Earthline Journal of Mathematical Sciences
E-ISSN: 2581-8147
Volume 14, Number 3, 2024, Pages 379-389
https://doi.org/10.34198/ejms.14324.379389

Some Useful Results on Fuzzy Differential Subordination of Multivalent Functions Defined by Borel Distribution Series

Bedaa Alawi Abd[1] and Abbas Kareem Wanas[2]

[1] Department of Mathematics, College of Science, University of Al-Qadisiyah, Iraq
e-mail: bedaaalawi@gmail.com

[2] Department of Mathematics, College of Science, University of Al-Qadisiyah, Iraq
e-mail: abbas.kareem.w@qu.edu.iq

Abstract

In this work, we define and study some families of multivalent analytic functions defined by the fuzzy subordination and Borel distribution. We discuss some interesting inclusion results and various other useful properties involving integral of these families.

1. Introduction

In 1965 Zadeh [21] proposed the fuzzy set concept, existing the way for a new dominant theory. The fuzzy set theory it has grown exponentially and now has applications in several scientific and technological fields. Differential subordination is the essential technique in Geometric Function Theory used by various researchers in studies to get important new results. The concept of differential subordination was introduced by Miller and Mocanu [8]. Many of authors worked on different ideas using the notion of differential subordination. In 2011, Oros and Oros [12] modified the concept of differential subordination to accommodate the notion of fuzzy. The basics of fuzzy differential subordination theory were set in 2012 [14]. This idea showed to be a significant application of fuzzy set theory in the field of Geometric Function Theory. The initial results supported the direction of the research, adapting the conventional theory of differential subordination to the novel features of fuzzy differential subordination, and providing methods for examining the dominants and best dominants of fuzzy differential

Received: January 6, 2024; Revised & Accepted: January 30, 2024; Published: February 14, 2024

2020 Mathematics Subject Classification: 30C20, 30C45, 30A20.

Keywords and phrases: analytic functions, multivalent functions, fuzzy differential subordination, Borel distribution series.

subordinations [13], without which the research would not have been able to proceed. Following that, the particular form of Briot-Bouquet fuzzy differential subordinations was studied [15]. In [3], the researcher adopted the concept and begun to look into the new conclusions on fuzzy differential subordinations. The idea of fuzzy differential subordination is a generalisation of the traditional idea of differential subordination that evolved in recent years as a result of incorporating the idea of fuzzy set into the field of Geometric Function Theory. Furthermore, the work of several authors about the fuzzy differential subordination is referred to the readers, for example, see [1,2,4,5,6,7,9,10,16,17,18,20].

Denote by $\mathcal{A}_p$ the family of multivalent analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ with the following series form:

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n, \quad (z \in U, p \in \mathbb{N}).$$

Particularly, $\mathcal{A}_p = \mathcal{A}$, for $p = 1$, where $\mathcal{A}$ be the family of normalized analytic functions in U.

The function $f \in \mathcal{A}_p$ is said to be p-valent starlike of order $\rho(0 \leq \rho < p)$ if it satisfies the condition:

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \rho \qquad (z \in U).$$

A function $f \in \mathcal{A}_p$ is said to be p-valent convex of order $\rho(0 \leq \rho < p)$ if it satisfies the condition

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \rho \qquad (z \in U).$$

The families of $\mathcal{A}_p$ of p-valent starlike functions and p-valent convex functions aredenoted by ST_p and CV_p, respectively.A discrete random variable x is said to have a Borel distribution if it takes the values $1,2,3, \ldots$ with the probabilities $\frac{e^{-\lambda}}{1!}, \frac{2\lambda e^{-2\lambda}}{2!}, \frac{9\lambda^2 e^{-3\lambda}}{3!}, \ldots$ respectively, where λ is called the parameter.

Very recently, Wanas and Khuttar [17] introduced the Borel distribution (BD) whose probability mass function is

$$P(x = r) = \frac{(\lambda r)^{r-1} e^{-\lambda r}}{r!}, \ \ r = 1,2,3, \dots .$$

Wanas and Khuttar [17] introduced a series whose coefficients are probabilities of the Borel distribution (BD)

$$\mathcal{N}_p(\lambda; z) = z^p + \sum_{n=p+1}^{\infty} \frac{\big(\lambda(n-p)\big)^{n-p-1} e^{-\lambda(n-p)}}{(n-p)!} z^n = z^p + \sum_{n=p+1}^{\infty} \Phi_{n,p}(\lambda)\, z^n,$$

where $0 < \lambda \leq 1$ and

$$\Phi_{n,p}(\lambda) = \frac{\big(\lambda(n-p)\big)^{n-p-1} e^{-\lambda(n-p)}}{(n-p)!}.$$

We consider a linear operator $D(p,\lambda)f\colon \mathcal{A}_p \longrightarrow \mathcal{A}_p$ defined by the convolution or Hadamard product

$$D(p,\lambda)f(z) = \mathcal{N}_p(\lambda; z) * f(z) = z^p + \sum_{n=p+1}^{\infty} \frac{\big(\lambda(n-p)\big)^{n-p-1} e^{-\lambda(n-p)}}{(n-p)!} a_n z^n, \quad (1.1)$$

where $a_n \geq 0, 0 < \lambda \leq 1$ and $z \in U$.

Here, we provide an overview of some important fundamental ideas connected to our work.

Definition 1.1 [8]**.** Denote by Q the set of functions q that are analytic and injective on $\overline{U} \setminus E(q)$, where

$$E(q) = \left\{ \xi \in \partial U \colon \lim_{z \to \xi} q(z) = \infty \right\}$$

and are such that $q'(\xi) \neq 0$ for $\xi \in \partial U \backslash E(q)$. Further, let the subclass of Q for which $q(0) = a$ be denoted by $Q(a), Q(0) = Q_0$ and $Q(1) = Q_1$.

Definition 1.2 [21]**.** Let X be a non-empty set. An application $F : X \to [0,1]$ is called fuzzy subset. An alternate definition, more precise, would be the following:

A pair (A, F_A), where $F_A : X \to [0,1]$ and $A = \{x \in X \colon 0 < F_A(x) \leq 1\} = supp(A, F_A)$ is called fuzzy subset. The function F_A is called membership function of the fuzzy subset (A, F_A).

Definition 1.3 [12]**.** Let two fuzzy subsets of X, (M, F_M) and (N, F_N). We say that the fuzzy subsets M and N are equal if and only if $F_M(x) = F_N(x), x \in X$ and we denote this by $(M, F_M) = (N, F_N)$. The fuzzy subset (M, F_M) is contained in the fuzzy subset (N, F_N) if and only if $F_M(x) \leq F_N(x)$, $x \in X$ and we denote the inclusion relation by$(M, F_M) \subseteq (N, F_N)$.

Definition 1.4 [12]**.** Let $D \subseteq \mathbb{C}$, $z_0 \in D$ be a fixed point, and let the functions $f, g \in \mathcal{H}(D)$. The function f is said to be fuzzy subordinate to g and write $f \prec_F g$ or $f(z) \prec_F g(z)$ if the following conditions are satisfied:

1- $f(z_0) = g(z_0)$,

2- $F_{f(D)}\big(f(z)\big) \leq F_{g(D)}\big(g(z)\big),\ z \in D$

where

$$f(D) = supp\big(f(D), F_{f(D)}\big) = \big\{f(z) : 0 < F_{f(D)}\big(f(z)\big) \leq 1, z \in D\big\}$$

and

$$g(D) = supp\big(g(D), F_{g(D)}\big) = \big\{g(z) : 0 < F_{g(D)}\big(g(z)\big) \leq 1, z \in D\big\}.$$

Lemma 1.1 [15]**.** *Let* $r_1, r_2 \in \mathbb{C}, r_1 \neq 0$, *and a convex function* g *satisfies*

$$Re(r_1 g(t) + r_2) > 0, t \in U.$$

If h *is analytic in* U *with* $h(0) = g(0)$, *and* $G(h(t), th'(t); t) = h(t) + \frac{th'(t)}{r_1 h(t) + r_2}$ *is analytic in* U *with* $G(g(0), 0; 0) = g(0)$, *then*

$$F_{G(\mathbb{C}^2 \times U)}\left[h(t) + \frac{th'(t)}{r_1 h(t) + r_2}\right] \leq F_{g\,(U)}\big(g(t)\big)$$

implies

$$F_{p(U)}\big(h(t)\big) \leq F_{g\,(U)}\big(g(t)\big),\ \ t \in U.$$

That is, $h(z) \prec_F g(z)$.

2. Main Results

Let G be the family of functions g with $g(0) = 1$, which are analytic and convex univalent in U and $Re\big(g(z)\big) > 0$. Now, for $g(z) \in G$, $F : \mathbb{C} \to [0,1]$, we define the following family:

Definition 2.1. A function $f \in \mathcal{A}_p$ is in the family $M_F(\alpha, \lambda, p; g)$ if it satisfies the fuzzy subordination:

$$\frac{(1-\alpha)}{p}\frac{z\left(D(p,\lambda)f(z)\right)'}{D(p,\lambda)f(z)}+\frac{\alpha}{p}\frac{\left(z\left(D(p,\lambda)f(z)\right)'\right)'}{\left(D(p,\lambda)f(z)\right)'}\prec_F g(z),$$

where $0 \leq \alpha \leq 1$, $p \in \mathbb{N}$ and $0 < \lambda \leq 1$.

In particular, if we choose $\alpha = 0$ in Definition 2.1, the family $M_F(\alpha, \lambda, p; g)$ reduces to the family $ST_F(\lambda, p; g)$ of the fuzzy p-valent starlike functions which satisfying the following fuzzy subordination:

$$\frac{z\left(D(p,\lambda)f(z)\right)'}{pD(p,\lambda)f(z)}\prec_F g(z).$$

If we choose $\alpha = 1$ in Definition 2.1, the family $M_F(\alpha, \lambda, p; g)$ reduces to the family $CV_F(\lambda, p; g)$ of the fuzzy p-valent convex functions which satisfying the following fuzzy subordination:

$$\frac{\left(z\left(D(p,\lambda)f(z)\right)'\right)'}{p\left(D(p,\lambda)f(z)\right)'}\prec_F g(z).$$

Theorem 2.1. *Let* $g \in G, p \in \mathbb{N}, 0 < \lambda \leq 1, 0 \leq \alpha \leq 1$. *Then*,

$$M_F(\alpha, \lambda, p; g) \subset ST_F(\lambda, p; g).$$

Proof. Let $f \in M_F(\alpha, \lambda, p; g)$ and consider

$$B(z)=\frac{z\left(D(p,\lambda)f(z)\right)'}{pD(p,\lambda)f(z)}. \tag{2.1}$$

It is clear that B is analytic in U and $B(0) = 1$.

We take logarithmic differentiation of (2.1) to obtain

$$\frac{\left(z\left(D(p,\lambda)f(z)\right)'\right)'}{z\left(D(p,\lambda)f(z)\right)'}-\frac{\left(D(p,\lambda)f(z)\right)'}{D(p,\lambda)f(z)}=\frac{B'(z)}{B(z)}.$$

Equivalently,

$$\frac{\left(z\left(D(p,\lambda)f(z)\right)'\right)'}{p\left(D(p,\lambda)f(z)\right)'}=B(z)+\frac{1}{p}\frac{zB'(z)}{B(z)}. \tag{2.2}$$

Since $f \in M_F(\alpha, \lambda, p; g)$, from (2.1) and (2.2), we get

$$\frac{(1-\alpha)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha}{p}\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'} = B(z) + \frac{\alpha}{p}\frac{zB'(z)}{B(z)} \prec_F g(z)\,. \quad (2.3)$$

By making use of (2.3) along with Lemma 1.1, we have $B(z) \prec_F g(z)$. Hence, $f \in ST_F(\lambda, p; g)$ and the proof is complete.

Theorem 2.2. *Let* $g \in G, \alpha > 1, 0 < \lambda \leq 1, p \in \mathbb{N}$. *Then*,

$$M_F(\alpha, \lambda, p; g) \subset CV_F(\lambda, p; g).$$

Proof. Let $f \in M_F(\alpha, \lambda, p; g)$. We write

$$E(z) = \frac{(1-\alpha)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha}{p}\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'}\,. \quad (2.4)$$

Then, by Definition 2.1 and (2.4), we deduce that $E(z) \prec_F g(z)$.

Now,

$$\begin{aligned}
&\frac{\alpha}{p}\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'} \\
&\qquad = \frac{(1-\alpha)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha}{p}\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'} \\
&\qquad + \frac{(\alpha-1)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} = \frac{(\alpha-1)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + E(z).
\end{aligned}$$

This implies

$$\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{p\big(D(p,\lambda)f(z)\big)'} = \frac{1}{\alpha}E(z) + \left(1 - \frac{1}{\alpha}\right)\frac{z\big(D(p,\lambda)f(z)\big)'}{pD(p,\lambda)f(z)} = \frac{1}{\alpha}E(z) + \left(1 - \frac{1}{\alpha}\right)B(z),$$

where B is given by (2.1). Since $E, B \prec_F g(z)$,

$$\frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{p\big(D(p,\lambda)f(z)\big)'} \prec_F g(z).$$

This gives the required result.

Theorem 2.3. *Let* $g \in G, 0 \leq \alpha_1 < \alpha_2, 0 < \lambda \leq 1, p \in \mathbb{N}$. *Then,*

$$M_F(\alpha_2, \lambda, p; g) \subset M_F(\alpha_1, \lambda, p; g).$$

Proof. For $\alpha_1 = 0$, it is obviously true from Theorem 2.2.

Let $f \in M_F(\alpha_2, \lambda, p; g)$. We write

$$L(z) = \frac{(1-\alpha_2)}{p} \frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha_2}{p} \frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'}. \tag{2.5}$$

Then, by Definition 2.1 and (2.5), we find that

$$L(z) \prec_F g(z).$$

Now, we note that

$$\frac{(1-\alpha_1)}{p} \frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha_1}{p} \frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'}$$

$$= \frac{\alpha_1}{\alpha_2}\left[\frac{(1-\alpha_2)}{p} \frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha_2}{p} \frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'}\right]$$

$$+\left(1-\frac{\alpha_1}{\alpha_2}\right)\frac{z\big(D(p,\lambda)f(z)\big)'}{pD(p,\lambda)f(z)}$$

$$= \frac{\alpha_1}{\alpha_2} L(z) + \left(1-\frac{\alpha_1}{\alpha_2}\right) B(z), \tag{2.6}$$

where B is given by (2.1). Since $L, B \prec_F g(z)$,

$$\frac{(1-\alpha_1)}{p} \frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)} + \frac{\alpha_1}{p} \frac{\Big(z\big(D(p,\lambda)f(z)\big)'\Big)'}{\big(D(p,\lambda)f(z)\big)'} \prec_F g(z).$$

Therefore $f \in M_F(\alpha_1, \lambda, p; g)$ and we obtain the required result.

Remark 2.1. If $\alpha_2 = 1$, we have $f \in M_F(\alpha_1, \lambda, p; g) = CV_F(\lambda, p; g)$, then the previous results for Theorem 2.3 gives us $f \in M_F(\alpha_1, \lambda, p; g), 0 \leq \alpha_1 < 1$. Hence, by using Theorem 2.1, we have $CV_F(\lambda, p; g) \subset ST_F(\lambda, p; g)$.

Theorem 2.4. *Let* $f \in \mathcal{A}_p$. *Then,* $f \in M_F(\alpha, \lambda, p; g)$, $\alpha \neq 0$, *if and only if there exists* $\Upsilon \in ST_F(\lambda, p; g)$ *such that*

$$D(p,\lambda)f(z)=\frac{1}{\alpha}\left[\int_0^t t^{\frac{1}{\alpha}-1}\left(\frac{D(p,\lambda)\Upsilon(t)}{t}\right)^{\frac{1}{\alpha}}dt\right]^{\alpha}. \tag{2.7}$$

Proof. Let $f \in M_F(\alpha,\lambda,p;g)$. Then,

$$\frac{(1-\alpha)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)}+\frac{\alpha}{p}\frac{\left(z\big(D(p,\lambda)f(z)\big)'\right)'}{\big(D(p,\lambda)f(z)\big)'}\prec_F g(z). \tag{2.8}$$

Simple calculations of (2.7), we obtain

$$z\big(D(p,\lambda)f(z)\big)'.\big(D(p,\lambda)f(z)\big)^{\frac{1}{\alpha}}=\big(D(p,\lambda)\Upsilon(z)\big)^{\frac{1}{\alpha}}. \tag{2.9}$$

Differentiating (2.9) logarithmically with respect to z, we have

$$\frac{z\big(D(p,\lambda)\Upsilon(z)\big)'}{pD(p,\lambda)\Upsilon(z)}=\frac{(1-\alpha)}{p}\frac{z\big(D(p,\lambda)f(z)\big)'}{D(p,\lambda)f(z)}+\frac{\alpha}{p}\frac{\left(z\big(D(p,\lambda)f(z)\big)'\right)'}{\big(D(p,\lambda)f(z)\big)'}. \tag{2.10}$$

From (2.8) and (2.10), we get

$$\frac{z\big(D(p,\lambda)\Upsilon(z)\big)'}{pD(p,\lambda)\Upsilon(z)}\prec_F g(z).$$

Thus $\Upsilon \in ST_F(\lambda,p;g)$.

Theorem 2.5. *Let $f \in M_F(\alpha,\lambda,p;g)$ and define $I_{v,p}$ by*

$$I_{\eta,p}(z)=\frac{\eta+p}{z^{\eta}}\int_0^z t^{\eta-1}f(t)dt. \tag{2.11}$$

Then $I_{\eta,p} \in ST_F(\lambda,p;g)$.

Proof. Let $f \in M_F(\alpha,\lambda,p;g)$ and set $I_{\eta,p}^{\lambda}(z)=D(p,\lambda)\left(I_{\eta,p}(z)\right)$. We assume

$$K(z)=\frac{z\left(I_{\eta,p}^{\lambda}(z)\right)'}{pI_{\eta,p}^{\lambda}(z)}. \tag{2.12}$$

We see that K is analytic in U with $K(0)=1$. From (2.11), we obtain

$$\frac{\left(z^{\eta} I_{\eta,p}(z)\right)'}{\eta+p} = z^{\eta-1} f(z).$$

This implies

$$z\left(I_{\eta,p}(z)\right)' = (\eta+p) f(z) - \eta I_{\eta,p}(z). \tag{2.13}$$

It follows from (1.1), (2.12) and (2.13), we find that

$$K(z) = (1+\eta)\frac{zD(p,\lambda)f(z)}{pI_{\eta,p}^{\lambda}(z)} - \frac{\eta}{p}\,. \tag{2.14}$$

If we take logarithmic differentiation to (2.14), we conclude that

$$\frac{z\left(D(p,\lambda)f(z)\right)'}{pD(p,\lambda)f(z)} = K(z) + \frac{zK'(z)}{pK(z)+\eta}\,. \tag{2.15}$$

Since $f \in M_F(\alpha,\lambda,p;g)$ and $M_F(\alpha,\lambda,p;g) \subset ST_F(\lambda,p;g)$, then $f \in ST_F(\lambda,p;g)$ and so

$$\frac{z\left(D(p,\lambda)f(z)\right)'}{pD(p,\lambda)f(z)} \prec_F g(z). \tag{2.16}$$

By using (2.15) and (2.16), we obtain

$$K(z) + \frac{zK'(z)}{pK(z)+\eta} \prec_F g(z).$$

An application of Lemma 1.1, we find $K(z) \prec_F g(z)$. From (2.12), we obtain $\frac{z\left(I_{\eta,p}^{\lambda}(z)\right)'}{pI_{\eta,p}^{\lambda}(z)}$, this implies $I_{\eta,p} \in ST_F(\lambda,p;g)$.

3. Conclusions

The primary objective was to use the concept of a fuzzy subset to create some families of multivalent functions associated with Borel distribution. We have introduced several properties such as, inclusion properties and properties involving integral are examined. As future research directions, the symmetry properties of this newly introduced distribution can be studied.

References

[1] Altinkaya, S., & Wanas, A. K. (2020). Some properties for fuzzy differential subordination defined by Wanas operator. *Earthline Journal of Mathematical Sciences*, 4(1), 51-62. https://doi.org/10.34198/ejms.4120.5162

[2] Azzam, A. A., Shah, S. A., Catas, A., & Cotîrla, L.-I. (2023). On fuzzy spiral-like functions associated with the family of linear operators. *Fractal Fract.*, 7, 145. https://doi.org/10.3390/fractalfract7020145

[3] Haydar, E. A. (2015). On fuzzy differential subordination. *Math. Moravica*, 19, 123-129. https://doi.org/10.5937/MatMor1501123H

[4] Lupas, A. A. (2013). A note on special fuzzy differential subordinations using generalized Salagean operator and Ruscheweyh derivative. *J. Comput. Anal. Appl.*, 15, 1476-1483.

[5] Lupaş, A. A. (2017). On special fuzzy differential subordinations using multiplier transformation. *J. Comp. Anal. Appl.*, 23(6), 1029-1035.

[6] Lupas, A. A. (2018). A note on special fuzzy differential subordinations using multiplier transformation and Ruscheweyh derivative. *J. Comput. Anal. Appl.*, 25, 1116-1124.

[7] Lupas, A. A., & Cãtas, A. (2021). Fuzzy differential subordination of the Atangana-Baleanu fractional integral. *Symmetry*, 13, 1929. https://doi.org/10.3390/sym13101929

[8] Miller, S. S., & Mocanu, P. T. (2000). *Differential Subordinations: Theory and Applications*. Marcel Dekker Inc.

[9] Noor, K. I., & Noor, M. A. (2022). Fuzzy differential subordination involving generalized Noor-Sălăgean operator. *Inf. Sci. Lett.*, 11(6), 1-7. https://doi.org/10.18576/isl/110606

[10] Naik, U. H., Shaikh, R. M., Gophane, M. T., & Wanas, A. K. (2022). Some differential subordinations and fuzzy differential subordinations using generalized integral operator. *Italian Journal of Pure and Applied Mathematics*, 48, 830-842.

[11] Masih, V. S., Cheshmavar, J., & Maghsoudi, S. (2023). On the behavior of analytic representation of multivalent α-convex functions. *Indian Journal of Pure and Applied Mathematics*. https://doi.org/10.1007/s13226-023-00509-9

[12] Oros, G. I., & Oros, Gh. (2011). The notion of subordination in fuzzy set theory. *General Mathematics*, 19(4), 97-103.

[13] Oros, G. I., & Oros, Gh. (2012). Fuzzy differential subordination. *Acta Universitatis Apulensis*, 30, 55-64.

[14] Oros, G. I., & Oros, Gh. (2012). Dominants and best dominants in fuzzy differential subordinations. *Studia Universitatis Babes-Bolyai Mathematica*, 57, 239-248.

[15] Oros, G. I., & Oros, Gh. (2012). Briot-Bouquet fuzzy differential subordination. *Analytic Universitatis Oradea Fasciculus Mathematica*, 19, 83-87.

[16] Srivastava, H. M., & El-deeb, S. M. (2021). Fuzzy differential subordinations based upon the Mittag-Leffler type Borel distribution. *Symmetry*, 13, 1023. https://doi.org/10.3390/sym13061023

[17] Wanas, A. K., & Khuttar, J. A. (2020). Applications of Borel distribution series on analytic functions. *Earthline Journal of Mathematical Sciences*, 4(2), 71-82. https://doi.org/10.34198/ejms.4120.7182

[18] Wanas, A. K., & Majeed, A. H. (2017). Fuzzy differential subordinations for prestarlike functions of complex order and some applications. *Far East Journal of Mathematical Sciences*, 102(8), 1777-1788. https://doi.org/10.17654/MS102081777

[19] Wanas, A. K., & Majeed, A. H. (2018). Fuzzy differential subordination properties of analytic functions involving generalized differential operator. *Science International* (*Lahore*), 30(2), 297-302.

[20] Wanas, A. K., & Majeed, A. H. (2019). Fuzzy subordination results for fractional integral associated with generalized Mittag-Leffler function. *Engineering Mathematics Letters*, Article ID 10, 1-13. https://doi.org/10.34198/ejms.1219.143155

[21] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X