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$\mathbb{T}-$Relative Fuzzy Linear Programming for $\mathbb{T}-$Relative Fuzzy Target Coverage Problems

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Abstract

Optimal set covering problems are commonplace in communication, remote sensing, logistics, image processing, and network fields [3]. Thus, studies on determining optimal covering sets (sensors) of points (targets) in a region have emerged recently. One characteristic of these studies is the consideration of cases where a target is considered fully covered when it falls within a coverage area ("Boolean" coverage). Consequently, optimality solutions/methods/algorithms founded on this coverage scheme are usually too restrictive and (or) precise and so are not suitable for many complex and real life situations, which are most times plagued with ambiguity, vagueness, imprecision and approximate membership of points and (or) covering sets.

Fuzzy structures have proven to be suitable for the representation and analysis of such complex systems with many successful applications. Although *fuzzy sets* generalizes a set, a more recent generalization for both and its related concepts is the *Relative fuzzy set* [1] which gives a dynamic fuzzy representation to sets. $\mathbb{T}-$Relative Fuzzy fixed points results of $\mathbb{T}-$Relative fuzzy maps were studied in [5] and recently, the concept of $\mathbb{T}-$Relative fuzzy linear programming [6] was introduced as a generalization of fuzzy linear programming. The results were applied to generalize the

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Boolean set based covering problems in literature to a $\mathbb{T}-$Relative fuzzy Boolean coverage one. Although, Shan et al. in [15] and others [16] - [21] have given a probabilistic coverage consideration but this lacks subjectivity in representing vagueness and imprecision inherent in most systems.

In this present article the Linear Programming (LP) formulation of "A Computational Physics-based Algorithm for Target Coverage Problems" by Jordan Barry and Christopher Thron is generalized by considering a fuzzy and relative fuzzy target coverage instead of the crisp set Boolean coverage. Also we introduce the Fuzzy Linear Programming (FLP) and the $\mathbb{T}-$Relative Fuzzy Linear Programming (RFLP) for the set coverage problem which allows for ascertaining dynamic optimality with aspiration levels.

1 Introduction

In literature, set coverage problem is a class of problems that have the following essential structure: there is a set of m objects and they are to be grouped into subsets, so some criterion is optimized. Such problems are categorized into; area coverage, target coverage (simple coverage, k-coverage or Q-coverage) and Barrier coverage [14]. Target coverage structure is best suited for wireless sensor networks problems which have found applications in security surveillance, military operations, traffic control, environmental risk monitoring among others. It refers to a situation where a finite set of points within a region must be covered by sets, so as to minimize a cost function which depends on the sets included in the cover. Many authors have studied this problem especially the recent results of [14] and [3]. Although a more general than the classical area covering problem called fuzzy area covering has been studied in [12] with successful application to LP problems in [13] but the fuzzy version of the target set covering has not been studied.

On the other hand, a more general fuzzy set called the relative fuzzy set was recently introduced by Osawaru, Olaleru and Olaoluwa [1]. The membership function of the relative fuzzy set expresses the dynamics of the membership values of elements of a set. It characterization, examples and applications were provided. Fixed point results of maps defined on a space equipped with the relative fuzzy set with application in fuzzy optimization has been studied by Osawaru, Olaleru and Akewe [5]. Recently relative fuzzy optimization was introduced by Osawaru and Olowu [6] with applications.

To the best of our knowledge, the fuzzy and relative fuzzy versions of the target set coverage problem with embedded fuzzy Boolean coverage have not been considered. In this article, we formulate, in the sense of [6], a RFLP problem for a relative fuzzy system equipped with a relative fuzzy Boolean coverage, thus generalizing the formulations of [3] and other related works. In addition, we deduce from our formulations, the FLP problem formulation for a target covering problem equipped with a fuzzy system and a fuzzy Boolean coverage.

2 Motivation

If a user engages a word search algorithm to find the word *catarrh*, then the Boolean based target coverage algorithm would give a minimal subset of word combination that contains the word *catarrh*. Now, suppose the user cannot recall the spelling correctly and spells *cater* instead, then the algorithm would give a minimal subset of word combination that contains the word *cater* which most likely may not include *catarrh*. Generalizing the Boolean coverage based algorithm to a relative fuzzy type solves this problem as it considers degrees of membership of elements of a set. Furthermore, introducing $\mathbb{T}-$relative fuzzy set allows for a varied fuzzy Boolean coverage. Thus, we can search a word relative to the origin, spelling, meaning, phonetic sound, images etc.

Also, suppose the position of target points and (or) sensors are dynamic, then the algorithm for target coverage problem in literature cannot guarantee a minimal coverage e.g. Target coverage results of sensors monitoring mobile particles in a medium say gas will be more effective if the algorithm is dynamically fuzzy based.

This study therefore seeks to propose a relative fuzzy based target coverage problem that gives degrees of minimal cover sets of targets even when the target points and (or) sensors are dynamic. Our results are also shown to generalize the crisp set based coverage problem in literature.

3 Preliminary Definitions

The following definitions and results would be needed in the sequel.

Definition 3.1. [1] ($\mathbb{T}-$**Relative Fuzzy Set**). Let R be the universal set of discourse, $\mathbb{T}$ any time scale, A a subset of R and $\sigma : \mathbb{T} \to \mathbb{T}$ a forward difference operator. Then a $\mathbb{T}-$Relative fuzzy set A of R with respect to $\mathbb{T}$ is a set equipped with the membership growth function $\mu_A : R \times \mathbb{T} \to [0, 1]$ such that

$$\mu_A(r, \sigma(t)) = \begin{cases} 0 & \text{if} \quad r \notin A \\ k \in (0, 1] & \text{if} \quad r \in A \text{ and } t \in \mathbb{T} \end{cases}$$

for all $r \in R$ and all $t \in \mathbb{T}$. The triple $(R, [0, 1], \mathbb{T})$ would denote a $\mathbb{T}$-Relative fuzzy structure.

Remark 3.1. [1,2]

(i) If instead of $[0, 1]$ in Definition 3.1, we have a lattice J (an ordered set containing its maximum and minimum elements), then the triple $(R, [0, 1], \mathbb{T}) = (R, J, \mathbb{T})$ would denote a lattice $\mathbb{T}$-Relative fuzzy structure.

(ii) We assume in this study that all t is dense in $\mathbb{T}$ so that $\mu_A(r, \sigma(t)) = \mu_A(r, t)$.

(iii) If for each $r \in A$, we have that $\mu_A(r, t) = c_r \in [0, 1]$ for all $t \in \mathbb{T}$, then A is a fuzzy set.

(iv) If for each $r \in A$, we have that $\mu_A(r, t) = 1$ for all $r \in A$ and $t \in \mathbb{T}$, then A is a set.

4 Problem Statement and Mathematical Background

Two Linear programming problem formulations are presented in this work:

(i) Formulation I: A crisp set based LP problem formulation of a target set coverage is transformed to a RFLP problem by relaxing the pretenses of optimization by means of a subjective gradation which can be modelled into relative fuzzy membership functions.

(ii) Formulation II: A fuzzy set based RFLP problem formulation of a fuzzy target set coverage is transformed to a $[0, 1]$ LP problem.

4.1 Formulation I

Let R be any set and $P(R)$ be any collection of sets $A^{'}$ of R. Let $\{F_i^{'}\}_{i=1}^{M} = \mathbb{F}^{'} \subset P(R)$ be the set of possible covering sets (sensors) i.e. $F_i^{'} \in \mathbb{F}^{'}$ for each i contains at least a target point of R. This is referred to as crisp (or full) target cover in literature. We define $c : \mathbb{F}^{'} \to [0, \infty)$ as the cost function associated with the sets in $\mathbb{F}^{'}$. Let $I_{F'} \equiv T \cap F_i^{'}$, where $T = \{\mathbf{t}_i\}_{i=1}^{n}$ the set of target points of R. Then $\{I_{F_i'}\}_{F_i' \in \mathbb{F}'}$ is finite.

Suppose $c_i = \inf_{F'=F_i'} c(F^{'})$ be the cost of $F_i^{'}$ and define a function $\mu_i : T \to \{0, 1\}$ such that $\mu_i(\mathbf{t}_i) = 0$ if $\mathbf{t}_i \notin I_{F'}$ and $\mu_i(\mathbf{t}_i) = 1$ if $\mathbf{t}_i \in I_{F_i'}$, for all $i = 1, 2, ..., n$. The problem is called target coverage problem with Boolean coverage. Thus we must find $\{a_i\}_{i=1}^{M} \in \{0, 1\}$ such that $a_i = 0$ if $I_{F_i'} \notin \{I_{F_i'}\}_{F_i' \in \mathbb{F}'}$ and $a_i = 1$ if $I_{F_i'} \in \{I_{F_i'}\}_{F_i' \in \mathbb{F}'}$ that satisfy the following LP problem:

$$\begin{aligned} \min\ Z &= \sum_{m=1}^{M} c_i a_i \\ s.t. \sum_{i=1}^{M} \mu_i(\mathbf{t}_n) a_i &\succ b \\ \{a_i\}_{i=1}^{M} \in \{0, 1\} & \end{aligned} \tag{1}$$

where $\tilde{\min}$ and $\succ$ denote the fuzzified ordinary *minimum* and $>$ respectively.

In other to transform the LP problem above to a RFLP problem formulation, i.e. represent the relative fuzzy goal, we stipulate that the objective function be essentially less than or equal to the aspiration level b_0 w.r.t. t, chosen by the decision maker (DM). Thus we consider the following problem

$$\begin{aligned} &\text{find } a_i \\ s.t. \sum_{i=1}^{M} c_{i_\alpha} a_i &\lesssim b_0 \\ \sum_{i=1}^{M} \mu_i(\mathbf{t}_n) a_i &\succ b \\ \{a_i\}_{i=1}^{M} &\in \{0,1\}. \end{aligned} \tag{2}$$

For treating relative fuzzy inequalities, we propose a linear membership function as follows:

$$\mu^o(a_{i_t}, t) = \begin{cases} 1 & , \quad \sum_{i=1}^{M} c_{m_t} a_{i_t} < b_0 \\ 1 - \frac{b_0}{1+p_0 e^{t c_{i_t} a_{i_t}}} & , \quad b_0 \le \sum_{i=1}^{M} c_{i_t} a_{i_t} \le b_0 - p_0 \\ 0 & , \quad \sum_{m=1}^{M} c_{i_t} a_{i_t} > b_0 - p_0 \end{cases}$$

$$\mu^c(a_{i_t}, t) = \begin{cases} 1 & , \quad \mu(\mathbf{t}_n) a_{i_t} > b \\ 1 - \frac{\mu(\mathbf{t}_n) a_{i_t} - b}{1+p e^{t\beta}} & , \quad \mu(\mathbf{t}_n) a_{i_t} \ge b + p \\ 0 & , \quad \mu(\mathbf{t}_n) a_{i_t} < b + p \end{cases}$$

$$\{a_i\}_{i=1}^{M} \in \{0,1\} \tag{3}$$

Thus for any $t \in \mathbb{T}$, we have

$$\mu^D(a_{i_t}, t) = \min\{\mu^c(a_{i_t}, t), \mu^o(a_{i_t}, t)\} \tag{4}$$

and for all $t \in \mathbb{T}$,

$$\begin{aligned} \mu^o(a_i) &= \min_{t \in \mathbb{T}} \mu^o(a_{i_t}, t) \\ \mu^c(a_i) &= \min_{t \in \mathbb{T}} \mu^c(a_{i_t}, t) \end{aligned} \tag{5}$$

So

$$\mu^D(a_i) = \min\{\mu^o(a_i), \mu^c(a_i)\} = \min_{t\in\mathbb{T}} \mu^D(a_{i_t}, t). \tag{6}$$

Let $\max\{\mu^D(a_i)\} = \alpha \in (0, 1]$. Then a_i is the set of relative fuzzy optimal solution for any $\beta \in \mathbb{R}$ for all $t \in \mathbb{T}$.

Remark 4.1.

4.1a: If Definition 3.1 is a fuzzy set i.e Remark 3.1(iii) holds, then instead of Equations (3)-(6) we have Equations (7)-(10) below representing the ordinary fuzzy version of the LP problem formulation.

$$\mu^o(a_{i_t}) = \begin{cases} 1 & , \quad \sum_{i=1}^{M} c_m a_i < b_0 \\ 1 - \frac{b_0}{1+p_0 e^{c_i a_i}} & , \quad b_0 \leq \sum_{i=1}^{M} c_i a_i \leq b_0 - p_0 \\ 0 & , \quad \sum_{m=1}^{M} c_i a_i > b_0 - p_0 \end{cases}$$

$$\mu^c(a_i) = \begin{cases} 1 & , \quad \mu(\mathbf{t}_n) a_i > b \\ 1 - \frac{\mu(\mathbf{t}_n) a_i - b}{1+pe^{\beta}} & , \quad \mu(\mathbf{t}_n) a_i \geq b + p \\ 0 & , \quad \mu(\mathbf{t}_n) a_i < b + p \end{cases}$$

$$\{a_i\}_{i=1}^{M} \in \{0, 1\} \tag{7}$$

$$\mu^D(a_i) = \min\{\mu^c(a_i), \mu^o(a_i)\} \tag{8}$$

So

$$\mu^D(a_i) = \min\{\mu^o(a_i), \mu^c(a_i)\} = \min \mu^D(a_i) \tag{9}$$

Let $\max\{\mu^D(a_i)\} = \alpha \in (0, 1]$, Then a_i is the set of fuzzy optimal solution for any $\beta \in \mathbb{R}$.

4.1b: $Z \leq Z_t$ for any $t \in \mathbb{T}$.

4.2 Formulation II

Let R be any set and $F(R)$ be any collection of lattice relative fuzzy subsets $A^{'}$ of R with $f_{A^{'}} : R \times \mathbb{T} \to J$ such that

$$f_{A^{'}}(r,t) \simeq \begin{cases} \inf\{J\} & , \quad r \notin A^{'} \\ k \in (\inf\{J\}, \sup\{J\}] & , \quad r \in A^{'} \text{ for any } t \in \mathbb{T} \end{cases}$$

where $(J, \preceq)$ is a complete distributive lattice.

Remark 4.2.

4.2a: If $J = [0,1]$ and $f_{A^{'}}(r,t)$ is constant for all $t \in \mathbb{T}$, then $F(R)$ is a collection of fuzzy sets of R.

4.2b: If $J = \{0,1\}$ and $f_{A^{'}}(r,t)$ is constant for all $t \in \mathbb{T}$, then $F(R)$ is a collection of subsets of R.

Let $\{F_i^{'}\}_{i=1}^{M} = \mathbb{F}^{'} \subset F(R)$ be the set of possible lattice relative fuzzy covering sets (sensors) i.e. $F_i^{'} \in \mathbb{F}^{'}$ for each i contains at least a target point of R with a membership grade of at least an $\alpha \in (\inf\{J\}, \sup\{J\}]$ for all $t \in \mathbb{T}$. We refer to this as relative fuzzy (or partial) target cover. We define α-level sets for each $F_i^{'}$ as $F_{i_\alpha}^{'} = \{r : f_{F_i^{'}}(r,t) \succeq \alpha \ , \forall t \in \mathbb{T}\}$ w.r.t. $\mathbb{T}$ and $F_{i_{\alpha_t}}^{'} = \{r : f_{i_{F^{'}}}(r,t) \succeq \alpha \ , \text{ for any } t \in \mathbb{T}\}$ w.r.t. $t \in \mathbb{T}$, where $\alpha \in (\inf\{J\}, \max\{J\}]$ and $c : \mathbb{F}^{'} \to [0, \infty)$ the cost function associated with the lattice relative fuzzy sets in $\mathbb{F}^{'}$. Let $I_{F_{i_\alpha}^{'}} \equiv T \cap F_{i_\alpha}^{'}$ where $T = \{\mathbf{t}_i\}_{i=1}^{n}$ the set of target points of R for any $t \in \mathbb{T}$. Then $\{I_{F_{i_\alpha}^{'}}\}_{F_i^{'} \in \mathbb{F}^{'}}$ for any $\alpha \in (\inf\{J\}, \max\{J\}]$ is finite.

Remark 4.3. [1, 3]

4.3a: If $J = \{0,1\}$, $\alpha = 1$ and $f_{A^{'}}(r,t)$ is constant for all $t \in \mathbb{T}$, then $\cup_{i=1}^{M} F_{i_\alpha}^{'} = S \in P(R)$. ($S$, as defined in [3]).

4.3b: $F_{i_\alpha}^{'} = \cap_{t \in \mathbb{T}} F_{i_{\alpha_t}}^{'}$ and so $I_{F_{i_\alpha}^{'}} = \cap_{t \in \mathbb{T}} I_{F_{i_{\alpha_t}}^{'}}$ for any $\alpha \in J$.

Suppose $c_{i_{\alpha_t}} = \inf_{F'=F'_{i_{\alpha_t}}} c(F')$ be the cost of $F'_{i\alpha t}$ and define a relative membership function $\mu_{i_{\alpha_t}} : T \times \mathbb{T} \to J$ such that

$$\mu_{i_{\alpha_t}}(\mathbf{t}_n, t) \simeq \begin{cases} \inf\{J\} & , \quad \mathbf{t}_n \notin I_{F'_{i_{\alpha_t}}} \\ k \in (\inf\{J\}, \sup\{J\}] & , \quad \mathbf{t}_n \in I_{F'_{i_{\alpha_t}}} \text{ for any } t \in \mathbb{T} \end{cases}$$

A specific case is when k is the relative membership value of $\mathbf{t}_n$ in $\in F'_{i_{\alpha_t}}$ for any $t \in \mathbb{T}$. So

$$\mu_{i_\alpha}(\mathbf{t}_n) \simeq \min_{t\in\mathbb{T}} \mu_{i_{\alpha_t}}(\mathbf{t}_n, t).$$

Remark 4.4.

4.4a: We refer to the target coverage problem as a lattice $\mathbb{T}$-relative fuzzy target coverage for a lattice $\mathbb{T}$-relative fuzzy set with lattice $\mathbb{T}$-relative fuzzy Boolean coverage expressed above:

4.4b: If $J = [0, 1]$ and $\mu_{i_\alpha}(\mathbf{t}_n, t)$ is constant for all $t \in \mathbb{T}$, then we have the fuzzy boolean target coverage.

4.4c: If $J = \{0, 1\}$, $\alpha = 1$ and $\mu_{i_\alpha}(\mathbf{t}_n, t)$ is constant for all $t \in \mathbb{T}$, then we have the boolean coverage in [3].

We must then find $\{a_{i_t}\}_{i=1}^{M} \in [\inf\{J\}, \sup\{J\}]$ for each $t \in \mathbb{T}$ with

$$a_{i_t} \simeq \begin{cases} \inf\{J\} & , \quad I_{F'_{i_{\alpha_t}}} \notin \{I_{F'_{i_\alpha}}\}_{F'_i\in\mathbb{F}'} \\ k \in (\inf\{J\}, \sup\{J\}] & , \quad I_{F'_{i_{\alpha_t}}} \in \{I_{F'_{i_\alpha}}\}_{F'_i\in\mathbb{F}'} \text{ for any } t \in \mathbb{T} \end{cases}$$

that satisfy the following L.P problems with lattice $\mathbb{T}$-relative fuzzy Boolean coverage:

For all $t \in \mathbb{T}$,

$$\begin{aligned} \tilde{\min}\ Z &= \sum_{m=1}^{M} c_{i_\alpha} a_i \\ s.t. \sum_{i=1}^{M} \mu_{i_\alpha}(\mathbf{t}_n) a_i &\succ b \\ a_{i_t} \in [\inf\{J\}, \sup\{J\}] \end{aligned} \tag{10}$$

For any $t \in \mathbb{T}$,

$$\begin{aligned} \tilde{\min}\ Z_t &= \sum_{i=1}^{M} c_{i_{\alpha_t}} a_{m_t} \\ s.t. \sum_{i=1}^{M} \mu_{i_{\alpha_t}}(\mathbf{t}_n, t) a_{i_t} &\succ b \\ a_{i_t} \in [\inf\{J\}, \sup\{J\}] \end{aligned} \tag{11}$$

where $\tilde{\min}$ and $\succ$ denote the fuzzified ordinary min and $>$ respectively.

In other to represent the relative fuzzy goal, we stipulate that the objective function be essentially less than or equal to the aspiration level b_0 w.r.t. t chosen by the decision maker (DM). Thus we consider the following problem:

For all $t \in \mathbb{T}$,

$$\begin{aligned} &\text{find } a_i \\ s.t. \sum_{i=1}^{M} c_{i_\alpha} a_i &\preceq b_0 \\ \sum_{i=1}^{M} \mu_{i_\alpha}(\mathbf{t}_n) a_i &\succ b \\ a_{i_t} \in [\inf\{J\}, \sup\{J\}]. \end{aligned} \tag{12}$$

For any $t \in \mathbb{T}$,

$$\begin{aligned} \text{find } a_{i_t} \\ s.t. \sum_{i=1}^{M} c_{i_{\alpha_t}} a_{i_t} &\preceq b_0 \\ \sum_{i=1}^{M} \mu_{i_{\alpha_t}}(\mathbf{t}_n, t) a_{i_t} &\succ b \\ a_{i_t} \in [\inf\{J\}, \sup\{J\}]. \end{aligned} \tag{13}$$

Here the problem is to find an α-cover optimality condition. Then the formulated RFLP problem can be generally formulated in the sense of [13] as the following mathematical programming.

For all $t \in \mathbb{T}$,

$$\begin{aligned} &\text{Min } Z = \textstyle\sum_{m=1}^{M} c_i a_i \\ &\text{s.t} \qquad \left[1 - \prod_{j=j}^{n}(1 - \sum_{i=1}^{M} \mu_i(\mathbf{t}_n) a_i)\right] \succeq \alpha, i = 1.2. \ldots, m \\ &\qquad x_j \in [\inf\{J\}, \sup\{J\}], j = 1, 2, \ldots, n, \alpha \in (\inf\{J\}, \sup\{J\}]. \end{aligned} \tag{14}$$

For any $t \in \mathbb{T}$,

$$\begin{aligned} &\text{Min } Z_t = \textstyle\sum_{m=1}^{M} c_{i_t} a_{i_t} \\ &\text{s.t} \qquad \left[1 - \prod_{j=j}^{n}(1 - \sum_{i=1}^{M} \mu_{i_t}(\mathbf{t}_n) a_{i_t})\right] \succeq \alpha, i = 1.2. \ldots, m \\ &\qquad x_j \in [\inf\{J\}, \sup\{J\}], j = 1, 2, \ldots, n, \alpha \in (\inf\{J\}, \sup\{J\}]. \end{aligned} \tag{15}$$

We refer to the above as the lattice RFLP formulation for a lattice relative fuzzy set target problem with lattice relative fuzzy Boolean coverage.

Remark 4.5.

4.5a: If in the formulation above we have that $J = [0, 1]$ then the above formulation is the RFLP formulation for a relative fuzzy set target problem with relative fuzzy Boolean coverage.

4.5b: If in the formulation above we have that $J = [0, 1]$ and that $\mu_{i_{\alpha_t}}(\mathbf{t}_n, t) = c_t$ for all $t \in \mathbb{T}$, then the above formulation is the RFLP formulation for a relative fuzzy set target problem with fuzzy Boolean coverage.

4.5c: If in the formulation above we have that $J = \{0, 1\}$ and that $\mu_{i_{\alpha_t}}(\mathbf{t}_n, t) = c_t \in [0, 1]$ for all $t \in \mathbb{T}$, then the above formulation is the LP formulation for a set target problem with fuzzy Boolean coverage. This is a generalization of the formulation in [3] suggested by Barry and Thron.

4.5d: If in the formulation above we have that $J = [0, 1]$ and that $\mu_{i_{\alpha_t}}(\mathbf{t}_n, t) = c_t$ and $a_{i_{\alpha_t}} = d_t$ for all $t \in \mathbb{T}$, then the above formulation is the FLP formulation for a fuzzy set target problem with fuzzy Boolean coverage. This is a further generalization of the formulation in [3].

4.5e: If in the formulation above we have that $J = [0, 1]$, $a_{i_{\alpha_t}} = d_t$ for all $t \in \mathbb{T}$ and $\mu_{i_{\alpha_t}} \in \{0, 1\}$ for all $t \in \mathbb{T}$, then the above formulation is the FLP formulation for a crisp set Boolean coverage problem. This is the fuzzy target set covering problem formulation equivalent of the results of Hwang and Chiang in [13]. An example of the extension of the Example in [13] was given in [6] by Osawaru and Olaleru.

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