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Forecasting Method for Optimal Diversification

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Abstract

Forecasting is a technique that uses historical data as inputs to make estimates that are predictive in determining the direction of future trends. The goal of investors is to make optimal choice that leads to minimization of risk and maximization of returns, but the method that leads to these objectives has been a challenge for investor. In this study, Black-Litterman model (BLM) is adopted and two forecasting methods; EGARCH and GARCH methods are used for two parameters of BLM; investor views and level of uncertainty. The aim of this paper is to investigate the best forecasting method to estimate BLM that would lead to minimum risk and maximum returns. The analysis of this paper shows that EGARCH method gives maximum expected returns and minimum risk.

1. Introduction

Asset allocation involves allotting investments among different assets, such as bonds, stocks and equities. It depends solely on individual goals, time horizon and risk tolerance. Asset allocation is used first for risk management and second for generating returns. It also observes that if an investor employs multiple asset classes that perform differently in any given market, the portfolio would be located in such a way in order to evade risk over time, as each piece of the pie moves in diverse directions or to a diverse degree. This

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phenomenon is called diversification. The BLM is an asset allocation which was developed in 1990 by Fischer Black and Robert Litterman at Goldman Sachs. The model combines knowledge from Capital Asset Pricing Model (CAPM) and Markowitz mean variance optimization model offers a way for investors to estimate the optimal portfolio weights under given parameters.

Forecasting is fundamental to the risk management process in order to price assets' derivatives, hedging strategies and estimating the financial risk of a firm's portfolio. In recent years, Autoregressive Conditional Heteroscedasticity (ARCH) type models have become popular as a means of capturing observed characteristics of financial returns like thick tails and volatility clustering. These models use time series data on returns to model conditional variance. An alternative way to estimate future volatility is to use options prices, which reflect the market's expectation of volatility. Analytical option pricing models can be used to back out implied volatility over the remaining life of the option given the observed market price. In the construction of volatility forecasts, energy market participants would like to know which model produces the most accurate forecasts, as well as, whether the complex time series models enhance any significant volatility information beyond what contained in option prices. [4] compared the relative information content and predictive power of implied volatility and ARCH family forecasts for asset futures. A similar study by [7] studied analysis for financial management of efficiency of options market in predicting volatility. [2] examined the prediction of financial volatilities for crude oil, gold and natural gas markets.

The aim of the paper is to test the best forecasting method to estimate BLM that would lead to minimum risk and maximum returns. The remainder of the paper proceeds as follows: Section 2 elucidates the literature review, Section 3 presents the methodology, Section 4 describes the data analysis, Section 5 discusses the results and Section 6 concludes.

2. Literature Review

Black and Litterman (BL) improved on the original MV model by combining mean-variance optimization of Markowitz and CAPM [1]. The original model was first developed in 1990 and a year later they elaborate on the strategic asset allocation that is embedded with investor's views in a global sense. The model does not consider the assumption that expected returns are always at equilibrium with CAPM. Rather as expected returns deviate from the mean, imbalances in the markets will attempt to drive them back. Therefore, it is observed that investors would make more returns by combining their views about returns with the information in the equilibrium [1]. Moreover, additional vital feature of the BL structure is that investors should be willing to take risk according to their views and this should be done when they have strong evidence to support their views [2]. BLM uses Bayesian approach to syndicate the views from the investor with respect to the expected returns of one or more assets with the market equilibrium vector of expected returns to provide a new mixed estimate of expected returns. The new vector of returns results to intuitive portfolio gives a reasonable portfolio weight [5]. Hence, the model produces better stable result than classical mean-variance optimization. Some researchers have tried to study the asset distribution and model simplification [6], [3], [8], [7].

[9] made effort to expand the model by demystifying it; they applied the conditional distribution theory straight to the return vector. The authors amended both the return vector and the covariance matrix. The outcome is the mean vector returns that are the same to those of BL, whereas the conditional covariance matrix is new. It also minimizes the sensitivity of the mean variance optimization to an investor's volatility estimates. [10] introduced new method for quantitative views, taking the form of linear inequalities attached to MV portfolio optimization. The authors evaluated the risk-adjusted measure (expected alpha) conditioned on qualitative views that in turn can be combined with a degree of confidence. [11] criticized all the previous studies on BLM and included behavioural finance in her study. She explained that an investor with home bias would have less or no confidence in the views about foreign assets than domestic assets. This would make the weights of assets closer to the benchmark weights when compared with the weights of the domestic assets. [12] introduced another different method to measure the weight of the weights to the eigenvalues from the prior covariance matrix. [8] criticized the Alternative Reference Model. Actually, it is a combination of opinion and fact that the only valid prior estimate is from statistical model. The authors arguments only based on Alternative reference model which is not relevant to the canonical Reference Model. The focus of their article is on basic statistical properties of time series. [13] stressed that the generated views may ooze from fundamental analysis, quantitative models or blind belief.

Economists have long thought that forecasts are potentially useful as decision aids, and have devoted considerable efforts to develop and assess forecasting methods [6]. Forecasts can provide decision makers with technical and market support to help execute policies. In stock markets, forecasts are typically made for the prices of assets commodity outputs. Less work has been done on forecasting the primary inputs needed to produce the commodities. However, most important variable inputs are crude oil and gold, since crude oil and gold are used as measurement in stock market [7]. With the recent price volatility in the fuel market, making wrong decisions in fuel purchasing can have a significant impact on the bottom line for farming firms or fuel providers. While the ability to anticipate short term fuel prices may be useful, very little work has been done to evaluate the ability to forecast asset price. The dearth of research on this topic requires examining the energy forecasting literature in order to design an approach to forecast asset prices.

3. Methodology

Two methods of forecasting are considered, therefore three methodologies are used in the study; GARCH, EGARCH and BLM:

3.1.1. GARCH model

The model is based on the assumption that forecasts of variance changing in time depends on the lagged variance of capital assets. An unexpected increase or fall in the returns of an asset at time t, will generate an increase in the variability expected in the period to come.

$$\sigma_t^2 = \varpi_i + \sum_{j=1}^{q} \beta_j \varepsilon_{t=j}^2 + \sum_{i=1}^{p} \alpha_i \gamma_{t-i}^2 , \tag{3.1}$$

where $i = 0,1,\ldots,p$ is conditional volatility, ϖ_i, α_i and β_j are non-negative constants with $\alpha_i + \beta_j < 1$ it should be closer to unity for accuracy, ε_{t-j} is residual and lagged conditional volatility $\beta_j \varepsilon_{t-i}^2$ are ARCH component and α_i and γ_{t-i}^2 are GARCH component.

3.1.2. EGARCH model

Asymmetric relationship in financial time series is called leverage effect which stated that volatility operates in diverse manner depending on the positivity and negativity of volatility that occur. In order to eradicate this asymmetry EGARCH model (Exponential generalized autoregressive conditional heteroskedacity) was developed for optimal forecasting. EGARCH model is divided into two parts; mean and variance models:

Mean equation:

$$\mu_t = \gamma_t + \varepsilon_t \tag{3.2}$$

Variance equation:

$$Log\sigma_t^2 = \alpha + \beta \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right| + \omega\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}^2}\right) + \phi\, log(\sigma_{t-1}^2) + \theta(z_t), \quad (3.3)$$

where μ_t is the dependent variable (excess return), σ_t is a vector of exogenous variables, ε_{t-1} is an error term, the one step ahead forecast variance σ_t^2 (conditional variance) depends on the mean, $\alpha, \beta, \omega, \phi$ and θ are the coefficients to be estimated and z_t is regressor term.

3.1.3. Black-Litterman model

A portfolio of *n* assets is represented by a vector $x \in R^n$ with $\sum_{i=1}^n x_i = 1$. Let the returns of an asset i be denoted by r_i and expected return (ER) of asset i be $E(r_i)$. Then the ER vector is $E(r) = col\{E(r_i)\} \in R^n, (i = 1, \ldots, n)$. The covariance matrix is denoted by $\Sigma \in R^{n\times n}$. The covariance of assets i and j is given as σ_{ij}. The return r_p of portfolio is estimated by:

$$r_p = \sum_{i=1}^n x_i r_i$$

$$E(r_p) = E\left(\sum_{i=1}^n x_i r_i\right)$$

$$\sum_{i=1}^n E(x_i r_i) = \sum_{i=1}^n x_i E(r_i)$$

$$= x' E(r). \quad (3.4)$$

The variance of return of the portfolio can be computed as:

$$\sigma_p^2 = \sigma_i^2\left(\sum_{i=1}^n x_i r_i\right)$$

$$= \sum_{i=1}^n \sum_{j=1}^m \sigma_{ij}\left(x_i r_i, x_j r_j\right)$$

$$= \sum_{i=1}^n \sum_{j=1}^m x_i x_j \sigma_{ij}(r_i r_j)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} x_i x_j \sigma_{ij}$$

$$= x' \Sigma x. \tag{3.5}$$

The ER of equilibrium portfolio is estimated as:

$$\prod = \delta \Sigma x_m \, , \tag{3.6}$$

where $\prod$ is the expected return of market equilibrium, δ is the risk aversion and x_m is the market weight.

Let the mean $E(r) = \prod$, the variance be assumed to be proportional to Σ, with factor of uncertainty τ, $E(r) \rightarrow N(\prod, r\Sigma)$. Therefore, $E(r)$ is given as follows:

$$E(r) = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\prod + P'\Omega^{-1}Q\right], \tag{3.7}$$

where there is no view from the investment $P = Q = 0$ and $E(r) = \prod$, therefore market is equilibrium.

4. Data Analysis

The sample data were explored from monthly data of Aluminium, Iron ore and Copper from yahoo finance DataStream. The data spans from 2016 to 2021. The actual data for this study are non-stationary. The non-stationary data were transformed to stationary by first differencing. In view of this, the data were used to check the best platform for investors to invest in order to minimise risk and maximise return.

5. Results and Discussions

As stated earlier that this study investigates the best platform for investors to invest. This is done by forecasting the two important parameters of BLM; Investor's views and Level of uncertainty using GARCH and EGARCH models. Moreover, estimating BLM divulges expected returns and risk which would determine the best platform for investors.

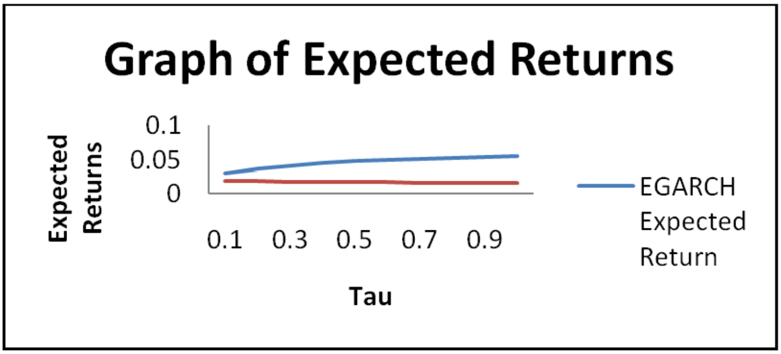

Figure 5.1: Graph of Expected Returns.

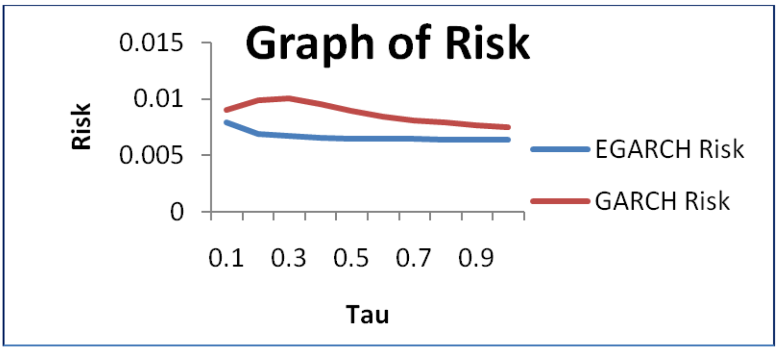

Figure 5.2: Graph of Risk.

Table 5.1: Results of Expected Returns and Risk.

Tau	EGARCH Expected Return	GARCH Expected Return	EGARCH Risk	GARCH Risk
0.1	0.0304	0.0195	0.0079	0.0090
0.2	0.0370	0.0191	0.0069	0.0098
0.3	0.0416	0.0186	0.0067	0.0100
0.4	0.0450	0.0183	0.0066	0.0095
0.5	0.0477	0.0179	0.0065	0.0089
0.6	0.0497	0.0176	0.0065	0.0084

0.7	0.0514	0.0173	0.0065	0.0081
0.8	0.0527	0.0171	0.0064	0.0079
0.9	0.0539	0.0168	0.0064	0.0077
1.0	0.0549	0.0166	0.0064	0.0075

Table 5.1 gives the results of Expected Returns and Risk. The value of Tau is calibrated into ten, ranges from 0.1 to 1.0. EGARCH and GARCH give their values of Expected returns and risk respectively. Looking at the Table, It is vividly noticed that EGARCH expected returns increases as Tau increases, while GARCH decreases.

However, EGARCH risk deceases as Tau increases while GARCH increases. From this study it is vividly seeing that EGARCH is the best platform for investors to invest resources.

6. Conclusion

The aim of this paper is to investigate the best forecasting method for estimating BLM, in order for investors to invest. The goal of every investor is to maximize returns while minimizing risk. Considering the results of Table 5.1, Tau 1.0 of EGARCH gives the highest expected returns; 5.5% compare with GARCH; 1.7%. Moreover, Tau 0.8 of EGARCH gives the same risk to 1.0 which is the lowest risk; 0.6% compare with GARCH; 0.8%. EGARCH is the best forecasting method for BLM to diversify assets.

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