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Rural Electrification of Selected Areas in the Northern Region of Ghana Viewed as a Minimum Spanning Tree Problem

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Abstract

The minimum spanning tree problem usually finds a spanning tree with the least total weight in a connected undirected graph. Minimum spanning trees have direct applications in the design of networks, including computer networks, telecommunications networks, transportation networks, water supply networks, and electrical grids. In this study, the concept of Minimum Spanning Tree Problem has been successfully used to analyze rural electrification of selected areas in the Savelugu Municipality and Mion Districts in the Northern Region of Ghana. Secondary data was collected from Northern Electricity Distribution Company (NEDCO) in Tamale, Ghana. Networks of the selected areas in the Savelugu and Mion Districts in the Northern Region of Ghana were constructed. Kruskal's algorithm has been employed to obtain the optimal electrification routes for the selected areas and Management Scientist Version 5 software used to confirm the optimal solutions. Post-optimality analysis has also been conducted to determine how variations of the distances between villages in the considered cases studies affect the optimal lengths of electrification routes. The government of Ghana should use the determined optimal routes as a guide to minimize the total cost of cables for future electrification of the selected areas. Other countries should apply techniques like the Kruskal's algorithm to minimize the cost of cables involved in their electrification processes.

1. Introduction

Rural electrification is the process or method of taking electricity or electrical power

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to non-urban and distant places or areas. Availability of electricity expedites sustainable or viable social and economic development. First of all, there will be an increase in educational achievement. Additionally, rural electrification will ensure greater efficiency and productivity. Businesses can remain opened for longer hours and accrue extra proceeds or revenues. Modernized techniques or methods for farming such as irrigation, crop processing, and food preservation can be assessed by farmers. Expanding the electrical grid creates a lot of jobs which in turn help to alleviate or reduce poverty. The availability of electricity or access to electrical power can drastically or hugely increase the quality of healthcare delivery. The service time for patients will increase as a result of improved lighting. Refrigerators or fridges can be used to preserve blood and vaccines. Sterilization or disinfection measures or processes will be enhanced and the implementation or execution of advanced machines or equipment such as ultrasound scanners or X-rays can offer nurses and doctors with the requisite tools or apparatuses needed to operate effectively.

Hydro generation and thermal generation fueled by crude oil, natural gas, and diesel are still the main sources of Ghana's power supply. The state is still heavily involved in the energy sector, with state entities having a controlling presence in the entire value chain. In the generation phase, "the entire hydroelectricity component is controlled by the Volta River Authority (VRA) and Bui Power Authority (BPA), with VRA also involved in some aspects of thermal generation along with Independent Power Producers (IPP). State-owned Ghana Grid Company (GRIDCO) is still solely responsible for transmission throughout the entire country". The final leg of distribution is "mainly controlled by the state-owned entities Electricity Company of Ghana (ECG) and Northern Electricity Distribution Company (NEDCO)". A private entity, Enclave Power Company, plays a minor role in the distribution chain [1]. Rural communities are suffering from colossal market failures as the national grids fall short of their electricity demand. As of 2017, over 1 billion people worldwide lacked household electric power – 14% of the global population. The electricity access rate stood at 86.63 percent in 2021, with 50 percent of rural residents and 91 percent of urban residents connected to the electricity grid. 50 percent of rural residents in Ghana do not have access to electricity. Extension of electricity to the affected rural areas has to be done scientifically. The minimum spanning tree problem is to find a spanning tree with the least types of labels if each edge in a graph is associated with a label from a finite label set instead of a weight [2]. Minimum spanning trees have direct applications in the design of networks, including computer networks, telecommunications networks, transportation networks, water supply networks, and electrical grids [3]. Based on the literature reviewed so far and to the best of our knowledge, rural electrification in Ghana viewed as a minimum spanning tree problem appears non-existent. Generally, the study therefore sought to analyze rural electrification of selected areas in the Savelugu and Mion districts in the Northern Region of Ghana as a Minimum Spanning Tree Problem. Specially, networks of the selected areas for electrification were constructed and some pre-optimality electrification routes determined by inspection. The optimal electrification routes for the selected areas were also determined using Kruskal's algorithm and some post-optimality analyses performed.

2. Literature Review

The minimum spanning tree (MST) problem originated in the 1920s when Boruvka identified and solved the problem during the electrification of Moravia. This graph theory problem and its numerous applications have inspired many others to look for alternate ways of finding a spanning tree of minimum weight in a weighted, connected graph since Boruvka's time. In the 1950s, many people contributed to the MST problem. Among them were R. C. Prim and J. B. Kruskal, whose algorithms are very widely used today.

The concept of Minimum Spanning Tree Problem has attracted the attention of mathematicians and researchers in various fields of study. This field has received this great attention because of the crucial role it plays in various endeavors of life and has therefore been applied by various researchers in different ways. [4] described a linear-time algorithm for verifying a minimum spanning tree. [5] presented a deterministic algorithm for computing a minimum spanning tree of a connected graph. [6] presented a variant of Boruvka's algorithm. They discussed the proof of the algorithm, compared it to the existing algorithm and presented an implementation of the procedure in Maple. [7] presented a new algorithm based on the distance matrix to solve the least cost minimum spanning tree problem. [8] proposed a new technique and its corresponding algorithm to construct the minimum spanning tree of a weighted graph. The algorithm was based on the weight matrix of a weighted graph. [9] examined spanning trees and minimum spanning trees and aimed at finding the minimum cost spanning tree using the matrix algorithm based on the weight matrix of the weighted graph (cost). [10] introduced the generalized minimum spanning tree game and studied some properties of this game. [11] formulated the minimum spanning tree problem with resource allocation as discrete and continuous optimization problem and presented algorithms to solve these problems to optimality. [12] presented a cycle detection based greedy algorithm to obtain a minimal spanning tree of a given input weighted undirected graph. [13] dealt with the construction of minimum spanning tree of city-city network in Nigeria. They employed Prim's algorithm to determine the minimum spanning tree of the cities. [14] presented the design of electrical grid or network on a least distance path between nodes which aimed at optimizing the electric grid network in the area of Upper Karnali Hydro Project using Kruskal's algorithm. [15] presented "an optimization process that minimizes the installation cost mix of generation sources for a rural mini-grid using a multi-objective particle swarm optimization (MOPSO) technique". [16] minimized the cost of a tree using three different methods named as column minima and row minima in matrix and quick sort approach among network nodes in computer science. [17] proposed a method to choose a less cost cable configuration providing a map of households and their maximum loads. The Prim's algorithm was used to find the minimum spanning tree of all nodes and distances between each pair of households as weights. [18] discussed "the mathematical properties of spanning tree and Kirchhoff's matrix tree theorem to find the total number of possible spanning trees". They also looked at the properties of minimum spanning tree, Kruskal's method and Prim's method to find minimum spanning tree. [19] examined minimum-weight spanning tree problem and compared the efficiency of modern MST algorithm like least cost MST with the classical algorithms such as Prim's, Kruskal's and Borukva's algorithm. [20] dealt with "minimum cost spanning forest/tree using different efficient algorithms based on graph isomorphism". They determined whether two graphs are isomorphic using Prim's and Kruskal's algorithm. [21] presented "a fast and efficient algorithm for the construction of minimum cost spanning trees based on the concept of travelling salesman problem (TSP) in a connected weighted graph in linear time". [22] studied the functioning of the internet of a company and found ways to reduce cost. They used the Kruskal's algorithm to establish whether it is a reliable method. Their finding showed that "the Kruskal's algorithm can be used to reduce cost". [23] suggested a matrix technique that uses the cost adjacency matrix to determine the minimum cost spanning tree of a given undirected connected graph. [24] proposed "a novel technique and effective method for studying the large scale of the problem and determining the minimum cost-spanning tree of a connected weighted undirected graph with fewer iterations using modified ant colony optimization algorithm (ACOA)". They compared this algorithm with the "Prim's and Kruskal's and had the same results". [25] developed a new algorithm for finding minimum spanning tree based on a deduced theorem of minimum spanning tree. The algorithm sorted edges by weight from greatest to smallest. The algorithm is equivalent to the inverse Kruskal's algorithm. Analysis concluded that, "the effectiveness of the algorithm proposed to be a better choice for finding minimum spanning tree of a sparse graph". Based on the literature reviewed so far and to the best of our knowledge, rural electrification in Ghana viewed as a minimum spanning tree problem appears non-existent. The study therefore sought to fill or bridge that knowledge gap.

3. Materials and Methods

The concept of Minimum Spanning Tree Problem under Graph Theory in Operations Research is the main theoretical framework of this study.

3.1. Some important definitions

A forest: It is a disjoint union of trees. Figure 1 is an example of a forest.

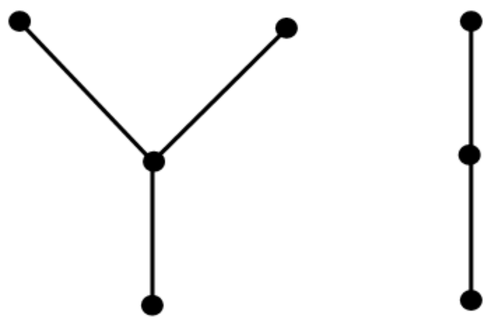


Figure 1: An example of a forest.

A tree: A tree is a connected forest (Wilson, 1996). Figure 2 is an example of a tree.

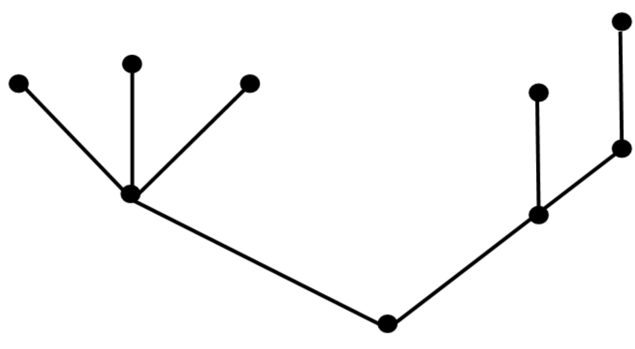
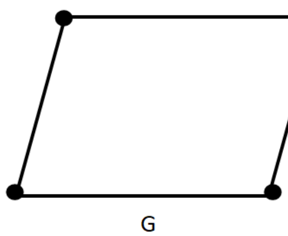


Figure 2: An example of a tree.

Spanning Tree: Let G be a connected graph with n vertices, a sub-graph of G is a spanning tree if the sub-graph has no cycles, contains all vertices of G and n-1 edges.



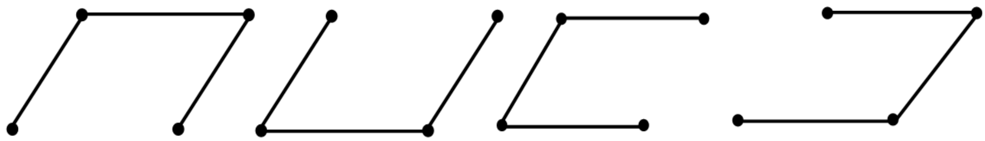


Figure 3: Spanning trees of a connected graph, G.

Weighted graph: A graph is considered weighted if "each edge of the graph is assigned a value or a weight". Figure 4 is an example.

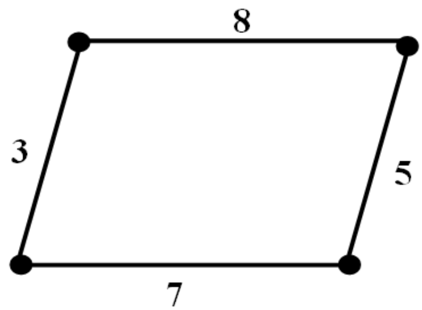


Figure 4: An example of a weighted graph.

A Minimum Spanning Tree (MST) is "a subset of the edges of a connected, undirected graph that connects all the vertices together, without any cycles and with the minimum possible total edge weight". It is a way to connect all the vertices in a graph in a way that minimizes the total weight of the edges in the tree.

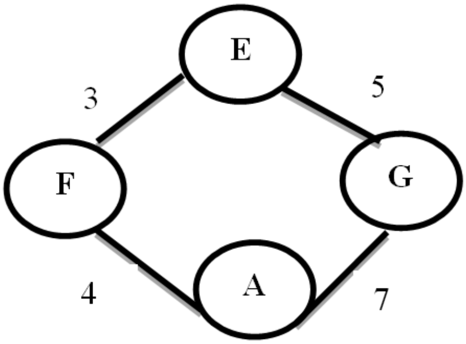
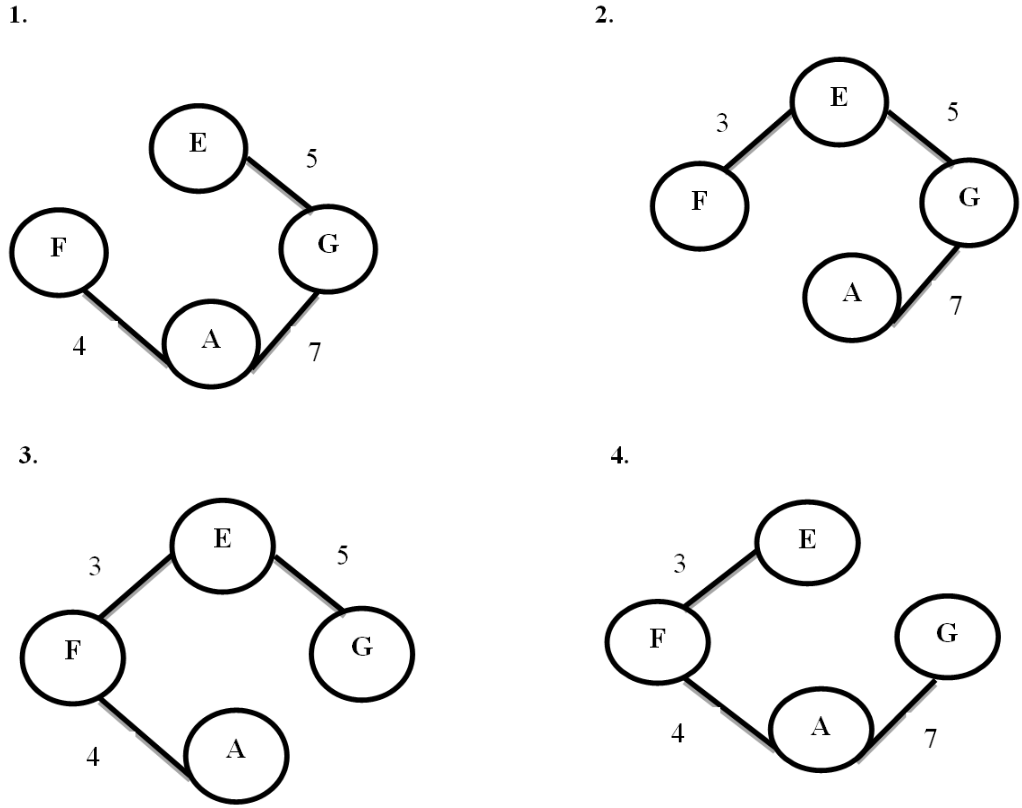


Figure 5: Connected graph for illustration of minimum spanning tree.



Spanning Trees from Figure 5

From the graph of G, the total weight of the paths in the sub-graph 1 is 16, sub-graph 2 is 15, sub-graph 3 is 12 and sub-graph 4 is 14, hence minimum spanning tree is the sub-graph 3 since it has the minimal total weight among the spanning trees. If a graph is a complete graph with n vertices $(n \geq 1)$, the total number of spanning trees is $n^{(n-2)}$, where n is the total number of vertices in the entire graph [26]. The above graph which has 4 vertices has a total of $4^{(4-2)}$= 4^2=16 spanning trees.

3.2. Formulation of minimum spanning tree (MST) problem

The general MST problem can be formulated as follows.

Given an undirected graph, $G = (V, E)$, where $\{v_1,v_2,...,v_n\}$ is a set of vertices and $\{e_1,e_2,...,e_m\}$ is a set of edges. If weight, W_e is allocated to each edge and C_e represents the cost of each edge, thus we want to find an acyclic subset that connects all of the vertices and minimizes the total weight (cost). The problem is then formulated as follows:

Minimise

$$Z = \sum_{e \in T} C_e$$

Subject to;

$$\sum_{e \in T} w_e \leq W$$

$$T \in \tau$$

τ is the set of all spanning trees in G.

We denote the cost of a tree T as $C(T) = \sum_{e \in T} C_e$ and the weight of a tree as $W(T) = \sum_{e \in T} W_e$. We call this problem weight-constrained minimal spanning tree problem [27].

3.3. Solution methods or algorithms for MST problems

A number of methods or algorithms abound for solving minimum spanning tree problems. Among them are the Boruvka's Algorithm, Kruskal's Algorithm, Prim's Algorithm, Reverse Delete Algorithm, Least Cost Algorithm etc. That notwithstanding, the Kruskal's algorithm employed in the study is discussed as follows.

- **Kruskal's Algorithm**

The algorithm was developed in 1957 by Joseph Kruskal. This algorithm uses the greedy approach for finding a minimum cost spanning tree.

The Kruskal's algorithm is as follows:

Step 1: Always select a minimum cost edge in the entire graph as the initial.

Step 2: Add edges to the spanning tree from the edges of the graph with the smallest weight until a minimum spanning tree covers all the vertices of the graph.

Step 3: Do not include an edge forming a loop or cycle.

The Kruskal's algorithm is illustrated as follows:

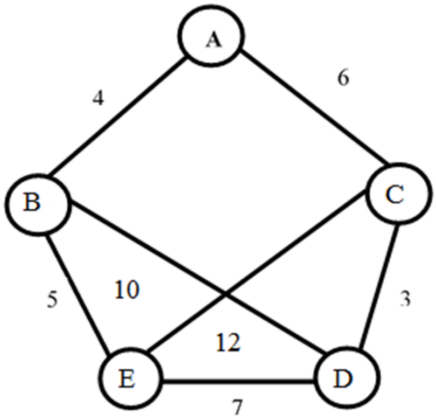
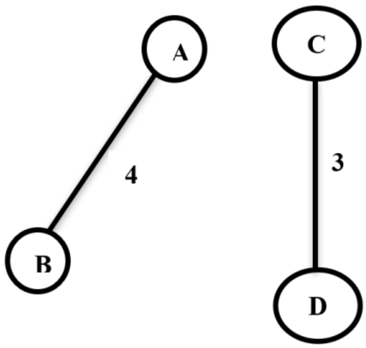


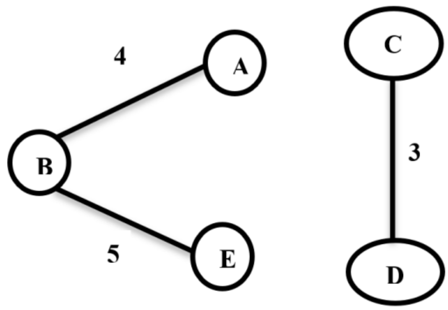
Figure 6: Connected graph for illustration of Kruskal's algorithm.

Step 1: Select the minimum cost edge in the entire graph

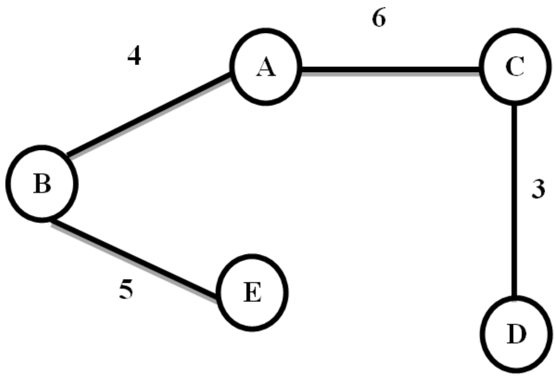
Step 2: Select the next minimum cost edge in the entire graph



Step 3: Continue to sort the minimum cost edge to make a tree



Step 4: Add the final edge since we have five (5) vertices from the graph



3.4. Data collection and analysis procedures

Secondary data was collected from Northern Electricity Distribution Company (NEDCO) in Tamale. Kruskal's was employed to obtain the optimal solutions and Management Scientist Version 5 software developed by [28] was used to confirm the solutions.

4. Results and Discussions

4.1. The MST problem in the case of Mion District

Table 1 gives the coordinates (latitudes and longitudes) of 20 villages in the Mion District in the Northern Region of Ghana which do not have electricity. This information was used to generate and construct a network of the respective villages using Google Earth Pro and to find the distance between any two villages.

Table 1: Latitudes and longitudes of villages without electricity in the Mion District.

No.	**Village**	**Latitude**	**Longitude**
1	Akinajili	9.53525672°N	-0.120887508°W
2	Balo	9.4782209°N	-0.0968813°W
3	Bogoyili	9.1984282°N	-0.1822898°W
4	Denie	9.267562981°N	-0.166432253°W
5	Djego	9.3456745°N	-0.4291087°W
6	Kwani	9.309231288°N	-0.533162257°W
7	Labaringa	9.4882015°N	-0.5820217°W
8	Palari	9.1879146°N	-0.3014437°W
9	Palguni	9.324695832°N	-0.460454205°W
10	Sampiaman	9.72038692°N	-0.195319636°W
11	Sanduli	9.5863102°N	-0.144916°W
12	Zanduli	9.5782392°N	-0.1780394°W
13	Zilimo	9.2788362°N	-0.3297182°W
14	Kaiyong	9.3309911°N	-0.3315932°W
15	Zuro	9.42°N	-0.56°W
16	Suri	9.45°N	-0.50°W
17	Sakoya	9.48°N	-0.50°W
18	Parishenaaya	9.41°N	-0.44°W
19	Kpablaa	9.35°N	-0.50°W
20	Jashee	9.40°N	-0.49°W

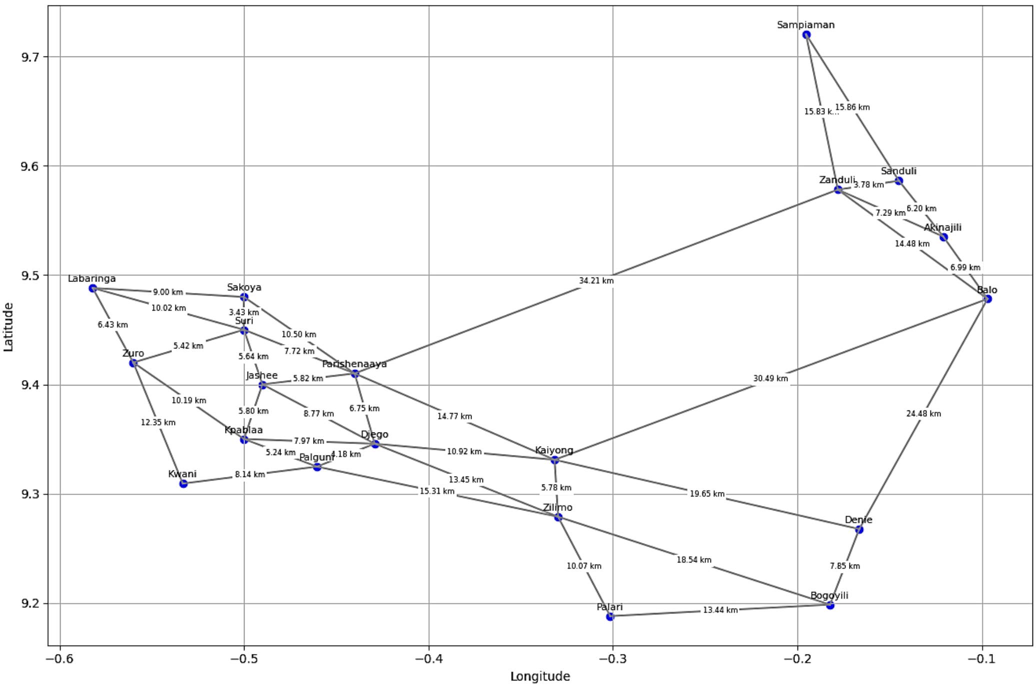


Figure 7: A network of the villages without electricity in Mion District.

Each of the 20 villages without electricity in the Mion District was assigned a specific number for computational purposes as given in Table 2.

Table 2: Villages in the Mion District and their assigned numbers.

Villages	Assigned Numbers
Sampiaman	1
Zanduli	2
Sanduli	3
Akinajili	4
Balo	5
Denie	6
Bogoyili	7
Palari	8
Zilimo	9
Kaiyong	10
Parishenaaya	11

Sakoya	12
Suri	13
Jashee	14
Djego	15
Palguni	16
Kwani	17
Zuro	18
Labaringa	19
Kpablaa	20

Table 3: Distances in kilometres between villages without electricity in the Mion District.

No.	From	To	Distance (in km)
1	Labaringa (19)	Sakoya (12)	9
2	Zuro (18)	Kwani (17)	12.35
3	Kpablaa (20)	Jashee (14)	5.80
4	Labaringa (19)	Zuro (18)	6.43
5	Labaringa (19)	Suri (13)	10.02
6	Sakoya (12)	Suri (13)	3.43
7	Suri (13)	Jashee (14)	5.64
8	Kpablaa (20)	Zuro (18)	10.19
9	Sakoya (12)	Parishenaaya (11)	10.02
10	Zuro (18)	Suri (13)	5.42
11	Jashee (14)	Djego (15)	8.77
12	Kwani (17)	Palguni (16)	8.14
13	Palguni (16)	Djego (15)	4.18
14	Djego (15)	Parishenaaya (11)	6.75
15	Parishenaaya (11)	Kaiyong (10)	14.77
16	Djego (15)	Zilimo (9)	13.45
17	Zilimo (9)	Kaiyong (10)	5.78
18	Djego (15)	Kaiyong (10)	10.92
19	Zilimo (9)	Palari (8)	10.07

20	Zilimo (9)	Bogoyili (7)	18.54
21	Palari (8)	Bogoyili (7)	13.44
22	Bogoyili (7)	Denie (6)	7.85
23	Denie (6)	Kaiyong (10)	19.65
24	Denie (6)	Balo (5)	24.48
25	Balo (5)	Kaiyong (10)	30.49
26	Balo (5)	Akinajili (4)	6.99
27	Sanduli (3)	Akinajili (4)	6.2
28	Akinajili (4)	Zanduli (2)	7.29
29	Balo (5)	Zanduli (2)	14.48
30	Zanduli (2)	Sanduli (3)	3.78
31	Zanduli (2)	Sampiaman (1)	15.83
32	Sampiaman (1)	Sanduli (3)	15.86
33	Zanduli (2)	Parishenaaya (11)	34.21
34	Kpablaa (20)	Palguni (16)	5.24
35	Kpablaa (20)	Djego (15)	7.97
36	Suri (13)	Parishenaaya (11)	7.72
37	Palguni (16)	Zilimo (9)	15.31
38	Jashee (14)	Parishenaaya (11)	5.82

Table 4: Simplified description of the Mion District MST problem.

Edge	Starting Node	Ending Node	Distance
1	19	12	9.00
2	18	17	12.35
3	20	14	5.80
4	19	18	6.43
5	19	13	10.02
6	12	13	3.43
7	13	14	5.64
8	20	18	10.19
9	12	11	10.02
10	18	13	5.42

11	14	15	8.77
12	17	16	8.14
13	16	15	4.18
14	15	11	6.75
15	11	10	14.77
16	15	9	13.45
17	9	10	5.78
18	15	10	10.92
19	9	8	10.07
20	9	7	18.54
21	8	7	13.44
22	7	6	7.85
23	6	10	19.65
24	6	5	24.48
25	5	10	30.49
26	5	4	6.99
27	3	4	6.20
28	4	2	7.29
29	5	2	14.48
30	2	3	3.78
31	2	1	15.83
32	1	3	15.86
33	2	11	34.21
34	20	16	5.24
35	20	15	7.97
36	13	11	7.72
37	16	9	15.31
38	14	11	5.82

4.1.1. Some pre-optimality electrification routes for the case of Mion District

The following are some randomly selected routes to traverse in the electrification process which are not based on any scientific method or approach.

Route 1

12-13 → 13-18 → 18-19 → 18-17 → 13-14 → 14-11 → 14-20 → 20-15 → 15-16 → 15-10 → 10-9 → 9-8 → 8-7 → 7-6 → 6-5 → 5-4 → 4-3 → 3-2 → 2-1,

with the corresponding weights as;

3.43 + 5.42 + 6.43 + 12.35 + 5.64 + 5.82 + 5.80 + 5.24 + 4.18 + 10.92 + 5.78 + 10.07 + 13.44 + 7.85 + 24.48 + 6.99 + 6.20 + 3.76 + 15.83 = **159.63km**.

Route 2

19-12 → 12-13 → 13-18 → 18-20 → 20-14 → 14-11 → 11-15 → 15-16 → 16-17 → 16-9 → 9-10 → 9-8 → 8-7 → 7-6 → 6-5 → 5-4 → 4-3 → 3-2 → 2-1,

with corresponding weights as;

9.00 + 3.43 + 5.42 + 10.19 + 5.8 + 5.82 + 6.75 +4.18 + 8.14 + 15.31 + 5.78 + 10.07 + 13.44 + 7.85 + 24.48 +6.99 +6.20 + 3.78 + 15.83 = **168.46km**

Route 3

19-18 → 18-13 → 13-12 → 13-14 → 14-20 → 14-15 → 15-11 → 15-16 → 16-17 → 15-10 → 10-9 → 9-8 → 8-7 → 7-6 → 6-5 → 5-4 → 4-3 → 3-2 → 2-1,

with corresponding weights as;

6.43 + 5.42 + 3.43 + 5.64 +5.8 +8.77 + 6.75 +4.18 +8.14 +10.92 +5.78 + 10.07 + 13.44 +7.85 + 24.48 + 6.99 + 6.20 +3.78 + 15.83 = **159.90km**

4.1.2. Optimality analysis for the case of Mion District

It should be recalled that, the total number of spanning trees in a connected graph with n vertices is n^{n-2}. In the case of this problem, the total number of spanning trees will be very big. It was therefore practically impossible to determine all these numerous numbers of spanning trees and pick the optimal tree out of them. Kruskal's algorithm was therefore employed to obtain the optimal solution and Management Scientist Version 5 software was used to confirm the optimal solution.

Applying Kruskal's Algorithm to Figure 7,

Step 1: Select the minimum cost edge in the entire graph.

Step 2: Sort and add edges with least costs

Step 3: Do not add edges the form a circuit or cycle

Table 5: The steps in determination of optimal electrification routes using Kruskal's algorithm.

Step	**Edge**	**Weight (in km)**	**What to do**
1	12-13	3.43	Start with edge
2	2-3	3.78	Add edge
3	15-16	4.18	Add edge
4	16-20	5.24	Add edge
5	13-18	5.42	Add edge
6	13-14	5.64	Add edge
7	9-10	5.78	Add edge
8	14-20	5.80	Add edge
9	14-11	5.82	Add edge
10	3-4	6.20	Add edge
11	18-19	6.43	Add edge
12	11-16	6.75	Ignore, it forms a cycle
13	4-5	6.99	Add edge
14	2-4	7.29	Ignore, it forms a cycle
15	13-11	7.72	Ignore, it forms a cycle
16	6-7	7.85	Add edge
17	15-20	7.97	Ignore, it forms a cycle
18	16-17	8.14	Add edge
19	14-15	8.77	Ignore, it forms a cycle
20	12-19	9.0	Ignore, it forms a cycle

21	13-19	10.02	Ignore, it forms a cycle
22	8-9	10.07	Add edge
23	18-20	10.19	Ignore, it forms a cycle
24	10-15	10.92	Add edge
25	17-18	12.35	Ignore, it forms a cycle
26	7-8	13.44	Add edge
27	9-15	13.45	Ignore, it forms a cycle
28	2-5	14.48	Ignore, it forms a cycle
29	10-11	14.77	Ignore, it forms a cycle
30	9-16	15.31	Ignore, it forms a cycle
31	1-2	15.83	Add edge
32	1-3	15.86	Ignore, it forms a cycle
33	7-9	18.54	Ignore, it forms a cycle
34	6-10	19.65	Ignore, it forms a cycle
35	5-6	24.48	Add edge

From Table 5, the optimal electrification routes of the 20 villages in Mion District in the Northern Region of Ghana using Kruskal's algorithm is as follows:

Optimal Route:

12-13 ➡ 2-3 ➡ 15-16 ➡ 6-10 ➡ 13-18 ➡ 13-14 ➡ 9-10 ➡ 14-20 ➡ 14-11 ➡

3-4 ➡ 18-19 ➡ 4-5 ➡ 6-7 ➡ 16-17 ➡ 8-9 ➡ 10-15 ➡ 7-8 ➡ 1-2 ➡ 5-6,

with their corresponding weights as

3.43 + 3.78 + 4.18 + 5.24 + 5.42 + 5.64 + 5.78 + 5.80 + 5.82 + 6.20 + 6.43 +6.99 + 7.85

+ 8.14 + 10.07 + 10.92 + 13.44 +15.83 + 24.48 = **155.44km.**

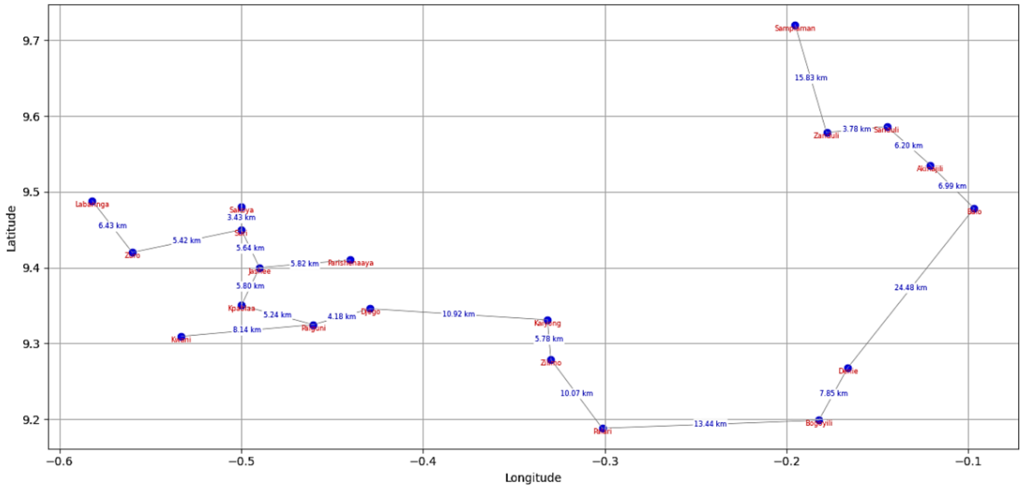


Figure 8: Optimal electrification routes in the case of Mion District using Kruskal's algorithm.

Clearly, Kruskal's algorithm has given the optimal length of electrification routes for the 20 villages in the Mion District to be 155.44 km. It must be stated here that, the optimal solution has been confirmed with Management Scientist Version 5 software.

4.2. The MST problem in the case of Savelugu Municipality

Table 6 also gives the coordinates (latitudes and longitudes) of 15 villages in the Savelugu Municipality in the Northern Region of Ghana that do not have electricity.

Table 6: Latitudes and longitudes of the villages in Savelugu Municipality that do not have electricity.

No.	Village	Latitude	Longitude
1	Daabei	9.623495°N	-0.852281°W
2	Kukuo	9.638797°N	-0.836655°W
3	Tinani	9.644018°N	-0.837509°W
4	Kpaliyogu Kukuo	9.638511°N	-0.845859°W
5	Kpalani	9.5939986°N	-0.822151°W
6	Tuuteenyili	9.644197°N	-0.844130°W
7	Bulung Yapalisi	9.587828°N	-0.800620°W

8	Bilisitua	9.613861°N	-0.808052°W
9	Chehayili	9.6447°N	-0.80963°W
10	Bunjee	9.6148°N	-0.81078°W
11	Zaachitansi	9.6342°N	-0.80675°W
12	Wulateesa	9.633747°N	-0.817311°W
13	Wayaayo	9.732173°N	-0.928247°W
14	Silinboma	9.724295°N	-0.942283°W
15	Zonchegu	9.711457°N	-0.946268°W

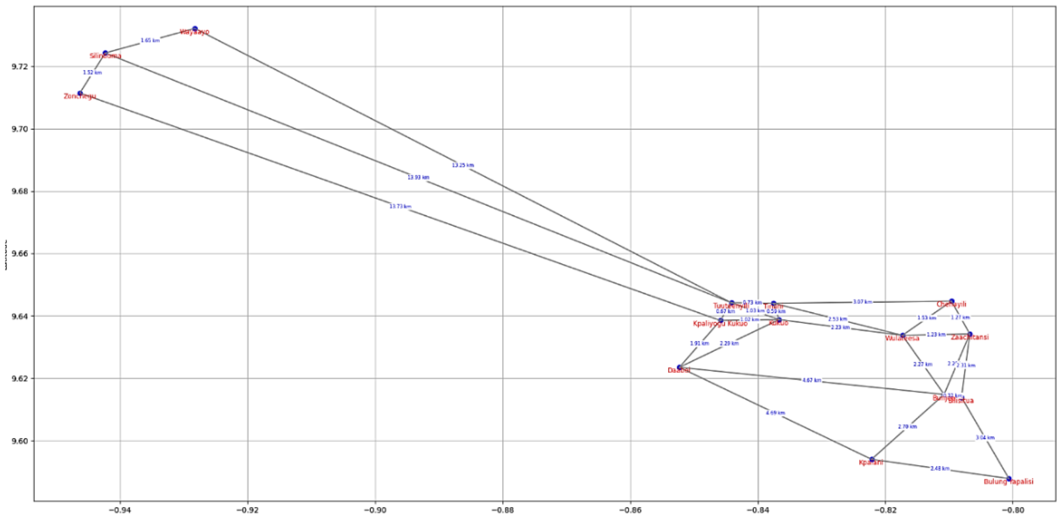


Figure 9: A network of the villages without electricity in Savelugu Municipality.

Each of the 15 villages without electricity in the Savelugu Municipality was assigned a specific number for computational purposes.

Table 7: Villages in the Savelugu Municipality and assigned numbers.

Village	**Assigned Number**
Wayaayo	1
Silinboma	2
Zonchegu	3
Kpaliyogu Kukuo	4

Tuuteenyili	5
Tinani	6
Chehayili	7
Zachitansi	8
Bilisitua	9
Bunlung Yapalisi	10
Kpalani	11
Bunjee	12
Wulateesa	13
Kukuo	14
Daabei	15

Table 8 gives the distances in kilometres between villages in the Savelugu Municipality that do not have electricity.

Table 8: Distances in kilometres between villages in the Savelugu Municipality that do not have electricity.

No.	**From**	**To**	**Distance (in km)**
1	Silinboma (2)	Zonchegu (3)	1.52
2	Silinboma (2)	Wayaayo (1)	1.65
3	Zonchegu (3)	Kpaliyogu kukuo (4)	13.73
4	Silinboma (2)	Tuuteenyili (5)	13.93
5	Wayaayo (1)	Tuuteenyili (5)	13.25
6	Tuuteenyili (5)	Kpaliyogu kukuo (4)	0.67
7	Kpaliyogu kukuo (4)	Daabei (15)	1.91
8	Tuuteenyili (5)	Tinani (6)	0.73
9	Tinani (6)	Chehayili (7)	3.07
10	Tinani (6)	Kukuo (14)	0.59
11	Tuuteenyili (5)	Kukuo (14)	1.03
12	Kukuo (14)	Kpaliyogu kukuo (4)	1.02

13	Kukuo (14)	Daabei (15)	2.29
14	Kukuo (14)	Wulateesa (13)	2.23
15	Tinani (6)	Wulateesa (13)	2.53
16	Daabei (15)	Kpalani (11)	4.69
17	Daabei (15)	Bunjee (12)	4.67
18	Kpalani (11)	Bunjee (12)	2.7
19	Kpalani (11)	Bulung Yapalisi (10)	2.48
20	Bulung Yapalisi (10)	Bilisitua (9)	3.04
21	Bilisitua (9)	Bunjee (12)	0.32
22	Bunjee (12)	Wulateesa (13)	2.27
23	Bunjee (12)	Zaachitansi (8)	2.2
24	Wulateesa (13)	Zaachitansi (8)	1.23
25	Wulateesa (13)	Chehayili (7)	1.53
26	Zaachitansi (8)	Chehayili (7)	1.27
27	Bilisitua (9)	Zaachitansi (8)	2.31

Table 9: Simplified description of the Savelugu Municipality MST problem.

Edge	Starting Node	Ending Node	Distance
1	2	3	1.52
2	2	1	1.65
3	3	4	13.73
4	2	5	13.93
5	1	5	13.25
6	5	4	0.67
7	4	15	1.91
8	5	6	0.73
9	6	7	3.07
10	6	14	0.59
11	5	14	1.03

12	14	4	1.02
13	14	15	2.29
14	14	13	2.23
15	6	13	2.53
16	15	11	4.69
17	15	12	4.67
18	11	12	2.70
19	11	10	2.48
20	10	9	3.04
21	9	12	0.32
22	12	13	2.27
23	12	8	2.20
24	13	8	1.23
25	13	7	1.53
26	8	7	1.27
27	9	8	2.31

Table 9 gives a simplified description of the Savelugu Municipality MST problem.

4.2.1. Some Pre-optimality Electrification Routes for the Case of Savelugu Municipality

The following are some randomly selected routes to traverse in the electrification process which are not based on any scientific method or approach.

Route 1

3-2 → 2-1 → 1-5 → 5-6 → 5-4 → 4-15 → 6-14 → 6-13 → 13-8 → 8-7 →

8-9 → 9-12 → 15-11 → 11-10,

with the corresponding weights as,

1.52 + 1.65 + 13.25 + 0.73 + 0.67 + 1.91 + 0.59 + 2.53 + 1.23 + 1.27 + 2.20 + 0.32 + 4.69 + 2.48 = **35.04km.**

Route 2

12-9 → 9-8 → 8-13 → 13-7 → 7-6 → 6-14 → 14-4 → 4-5 → 5-1 → 1-2 → 2-3 → 14-15 → 15-11 → 11-10,

with corresponding weights as,

0.32 + 2.31 + 1.23 + 1.53 + 3.07 + 0.59 + 1.02 + 0.67 + 13.25 + 1.65 + 1.52 + 2.29 + 4.69 + 2.48 = **36.62km**.

Route 3

10-11 → 11-12 → 12-9 → 9-8 → 8-13 → 13-7 → 7-6 → 6-14 → 14-15 → 15-4 → 4-5 → 5-1 → 1-2 → 2-3,

with the corresponding weights as,

2.48 + 2.70 + 0.32 + 2.31 + 1.23 + 1.53 + 3.07 + 0.59 + 2.29 + 1.19 + 0.67+ 13.25 +1.65 + 1.52 = **34.8 km.**

4.2.2. Optimality analysis in the case of Savelugu Municipality

Kruskal's algorithm was once again employed to obtain the optimal solution and Management Scientist Version 5 software was used to confirm the optimal solution.

Table 10: The steps in determination of optimal electrification routes using Kruskal's algorithm.

Step	Edge	Weight (in km)	What to do
1	9-12	0.32	Start with edge
2	6-14	0.59	Add edge
3	4-5	0.67	Add edge
4	5-6	0.73	Add edge
5	4-14	1.02	Ignore edge, it forms a circuit
6	5-14	1.03	Ignore edge, it forms a circuit
7	8-13	1.23	Add edge
8	8-7	1.27	Add edge

9	2-3	1.52	Add edge
10	7-13	1.53	Ignore edge, it forms a circuit
11	2-1	1.65	Add edge
12	4-15	1.91	Add edge
13	8-12	2.20	Add edge
14	13-14	2.23	Add edge
15	13-12	2.27	Ignore edge, it forms a circuit
16	10-11	2.48	Add edge
17	13-6	2.53	Ignore edge, it forms a circuit
18	11-12	2.70	Add edge
19	9-10	3.04	Ignore edge, it forms a circuit
20	6-7	3.07	Ignore edge, it forms a circuit
21	12-15	4.67	Ignore edge, it forms a circuit
22	11-15	4.69	Ignore edge, it forms a circuit
23	1-5	13.25	Add edge

From Table 10, the optimal electrification routes of the 15 villages in Savelugu Municipality in the Northern Region of Ghana using Kruskal's algorithm is given below.

Optimal Route:

9-12 → 6-14 → 4-5 → 5-6 → 8-13 → 8-7 → 2-3 → 2-1 → 4-15 →

8-12 → 13-14 → 10-11 → 11-12 → 1- 5,

with their corresponding weights as;

0.32 + 0.59 +0.67 + 0.73 + 1.23 + 1.27 +1.52 + 1.65 + 1.91 + 2.20 + 2.23 + 2.48 + 2.70 +

13.25 = **32.75km**.

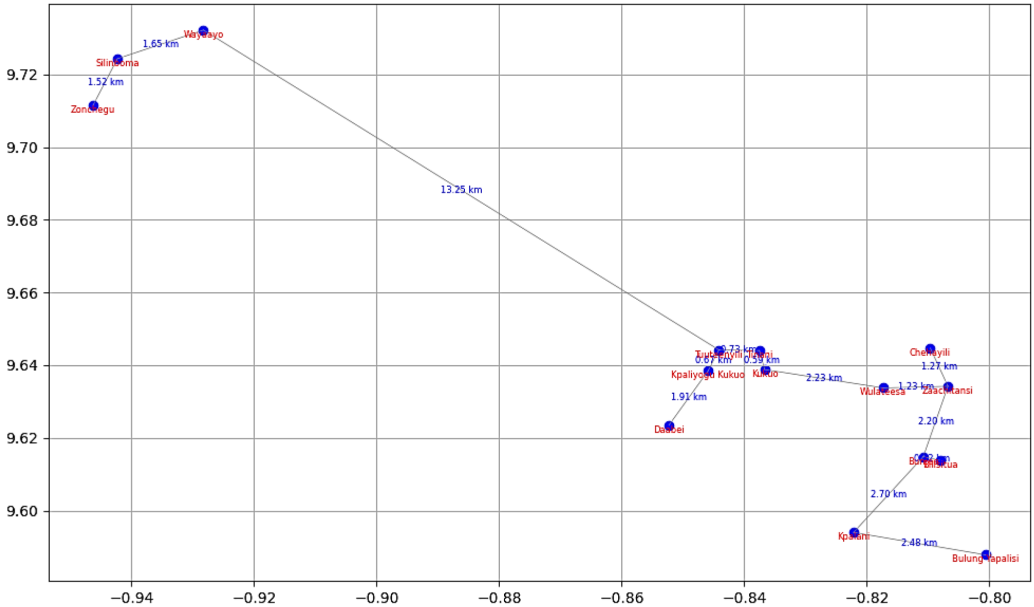


Figure 10: Optimal electrification routes in the case of Savelugu Municipality using Kruskal's algorithm

Clearly, Kruskal's algorithms has given the optimal length of electrification routes for the 15 villages in the Savelugu Municipality to be 32.75 km. It must be stated here that, the optimal soultion has been confirmed with Management Scientist Version 5 software.

4.3. Post optimality analysis

4.3.1. The case of MST problem of Mion District

Here the distance between every two villages in the MST problem of Mion District was varied a number of times and the corresponding optimal length of electrification routes calculated for each case as given in Table 11.

Table 11: Variation of the distances between villages in Mion District and the corresponding optimal lengths of electrification routes.

Type of Variation	Optimal Length (in km)
Original Distances (ODs)	155.44
ODs plus 50m (0.05km) each	156.39

ODs plus 100m (0.10km) each	157.34
ODs plus 150m (0.15km) each	158.29
ODs plus 200m (0.20 km) each	159.24
ODs plus 250m (0.25 km) each	160.19
ODs plus 300m (0.30 km) each	161.14
ODs plus 350m (0.35km) each	162.09
ODs plus 400m (0.40 km) each	163.04
ODs plus 450m (0.45km) each	163.99
ODs plus 500 m (0.50 km) each	164.94
ODs plus 1000 m (1.00 km) each	174.44

From Table 11, it can be observed that, as the distances of the original network are increased, the optimal lengths of electrification routes also increase.

4.3.2. The case of MST problem of Savelugu Municipality

In similar fashion to the case of Mion District, the distance between every two villages in the MST problem of Savelugu Municipality was varied a number of times and the corresponding optimal length of electrification routes calculated for each case as given in Table 12.

Table 12: Variation of the distances between villages in the Savelugu Municipality and the corresponding optimal lengths of electrification routes.

Type of Variation	Optimal Length (in km)
Original Distances (ODs)	32.75
ODs plus 10 m (0.01 km) each	32.89
ODs plus 20 m (0.02 km) each	33.03
ODs plus 30 m (0.03 km) each	33.17
ODs plus 40 m (0.04 km) each	33.31
ODs plus 50 m (0.05 km) each	33.45
ODs plus 60 m (0.06 km) each	33.59
ODs plus 100 m (0.10 km) each	34.15
ODs plus 120 m (0.12 km) each	34.43

ODs plus 150 m (0.15 km) each	34.85
ODs plus 180 m (0.18 km) each	35.27
ODs plus 200 m (0.20 km) each	35.55

From Table 12, it can be observed that, as the distances of the original network for the case of Savelugu Municipality are increased, the optimal lengths of electrification routes also increase.

4.4. Discussion of results

The constructed networks of the villages in the Mion District and Savelugu Municipality considered in the study can easily tell anyone about the electricity situation on the ground as far as those areas are concerned. Also, the pre-optimality analyses performed in both cases indicate clearly how expensive or cost-ineffective it will be to extend electricity to the selected villages unscientifically. Furthermore, the determined optimal electrification routes for the cases considered can go a long way to help the government of Ghana to know where to pass electrical cables when extending electricity to those areas in future so as to reduce cost. The determined optimal lengths of electrification routes of 155.44 km and 32.75 km for the cases of Mion District and Savelugu Municipality respectively will give the government of Ghana idea or knowledge about the lengths of electrical cables that will be required for electrification in those areas to prevent corruption in future. Moreover, the post-optimality analysis performed gives a clear picture of how future changes in the distances of the problems considered can affect the optimal lengths of electrification routes.

Finally, the Kruskal's algorithm has been demonstrated to be very powerful and effective algorithm or method for solving MST problems.

5. Conclusions

The concept of Minimum Spanning Tree Problem has been successfully used to analyze rural electrification of selected areas in the Savelugu and Mion Districts in the Northern Region of Ghana. Specifically, the study has successfully constructed networks of the selected areas in the Savelugu and Mion Districts in the Northern Region of Ghana. It has also determined some pre-optimality electrification routes by inspection. Again, the study has determined the optimal electrification routes for the selected areas using Kruskal's algorithm. Moreover, the study has determined how variations of the distances between villages in both case studies affect the optimal lengths of electrification routes by means of post-optimality analysis. It is recommended that the government of Ghana should use the determined optimal routes as a guide to minimize the total cost of cables for future electrification of the selected areas. Other countries shouldtake a cue from this study in order to minimize the cost of cables involved in their electrification processes.

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