$
$$+ (11 \times 2^{n+2} - 24) x^{(6^2+8^2)^2} + (2^{n+2} - 4) x^{(6^2+9^2)^2}$$
$$+ (3 \times 2^{n+2} - 8) x^{(6^2+12^2)^2} + (8 \times 2^{n+2} - 24) x^{(9^2+12^2)^2}$$
$$+ (2 \times 2^{n+2} - 5) x^{(12^2+12^2)^2}$$

$$= 2^{n+2} x^{169} + (2^{n+2} - 4) x^{2025} + 2^{n+2} x^{5329}$$
$$+ (7 \times 2^{n+2} - 16) x^{5184} + (11 \times 2^{n+2} - 24) x^{10000}$$
$$+ (2^{n+2} - 4) x^{13689} + (3 \times 2^{n+2} - 8) x^{32400}$$
$$+ (8 \times 2^{n+2} - 24) x^{50625} + (2 \times 2^{n+2} - 5) x^{82944}.$$

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