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Diversification of Stationary Data for Optimizing Risk of Portfolio

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Abstract

Diversification is a process of distributing capital that minimizes the exposure of each individual asset. The purpose of diversification is to minimize risk. This paper examines how these two strategies (stationary and non-stationary) can minimize portfolio risk. To achieve this, data on palm oil and copper from 2010 to 2016 are explored, and two methods are used: correlation and mean-variance. Diversification has different degrees, and correlation is used to check the degree of diversification of the two types of data, while the mean-variance method (MV) is used to estimate the risk of the two datasets after the correlation. The analyses in this paper show that the correlation of stationary data leads to moderately weak diversification, while non-stationary data results in very weak diversification. In estimating the risk of the two datasets, stationary data diversifies 93% of the risk, while non-stationary data diversifies 6%. This shows that stationary data minimizes portfolio risk more effectively than non-stationary data.

1. Introduction

Portfolio management is an art of policy-making using all existing data, in order to express a most likely situation for the future though harmonizing risk against enactment. In this article, Markowitz proposed that investors are risk bearer and that there is a trade-

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off among risk and return. Markowitz's context has further been developed by scholars and amongst the most influential contributions is the work of [1].

MV portfolio optimization practices a complete set of expected returns. Diversification is an advance by which a business moves its major corporate into other market result. Research exposes that corporate management actively employed diversification actions to minimize risk of their corporation. The advantage of investing in huge number of securities was obviously established in a more current study in [2]. Diversification is a significant modern investment theory that has been established.

Full diversification was introduced by [3] along with the concept of a Diversification Ratio (DR). Researchers such as [4] and [5] established some basic concepts of modern portfolio theory, called the efficient frontier and the capital market line as [6] established that modern portfolio theory provides a difficult understanding of what diversification is and how it works to improve investment opportunities. Modern Portfolio Theory (MPT) is an investment theory that acts to minimize risk of the portfolio. This [7] was the first to ascertain the MPT. However, portfolio in finance is a group of company's shares and other investment that owned by a particular person or organization, it is quite certain that investors holding diversified portfolio is the best generally accepted investment idea and the best practiced knowledge.

Moreover, [8] established that asset is diversifiable if it is imperfectly correlated with market and a hedge if it is negatively correlated with the other assets. It would be a safe haven if it is negatively correlated with the market as [9]. Assets are diversifiable, if it is negatively correlated with other asset classes in [9]. This paper examine show these two strategies (stationary and non-stationary data) can minimize risk of portfolio, using assets; palm oil and copper. The sample data is got from monthly data of Yahoo Finance, from 2010 to 2016.

This discovery gives birth to the theory of diversification as discussed in [10] and [11]. This paper is structured as follows: Section 2 describes the methodology, Section 3 discusses the results and Section 4 concludes the paper.

2. Materials and Methods

To understand the economics issues related with unit root and stationary, let us revise the conventional trend-cycle disintegration of a time series y_t using Augmented Dickey-Fuller (ADF) method.

$$y_t = \tau_t + \psi_t \tag{2.1}$$

$$\tau_t = k + \delta_t \tag{2.2}$$

$$\psi_t = \phi\tau_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2), \tag{2.3}$$

where y_t is dependent variable, τ_t is deterministic linear trend and ψ_t is an AR(1) process. If $y_t|\phi| < 1$, then y_t is 1(0) about the deterministic trend τ_t. If $\phi = 1$, then $\psi_t = \psi_{t-1} + \varepsilon_t$ is a stochastic trend.

2.1. Correlation method

Correlation, $\rho_{12,t}$ between two random variables γ_1 and γ_2 where each has mean, zero and is defined as:

$$\rho_{12,t} = \frac{E_{t-1}(\gamma_{1,t}, \gamma_{2,t})}{\sqrt{E_{t-1}(\gamma_{1,t}^2), E_{t-1}(\gamma_{2,t}^2)}}, \tag{2.4}$$

where E_{t-1} is the estimator of random variables. The conditional correlation relies entirely on prior period information with $-1 \leq \rho_{12,t} \leq 1$. To abridge the connectionin between conditional correlations and conditional variances, the returns are given as the conditional standard deviation times the standardized distribution:

$$h_{i,t} E_{t-1}(\gamma_{i,t}^2) \tag{2.5}$$

$$\gamma_{i,t} = \sqrt{h_{i,t}\varepsilon_{i,t}} \qquad i = 1,2, \ldots$$

ε is a standardized disturbance that has mean, zero and variance, one for each series. Now, substituting in equation (2.4) becomes

$$\rho_{12,t} = \frac{E_{t-1}(\varepsilon_{1,t}\varepsilon_{2,t})}{\sqrt{E_{t-1}(\varepsilon_{1,t}^2)E_{t-1}(\varepsilon_{2,t}^2)}} = E_{t-1}(\varepsilon_{1,t}\varepsilon_{2,t}). \tag{2.6}$$

The correlation is the equal to covariance between the standardized disturbances. The correlation estimator is defined for returns with a mean of zero as:

$$\rho_{12,t} = \frac{\sum_{s=t-n-1}^{t-1} \gamma_{1,s}\gamma_{2,s}}{\sqrt{(\sum_{s=t-n-1}^{t-1} \gamma_{1,s}^2)(\sum_{s=t-n-1} \gamma_{2,s}^2)}}. \tag{2.7}$$

The equal weight is for all observations less than *n* periods in the past and zero weight on older observations. The estimator lies in the interval $[-1, 1]$. The exponential smoother, is risk metrics that make use of weights depending on parameterλ.

$$\rho_{12,t} = \frac{\sum_{s=t-n-1}^{t-1} \lambda^{t-j-1} \gamma_{1,s}\gamma_{2,s}}{\sqrt{(\sum_{s=1}^{t-1} \lambda^{t-s-1}\gamma_{1,s}^2)(\sum_{s=t-n-1}^{t-1} \lambda^{t-s-1}\gamma_{2,s}^2)}}. \tag{2.8}$$

Multivariate context, to have positive correlation matrix, the same λ might be used for all assets. Covariance matrix returns is defined as:

$$E_{t-1}(\gamma_t\gamma_t^{'}) = H_t. \tag{2.9}$$

This estimator can be expressed in matrix form as:

$$H_t = \frac{1}{n}\sum_{j=1}^{n}(\gamma_{t-j}\gamma_{t-j}^{'}) \text{ and } H_t = \lambda(\gamma_{t-1}\gamma_{t-1}^{'}) + (1-\gamma)H_{t-1}. \tag{2.10}$$

To solve A, such that

$$A_r = y_t$$

$$E(y_t y_t^{'}) \equiv V.$$

Assuming that

$$E_{t-1}(y_t y_t^{'}) = V_t$$

$$H_t = A^{-1}V_t A^{-1}. \tag{2.11}$$

The matrix A can be assumed to be triangular and estimated by least squares where γ_1 is one element and residuals from a regression of γ_1 and γ_2 are the second. It is easy to run the regression as GARCH regressions in order to acquire the residual that are orthogonal in a generalized least square (GLS). [10] called the term *vec* model. The vec model parameterizes the vector of all covariances and variances identified as $vec(H_t)$. In the first order, it is given as:

$$vec(H_t) = vec(\Omega) + Avec(\lambda_{t-1}\lambda_1^{'}) + Bvec(H_{t-1}), \tag{2.12}$$

where α and β are $n^2 \times n^2$ matrices with structure following from the symmetry of H. The limits are obtained from the *BEKK* representation and can be stated as:

$$H_t = \Omega + A(\lambda_{t-1}\lambda_{t-1}^{'})A^{'} + BH_{t-1}B^{'}. \tag{2.13}$$

In general, *vec* model of Equation (2.11) can be expressed as:

$$vec(\Omega) = (1 - A - B)vec(s), \text{ where } S = \frac{1}{T}\sum(\gamma_t\gamma_t^{'}). \tag{2.14}$$

Equation (2.14) simplifies the scalar and diagonal *BEKK* cases. For the scalar *BEKK*, the intercept is:

$$\Omega = (1 - \alpha - \beta)S. \tag{2.15}$$

Looking at constant correlation estimator as:

$$H_t = D_t R_t D_t, \text{ where } D_t = diag\{h_{i,t}\}, \tag{2.16}$$

where R is a correlation matrix comprising the conditional correlations

$$E_{t-1}(\varepsilon_t \varepsilon_t^{'}) = D_t^{-1} H_t D_t^{-1} = R \text{ since } \varepsilon_t = D_t^{-1} \lambda_t. \tag{2.17}$$

The expressions of h are established on univariate GARCH models. Estimate R is the unconditional correlation matrix of the standardized residuals. The dynamic correlation model changes only in allowing R to be time varying:

$$H_t = D_t R_t D_t. \tag{2.18}$$

The matrix H_t is the residue of correlation matrix. The measurement for the correlation matrix is the exponential smoother which can be expressed as:

$$\rho_{i,j,t} = \frac{\sum_{s=1}^{t-1} \lambda^s \varepsilon_{i,t-s} \varepsilon_{j,t-s}}{\sqrt{\left(\sum_{s=1}^{t-1} \lambda^s \varepsilon_{i,t-s}^2\right)\left(\sum_{s=1}^{t-1} \lambda^s \varepsilon_{j,t-s}^2\right)}} [R_t]_{i,j}. \tag{2.19}$$

Equation (2.19) will produce a correlation matrix at each point. This method is used to produce correlation matrix of Table 3.1.

2.2. Mean Variance Model

Two assets model

The two assets (palm oil and copper) are used for the analysis of this paper. Therefore, we adopt two assets model.

$$R_p = w_1 r_1 + w_2 r_2 \tag{2.20}$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \gamma_{1,2} \sigma_1 \sigma_2 \tag{2.21}$$

$$\sigma_p = \sqrt{\sigma_p^2} \tag{2.22}$$

where R_p is return of portfolios, r_1 and r_2 are returns of palm oil and copper respectively, σ_p is standard deviation of portfolio, σ_p^2 is variance of portfolio, σ_1 and σ_2 are standard deviations of both assets, w_1 and w_2 are weights of the assets and $\gamma_{1,2}$ is correlation coefficient of the assets. Estimations of standard deviation and variance represent risk in investment.

3. Results and Discussion

The data used are Palm oil and Copper. The actual data used was non-stationary but transformed to stationary at first difference using ADF in equation (2.1). Two methods were adopted for the research, correlation and mean-variance. Hence, two results were obtained from the analyses.

3.1. Correlation result

Correlation is the evaluation of the degree of two assets move in relation to one another. This is to check the degree of diversification of two assets. This degree divides diversification into three categories; correlation from 0.6 to 1.0 it is called positive correlation, which means total weak diversification, if it ranges from 0.1 to 0.5 is still positive correlation but it implies averagely weak diversification. Moreover, if correlation is 0, it is called uncorrelation, this denotes strong diversification but if it is less than 0, it is negative correlation, which infers very strong diversification. However, this section compares the correlations of stationary and non-stationary data.

Table 3.1: Correlation matrix of Stationary and Non-stationary data.

Asset	Stationary Data		Non-stationary Data	
	Palm oil	Copper	Palm oil	Copper
Palm oil	1	0.3896	1	0.8808
Copper	0.3896	1	0.8808	1

Table 3.1 can be seen that the correlation of Stationary data is 0.3896 while that of Non-stationary data is 0.8808. Correlation of 0.3896 shows diversification is averagely weak, but correlation of 0.8808 means total weak diversification. The purpose of diversification is to minimize risk of portfolio. This result implies that averagely weak diversification should be better in minimizing risk than total weak diversification. This intrigues us to estimate the risk of each group.

3.2. Mean-Variance result

This section shall present the risk of the two data; Stationary and Non-stationary data and Mean-variance method is used for the estimation.

Table 3.2: Risk of Stationary and Non-stationary data.

Stationary Data	Non-stationary Data
0.1246	1.5861

Table 3.2 showed the risk of the two groups, Stationary is 0.1246 and Non-stationary is 1.5861. This means that Stationary portfolio contains 7.39% risk while Non-stationary portfolio has 94.10% risk. Moreover, this implies that Stationary portfolio minimized 92.61% risk of the portfolio but Non-stationary minimizes 5.9% risk of the portfolio.

4. Conclusion

The purpose of diversification is to minimize risk of portfolio but before this could achieve this purpose, it has to be based on certain measures. Correlation method checks the degree of diversification of two assets. This study examines how best could these two strategies (stationary and non-stationary) minimize risk of portfolio. The analyses of the paper were conducted using two methodologies; Correlation and mean-variance methods. Diversification has different degrees and correlation is used in this paper, to check the degrees of diversification of the two strategies, while mean variance method is used to estimate the risk of the two strategies after the estimation with correlation method. It is seen in Tables 3.1 and 3.2 that diversification would be better used for the purpose of minimizing risk if Stationary strategy is used. Stationary minimized 93% risk of the portfolio while Non-stationary only minimized 6% risk of its portfolio. This infers that Stationary data is worth adopting for diversification than Non-stationary data. This shows that stationary data optimizes risk better in portfolio than non-stationary data.

Declaration

Competing Interests

These authors declare that they have no competing interests.

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References

[1] Black, F., & Litterman, R. (1991). Global asset allocation with equities, bonds, and currencies. *Fixed Income Research Handbook, 2,* 15-28.

[2] Bhadrappa, H. (2021). Millennial and mobile-savvy consumers are driving a huge shift in the retail banking industry. *Journal of Advanced Research in Operational and Marketing Management, 4*(1), 17-19.

[3] Jayeola, D., Ismail, Z., & Sufahani, S. F. (2017). Effects of diversification of assets in optimizing risk of portfolio. *Malaysian Journal of Fundamental and Applied Sciences, 13*(4), 584-587. https://doi.org/10.11113/mjfas.v0n0.567

[4] Carrieri, F., Errunza, V., & Hogan, K. (2016). Characterizing world market integration through time. *Journal of Financial and Quantitative Analysis, 42*(4), 915-940. https://doi.org/10.1017/S0022109000003446

[5] Davis, M. H., & Lleo, S. (2016). A simple procedure for combining expert opinion with statistical estimates to achieve superior portfolio performance. *The Journal of Portfolio Management, 42*(4), 49-58. https://doi.org/10.3905/jpm.2016.42.4.049

[6] Sewando, P. T. (2022). Efficacy of risk reducing diversification portfolio strategies among agro-pastoralists in semi-arid areas: A modern portfolio approach. *Journal of Agriculture and Food Research, 7*(3), 55-67. https://doi.org/10.1016/j.jafr.2021.100262

[7] Isabel, A., Maria, J. C., Luis, M., & Armajac, R. (2021). Sport betting and the Black-Litterman model: A new portfolio management perspective. *International Journal of Sport Finance, 16*(4), 52-61. https://doi.org/10.32731/ijsf/164.112021.02

[8] Jayeola, D., & Ismail, Z. (2016). Effects of correlation on diversification of precious metals and oil. *Applied Mathematical Sciences, 10*(27), 1343-1352. https://doi.org/10.12988/ams.2016.6135

[9] Markowitz, H. (1959). *Portfolio selection: Efficient diversification of investment*. New York: John Wiley and Sons, Inc.

[10] Matteo, M., Brent, N., & Jesse, S. (2020). International currencies and capital allocation. *Journal of Political Economy, 128*(6), 2019-2066. https://doi.org/10.1086/705688

[11] Vinay, K. (2018). A simplified perspective of the Markowitz portfolio theory. *International Journal of Research and Analytical Reviews, 5*(3), 193-196.