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A New Distribution for Modeling both Blood Cancer Data and Median Effective Dose (ED50) of Artemether-Lumefantrine against P. falciparum

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Abstract

In this paper, a novel distribution was proposed for modeling data on Leukemia and median effective dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum. Plasmodium falciparum or P. falciparum is one of the protozoan species that causes malaria. In the treatment of malaria, especially in sub-Saharan Africa Artemether-Lumefantrine dominates the hospitals. The ED50 (median effective dose) is the dose of a medication that produces a specific effect in 50% of the population that takes that dose. The new distribution has three parameters that make it both flexible and tractable. The distribution is called the Gompertz-Lindley distribution. The model's hazard function behavior was presented together with the properties of the proposed distribution. The parameters were estimated using the method of maximum likelihood. From the analysis, the Gompertz-Lindley distribution is better than the competing standard distribution in the instances of the two data sets deployed.

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1 Introduction

Malaria is an acute or subacute infectious disease caused by any of the five protozoan species: Plasmodium falciparum, P. vivax, P. malariae, and P. ovale. [9] and [23] posited that infection caused by P. falciparum accounted for more than 90% of the world's malaria mortality and remains a threat to public health globally. [25] stated that the complexity of P. falciparum infection has severally prevented attempts at producing an effective vaccine. The most popular and advanced vaccine currently is licensed RTS, S subunit vaccine (RTS, S/ASO1), which targets the pre-erythrocytic stage of infection, hence preventing hepatocyte infection and parasite development, and regimenting red blood cell (RBC) invasion. It comprises a recombinant protein of the P. falciparum circumsporozoite protein (CSP) conjugated to the hepatitis B surface antigen, said [8]. RTS, S/ASO1, being the first malaria vaccine that has received regulatory approval for human use, was used to start the first routine malaria vaccination program in Africa - a pilot study in Malawi in April 2019, which has since expanded to Kenya and Ghana.

The duo of artemether and lumefantrine is essentially helpful in treating certain kinds of malaria infections. Artemether and lumefantrine are used to prevent malaria rather than treat the infection, hence it is in a class of medications called antimalarials. The ED50 (median effective dose) is the dose of a medication that produces a specific effect in 50% of the population that takes that dose. This number has common use as what a clinician or patient can expect for a drug effect, but clinicians may use a different dosage for their particular intended effect, depending on the balance of need or benefit of the drug versus the toxicity of the drug. The toxic dose in 50% of the population is called TD50; the ED50 should hopefully be much less than the TD50, as this would indicate an effective medication at a lower dose according to [6].

The ED50 for a particular medication derives from a dose-response curve, in which the ED50 is at the dosage (x-axis) where there is 50% of the desired response (y-axis). The ED50 is an important indicator that practitioners use it as a clinical starting point when prescribing medications, as adjustments are made to balance efficacy and toxicity. The E-max would be equivalent to the maximum effect the drug may have. It is important to remember that as dosages increase, the risk for toxicities will increase, and they may not be directly proportionate. Each patient requires individualized goals of treatment and should be monitored for such to ensure the lowest effective dose possible, especially for long-term treatment.

In addition, Leukemia is a malignant progressive disease in which the bone marrow and other blood-forming organs produce increased numbers of immature or abnormal leucocytes. These suppress the production of normal blood cells, leading to anaemia and other symptoms. Severally, studies have been done and is still being done in this aspect of human life. While some concentrate on the mortality rate and the risk of being infected others dwell on the preventive medicines against this disease, see [5].

Developing a suitable distribution for explaining the ED50 and Leukemia becomes imperative as huge investigations are ongoing in the fields of reliability engineering, health sciences, and mathematical statistics in this direction. Probability distributions play a key role in describing uncertainties surrounding life events. Several of these class of models has been proposed in the literature to fit lifetime occurrences such as COVID-19, HIV/AIDS, Cancer, and other diseases in both human and non-human beings, see [4, 7, 10, 12–20]. Other related models includes [21, 22, 24].

In this article, therefore, a novel lifetime distribution is proposed and demonstrated using the Leukemia and ED50 data. We show the tractability and flexibility of the model mathematical. The remainder of this work is organized as follows; in Section 2, the model is mathematically derived. In Section 3, the essential properties are obtained. In Section 4, the parameters are estimated using the maximum likelihood estimation. In Section 5, data on the lifetime (in days) of 40 Leukemia patients is used for the first demonstration. In Section 6, the Median Effective Dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum is used for the second demonstration. The article is concluded in Section 7.

2 Model Description

[2] developed the Gompertz-G family of distribution with additional two parameters a and b from the work of [3]. The cumulative distribution function (c.d.f) and probability density function (p.d.f) of the Gompertz-G family of distributions are respectively

$$G(x,a,b) = 1 - e^{\frac{a}{b}\left\{1-[1-F(x,\xi)]^{-b}\right\}} \tag{1}$$

and

$$g(x;a,b) = a\,[1-F(x;\xi)]^{-b-1}\, e^{\frac{a}{b}\left\{1-[1-F(x,\xi)]^{-b}\right\}} f(x;\xi) \tag{2}$$

where ξ is the vector of parameters of the baseline distribution with p.d.f and c.d.f respectively given as $f(x;\xi)$ and $F(x,\xi)$ [11] was the first to suggest mixing proportion procedure for developing new distributions. This led to what is now known as the Lindley distribution with c.d.f and p.d.f respectively given as

$$F(x,\theta) = 1 - \frac{\theta+1+\theta x}{\theta+1} e^{-\theta x} \tag{3}$$

and

$$f(x,\theta) = \frac{\theta^2}{\theta+1}(1+x)e^{-\theta x} \tag{4}$$

with a scale parameter (θ). Substituting eq 3 and 4 into eq 1 and 2, the c.d.f and p.d.f of the proposed Gompertz-Lindley distribution (GLD) is derived as follows

$$G(x;a,b,\theta) = 1 - e^{\frac{a}{b}\left\{1-\left(\frac{1+\theta+\theta x}{\theta+1}\right)^{-b} e^{\theta b x}\right\}}; \quad x>0, \quad a,b,\theta>0 \tag{5}$$

and

$$g(x;a,b,\theta) = a\theta^2(\theta+1)^b(1+x)(1+\theta+\theta x)^{-b-1} e^{\frac{a}{b}\left\{1-\left(\frac{1+\theta+\theta x}{\theta+1}\right)^{-b} e^{\theta b x}\right\}+\theta b x}. \tag{6}$$

The survival and failure rate functions are respectively

$$S(x;a,b,\theta)=e^{\frac{a}{b}\left\{1-\left[\left(1+\frac{\theta x(\theta x+2)}{\theta+2}\right)e^{-\theta x}\right]^{-b}\right\}} \tag{7}$$

and

$$h(x;a,b,\theta)=a\theta^{2}(1+\theta)^{b}(1+x)(1+\theta+\theta x)^{-b-1}e^{\theta bx}. \tag{8}$$

The monotonicity of the hazard function is obtained thus;

$$h(x;a,b,\theta)=\begin{cases}\frac{a\theta^{2}}{1+\theta}; & \text{for } x\to 0\\ \infty; & \text{for } x\to\infty.\end{cases} \tag{9}$$

Hence, it is strictly increasing and negatively skewed. Notice that for $x \to 0$, the hazard function is a function of only a and θ.

The Linear representation of the p.d.f is

$$\begin{aligned} g(x;a,b,\theta)= &\sum_{i,j,k,l,m}^{\infty}\sum_{h=0}^{k}\sum_{r=0}^{i+1}\frac{(-1)^{i+h+l+m}}{j!}\left(\frac{a}{b}\right)^{j}\binom{j}{k}\binom{k}{h}\binom{b+i}{i}\binom{i+1}{r}\\ &\times\binom{l-b+m-1}{m}\binom{b(k-h)+l-1}{l}\\ &\times\frac{\theta^{i+h+l-m+2}b^{2h}}{a^{h-1}}x^{h+l+r}e^{-(h-k)\theta bx}.\end{aligned} \tag{10}$$

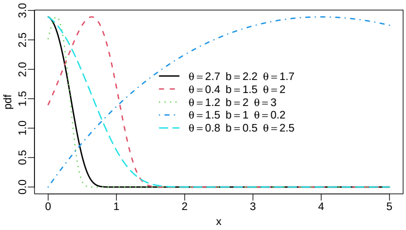

Figure 1: pdf of GLD.

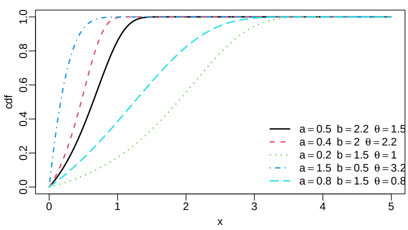

Figure 2: cdf of GLD.

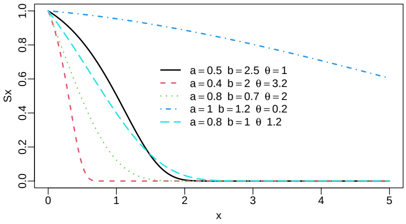

Figure 3: survival function of GLD.

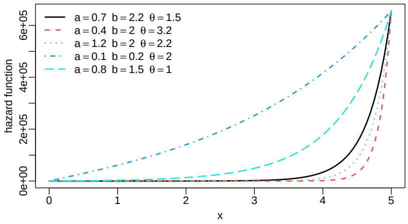

Figure 4: hazard function of GLD.

3 Properties

Definition 3.1. Let $X \sim$ GLD (a, b, θ), the s^{th} complete moment is

$$
\begin{aligned}
\mu_s^{'} = & \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^j \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
& \times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
& \times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \int_0^\infty x^{h+l+r+s} e^{-(h-k)\theta b x} dx \\
= & \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^j \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
& \times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
& \times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \frac{\Gamma[h+l+r+s+1]}{\{(h-k)\theta b\}^{h+l+r+s+1}} \\
= & \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^j \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
& \times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
& \times \frac{\theta^{i-m-r-s-+1} b^{h-l-r-s-1}}{a^{h-1}} \frac{\Gamma[h+l+r+s+1]}{(h-k)^{h+l+r+s+1}}; \quad s = 1, 2, ...
\end{aligned}
\tag{11}
$$

The first four crude moments $\mu, \mu_2^{'}, \mu_3^{'}$ and $\mu_4^{'}$ are obtained by replacing s with $1, 2, 3$ and 4 respectively in eq 11.

Definition 3.2 (Moment generating function)**.** Let $X \sim$ GLD (a, b, θ), the moment generating function $M_X(t)$ can be expressed as

$$\begin{aligned}
M_X(t) &= \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^{j} \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
&\times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
&\times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \int_{0}^{\infty} x^{h+l+r} e^{-\{(h-k)\theta b-t\}x} dx \\
&= \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^{j} \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
&\times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
&\times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \frac{\Gamma_{h+l+r+1}}{\{(h-k)\theta b-t\}^{h+l+r+1}}.
\end{aligned} \tag{12}$$

Definition 3.3 (Characteristic function)**.** Let $X \sim$ GLD (a, b, θ), the characteristic function $\Phi_X(it)$ can be expressed as

$$\begin{aligned}
\Phi_X(it) &= \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^{j} \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
&\times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
&\times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \int_{0}^{\infty} x^{h+l+r} e^{-\{(h-k)\theta b-it\}x} dx \\
&= \sum_{i,j,k,l,m}^{\infty} \sum_{h=0}^{k} \sum_{r=0}^{i+1} \frac{(-1)^{i+h+l+m}}{j!} \left(\frac{a}{b}\right)^{j} \binom{j}{k} \binom{k}{h} \binom{b+i}{i} \binom{i+1}{r} \\
&\times \binom{l-b+m-1}{m} \binom{b(k-h)+l-1}{l} \\
&\times \frac{\theta^{i+h+l-m+2} b^{2h}}{a^{h-1}} \frac{\Gamma_{h+l+r+1}}{\{(h-k)\theta b-it\}^{h+l+r+1}}.
\end{aligned} \tag{13}$$

4 Uncensored Sample Estimation

Definition 4.1 (The Maximum Likelihood). Suppose $x_1, x_2, ..., x_n$ are independent random samples of size n which assumes the GLD(a, b, θ), then the likelihood function of (a, b, θ) can be expressed as

$$L(x|a,b,\theta) = a^n \theta^{2n} (\theta+1)^{nb} e^{\sum\limits_{i=1}^{n} \frac{a}{b}\left\{1-\left(\frac{1+\theta+\theta x}{\theta+1}\right)^{-b} e^{\theta b x}\right\}+\theta b x} \prod_{i=1}^{n} (1+x)(1+\theta+\theta x)^{-b-1}. \tag{14}$$

The log-likelihood is

$$\begin{aligned} \ell = n\log a + 2n\log\theta + b\log(1+\theta) + \sum_{i=1}^{n}\left\{\frac{a}{b}\left[1-\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right)^{-b}\right] + \theta b x\right\} \\ + \sum_{i=1}^{n}\log(1+x) - (b+1)\sum_{i=1}^{n}\log(1+\theta+\theta x) \end{aligned} \tag{15}$$

differentiating partially with respect to a, b and θ yields

$$\frac{\partial \ell}{\partial a} = \frac{n}{a} + \sum_{i=1}^{n}\frac{1}{b}\left[1-\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right)^{-b}\right]. \tag{16}$$

Set $\frac{\partial \ell}{\partial a} = 0$, then

$$\hat{a} = -\frac{n}{\sum\limits_{i=1}^{n}\frac{1}{b}\left[1-\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right)^{-b}\right]}. \tag{17}$$

Similarly,

$$\begin{aligned} \frac{\partial \ell}{\partial b} = \log(1+\theta) + \sum_{i=1}^{n}\Bigg\{\theta x - \frac{a}{b^2}\left[1-\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right)^{-b}\right] \\ - \frac{a}{b}\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right)^{-b}\log\left(\frac{1+\theta+\theta x}{1+\theta}e^{-\theta x}\right) \\ - \sum_{i=1}^{n}\log(1+\theta+\theta x), \end{aligned} \tag{18}$$

and

$$\frac{\partial \ell}{\partial \theta} = -\frac{2n}{\theta} + \frac{b}{1+\theta} + -(b+1)\sum_{i=1}^{n}\left(\frac{1+x}{1+\theta+\theta x}\right)$$
$$+\sum_{i=1}^{n}\frac{a\left[(1+\theta)\left\{1+x-\theta(1+\theta+\theta x)\right\}-(1+\theta+\theta x\right](1+\theta+\theta x)^{-b-1}e^{\theta b x}}{(1+\theta)^{1-b}}. \tag{19}$$

Notice that eq 18 and 19 do not have closed-form solutions hence numerical iteration will provide the convergence in R.

5 Application to Blood Cancer (Leukemia) Data

The following data represent the lifetime of 40 patients suffering from blood cancer (leukemia) from one of the Ministry of Health Hospitals in Saudi Arabia studied by Abouammoh et al. [1]

Table 1: The lifetime of 40 patients suffering from blood cancer (leukemia) from one of the Ministry of Health Hospitals in Saudi Arabia.

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025	2.036	2.162	2.211	2.37	2.532
2.693	2.805	2.91	2.912	3.192	3.263	3.348	3.348	3.427	3.499	3.534	3.767	3.751
3.858	3.986	4.049	4.244	4.323	4.381	4.392	4.397	4.647	4.753	4.929	4.973	5.074
5.381												

The metrics of the performance of the distributions are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer von Mises (W^*), Anderson Darling (A^*), while the Kolmogorov-Smirnov (K-S) statistic and the p-value determine the fitness of the distribution to the data.

From Table 2, the proposed distribution has the largest p-value which is 0.9944 for the Leukemia data which demonstrates its goodness of fit. Similarly, it also has the minimum model performance statistics compared to its competitors.

Table 2: MLEs, Metrics of performance and fitness for the blood cancer data.

Distr	NLL	AIC	CAIC	BIC	HQIC	W^*	A^*	K-S	P-value
GLD	65.6	137.108	137.774	142.174	138.939	0.016	0.131	0.067	0.9944
Weibull	69.56	143.168	143.492	146.545	144.389	0.123	0.796	0.113	0.6835
Gamma	73.55	151.124	151.449	154.502	152.346	0.242	1.487	0.149	0.3396
LogNormal	79.03	162.247	162.571	165.625	163.468	0.405	2.385	0.191	0.1087
KumW	70.41	148.458	149.601	155.214	150.901	0.147	0.940	0.151	0.3179

Table 3: MLEs for the parameters of the fitted distributions using the blood cancer data.

Distr	a	b	θ	γ
GLD	2.3711	12.6129	0.1399	
Weibull	2.4475	3.5464		
Gamma	3.5188	0.9037		
LogNormal	0.9787	0.6178		
Kum-Weibull	1.0043	0.0685	2.0371	1.0841

Figures 5, and 6 indicate graphically how well the suggested distribution fits the Leukemia data.

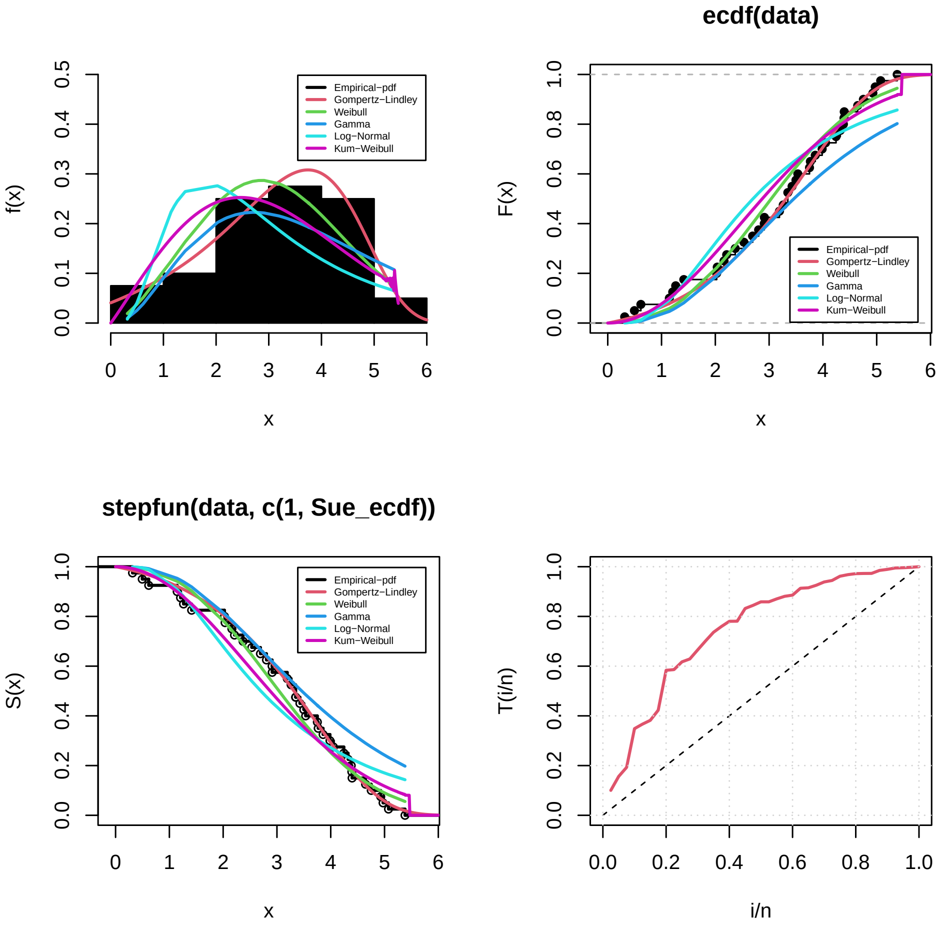

Figure 5: Density, cdf, survival function and TTT plots for blood cancer data.

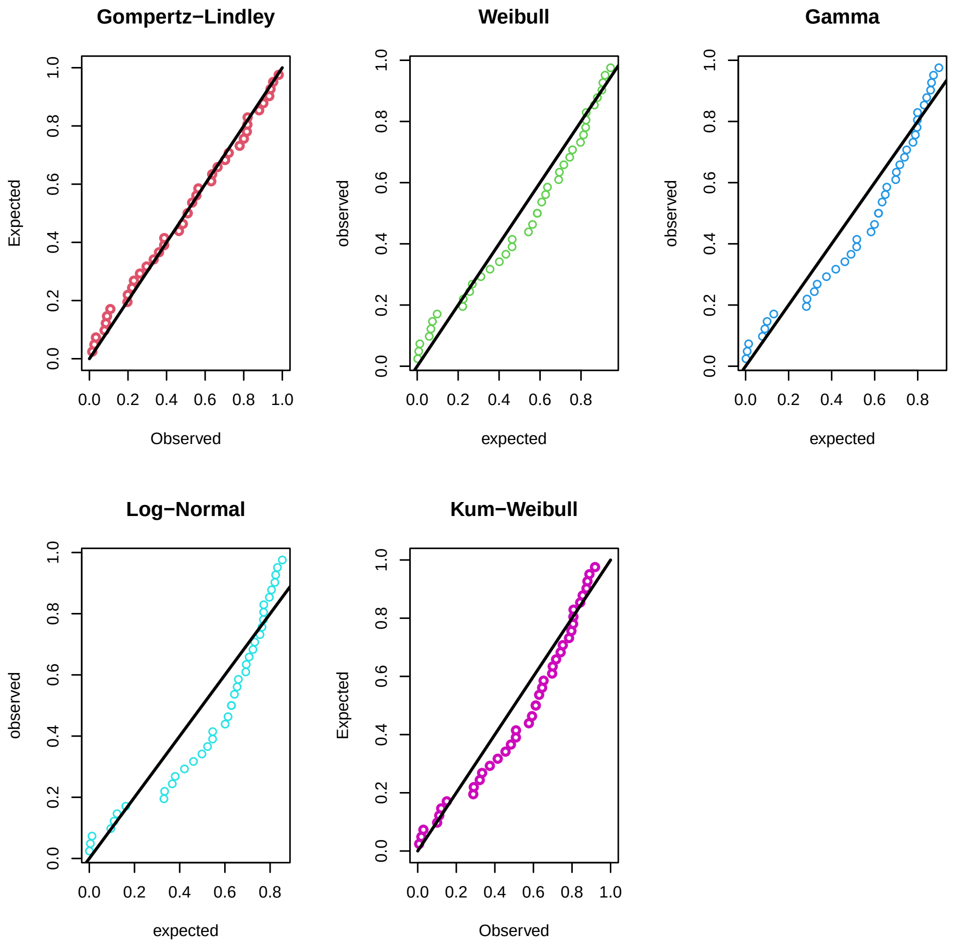

Figure 6: PP plots for blood cancer data.

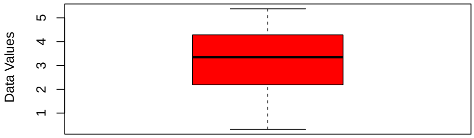

Figure 7: boxplot of the blood cancer data.

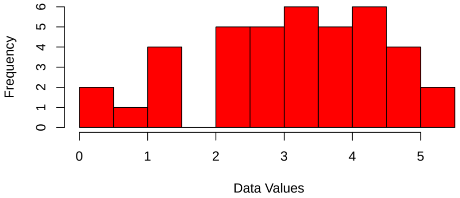

Figure 8: histogram of the blood cancer data.

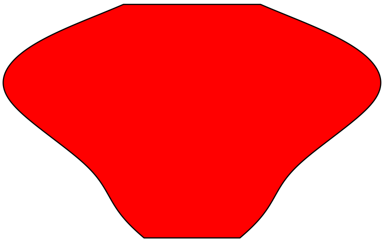

Figure 9: violin plot of the blood cancer data.

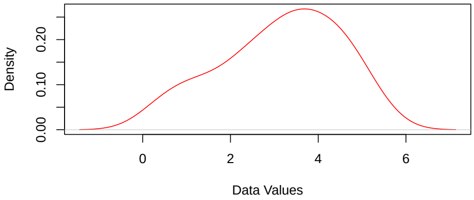

Figure 10: kernel density plot of the blood cancer data.

5.1 Application to Median Effective Dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum

The second application is on the median effective dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum in some countries reported by the World Health Organization. The data can be found in `https://www.who.int/teams/global-malaria-programme/case-management/drug-efficacy-and-resistance/antimalarial-drug-efficacy-database`.

Table 4: Median effective dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum in some countries.

4.5	0.7	1.8	5.0	3.3	1.1	4.9	1.2	2.8	1.9	2.0	5.0	2.8	7.6
1.8	2.4	0.9	0.1	1.5	1.9	2.7	5.3	1.4	8.5	2.7	1.8	1.4	1.5

As done in the first application, the indexes of the performance of the distributions are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), Hannan-Quinn information criterion (HQIC), Cramer von Mises (W^*), Anderson Darling (A^*), while the Kolmogorov-Smirnov (K-S) statistic and the p-value determine the fitness of the distribution to the data.

In Table 5, it is evident that the proposed model again best fits the data on Median Effective Dose (ED50) and the model performance statistics are also the least among others. This is the motivation for adopting this model for modeling disease spread and mortality rates.

Table 5: MLEs, metrics of model performance and fitness for Median Effective Dose (ED50) data.

Distr	NLL	AIC	CAIC	BIC	HQIC	W*	A*	K-S	P-value
GLD	54.24	114.470	115.470	118.466	115.691	0.107	0.600	0.133	0.7077
Burr	58.49	120.970	121.450	123.635	121.785	0.125	0.926	0.213	0.158
EIE	56.16	116.321	116.801	118.985	117.135	0.088	0.612	0.198	0.2242
Weibull	54.02	112.074	112.554	114.738	112.888	0.099	0.573	0.149	0.5598
Lomax	56.87	117.730	118.210	120.395	118.545	0.079	0.490	0.217	0.1417

Figures 11, and 12 indicate how well the proposed model fits the data on Median Effective Dose (ED50).

Table 6: MLEs for the parameters of the fitted distributions using the Median effective dose (ED50) of Artemether- Lumefantrine against Plasmodium falciparum data.

Distr	a	b	θ
GLD	5.9840	5.4719	0.0506
Burr	2.8717	0.3854	
EIE	0.1129	0.3003	
Weibull	4.2408	1.5128	
Lomax	0.7449	0.8575	

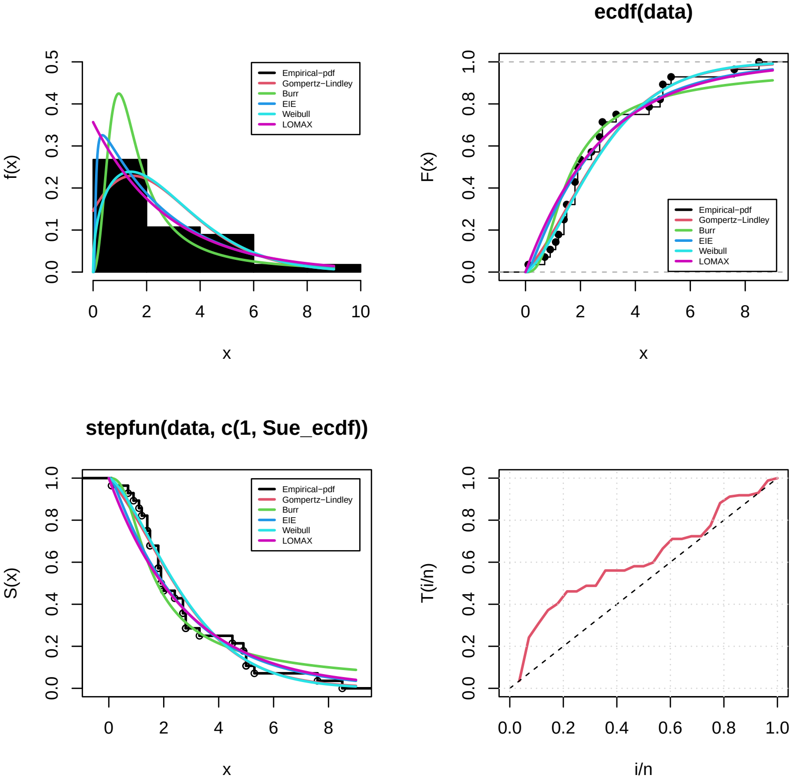

Figure 11: Density, cdf, survival function and TTT plots of Median Effective Dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum.

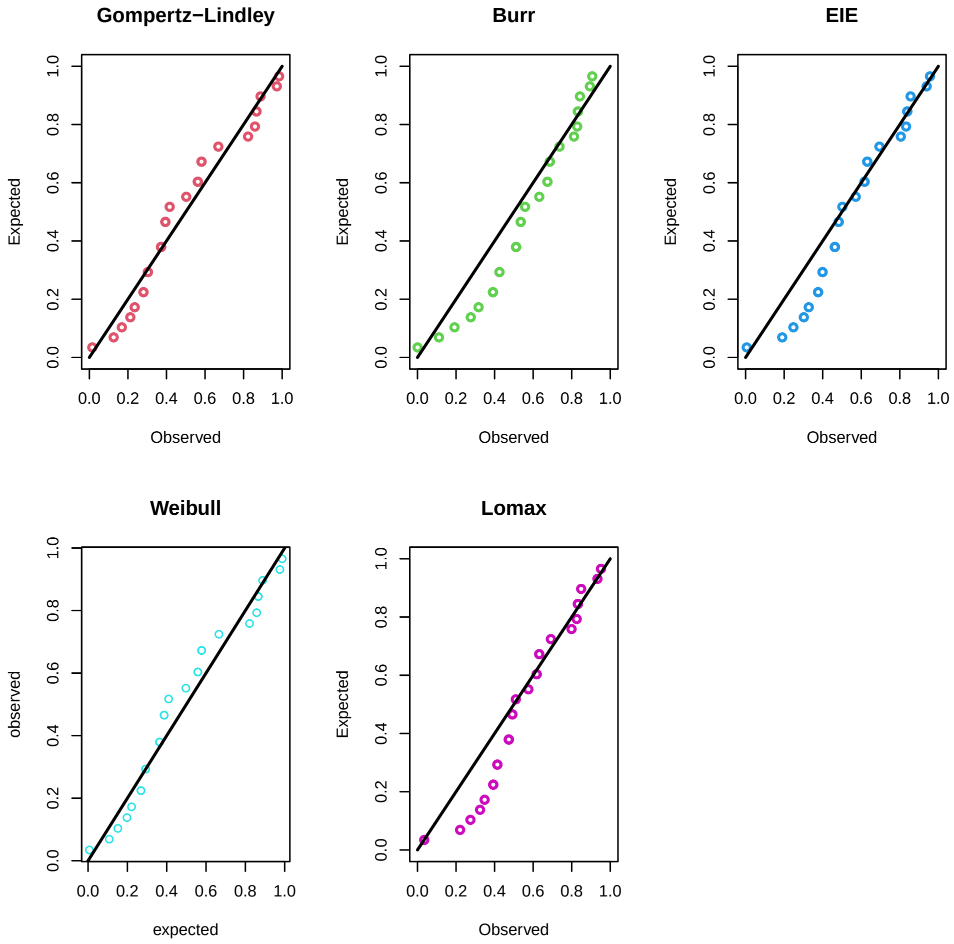

Figure 12: PP plots of the Median Effective Dose (ED50) of Artemether-Lumefantrine against Plasmodium falciparum.

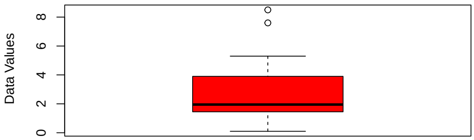

Figure 13: boxplot of the Median Effective Dose (ED50) data.

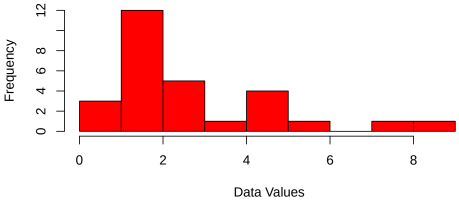

Figure 14: histogram of the Median Effective Dose (ED50) data.

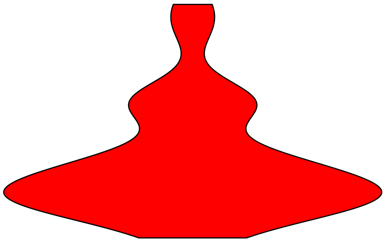

Figure 15: violin plot of the Median Effective Dose (ED50) data.

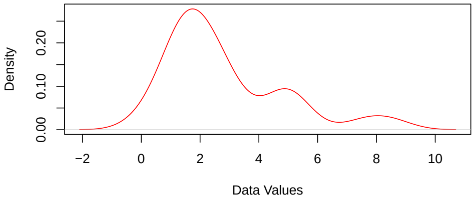

Figure 16: kernel density plot of the Median Effective Dose (ED50) data.

6 Conclusion

This article suggested a novel distribution for modeling Leukemia and Median effective dose (ED50) data. The properties of the suggested distribution were derived. A snap study of the hazard function was presented. The parameters were estimated using maximum likelihood. In the application of two-lifetime data sets, it was found that the proposed distribution is preferred to the competing distributions.

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Conflict of Interest

The authors declare that there is no conflict of interest.

References

[1] Abouammoh, A. M., Ahmad, R., & Khalique, A. (2000). On new renewal better than used classes of life distributions. *Statistics & Probability Letters*, *48*(2), 189-194. https://doi.org/10.1016/s0167-7152(99)00204-7

[2] Alizadeh, M., Cordeiro, G. M., Pinho, L. G. B., & Ghosh, I. (2017). The Gompertz-G family of distributions. *Journal of Statistical Theory and Practice*, *11*, 179-207. https://doi.org/10.1080/15598608.2016.1267668

[3] Alzaatreh, A., Lee, C., & Famoye, F. (2013). A new method for generating families of continuous distributions. *Metron*, *71*(1), 63-79. https://doi.org/10.1007/s40300-013-0007-y

[4] Anabike, I. C., Igbokwe, C. P., Onyekwere, C. K., & Obulezi, O. J. (2023). Inference on the parameters of Zubair-Exponential distribution with application to survival times of Guinea Pigs. *Journal of Advances in Mathematics and Computer Science*, *38*(7), 12-35. https://doi.org/10.9734/jamcs/2023/v38i71769

[5] Chennamadhavuni, A., Lyengar, V., Mukkamalla, S. K. R., & Shimanovsky, A. (2021). *Continuing education activity*. National Library of Medicine.

[6] Dimmitt, S., Stampfer, H., & Martin, J. H. (2017). When less is more-efficacy with less toxicity at the ED50. *British Journal of Clinical Pharmacology*, *83*(7), 1365. https://doi.org/10.1111/bcp.13281

[7] Etaga, H. O., Celestine, E. C., Onyekwere, C. K., Omeje, I. L., Nwankwo, M. P., Oramulu, D. O., & Obulezi, O. J. (2023). A new modification of Shanker distribution with applications to increasing failure rate data. *Earthline Journal of Mathematical Sciences*, *13*(2), 509-526. `https://doi.org/10.34198/ejms.13223.509526`

[8] Frimpong, A., Kusi, K. A., Ofori, M. F., & Ndifon, W. (2018). Novel strategies for malaria vaccine design. *Frontiers in Immunology*, *9*, 2769. `https://doi.org/10.3389/fimmu.2018.02769`

[9] Garcia, L. S. (2010). Malaria. *Clin Lab Med*, *30*(1), 93-129. `https://doi.org/10.1016/j.cll.2009.10.001`

[10] Innocent, C. F., Frederick, O. A., Udofia, E. M., Obulezi, O. J., & Igbokwe, C. P. (2023). Estimation of the parameters of the power size-biased Chris-Jerry distribution. *International Journal of Innovative Science and Research Technology*, *8*(5), 423-436.

[11] Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society. Series B (Methodological)*, 102-107. `https://doi.org/10.1111/j.2517-6161.1958.tb00278.x`

[12] Musa, A., Onyeagu, S. I., & Obulezi, O. J. (2023). Comparative study based on simulation of some methods of classical estimation of the parameters of exponentiated Lindley-Logarithmic distribution. *Asian Journal of Probability and Statistics*, *22*(4), 14-30. `https://doi.org/10.9734/ajpas/2023/v22i4489`

[13] Musa, A., Onyeagu, S. I., & Obulezi, O. J. (2023). Exponentiated Power Lindley-Logarithmic distribution and its applications. *Asian Research Journal of Mathematics*, *19*(8), 47-60. `https://doi.org/10.9734/arjom/2023/v19i8686`

[14] Obulezi, O., Igbokwe, C. P., & Anabike, I. C. (2023). Single acceptance sampling plan based on truncated life tests for Zubair-exponential distribution. *Earthline Journal of Mathematical Sciences*, *13*(1), 165-181. `https://doi.org/10.34198/ejms.13123.165181`

[15] Obulezi, O. J., Anabike, I. C., Okoye, G. C., Igbokwe, C. P., Etaga, H. O., & Onyekwere, C. K. (2023). The Kumaraswamy Chris-Jerry distribution and its applications. *Journal of Xidian University*, *17*(6), 575-591.

[16] Obulezi, O. J., Anabike, I. C., Oyo, O. G., Igbokwe, C., & Etaga, H. (2023). Marshall-Olkin Chris-Jerry distribution and its applications. *International Journal of Innovative Science and Research Technology*, *8*(5), 522-533.

[17] Obulezi, O. J., Chidimma, N. N., Igbokwe, C. P., & Anabike, I. C. (2023). Statistical analysis on diagnosed cases of malaria and typhoid fever in Enugu-Nigeria. *GSJ*, *11*(6).

[18] Omoruyi, F. A., Omeje, I. L., Anabike, I. C., & Obulezi, O. J. (2023). A new variant of Rama distribution with simulation study and application to blood cancer data. *European Journal of Theoretical and Applied Sciences*, *1*(4), 389-409. `https://doi.org/10.59324/ejtas.2023.1(4).36`

[19] Onuoha, H. C., Osuji, G. A., Etaga, H. O., & Obulezi, O. J. (2023). The Weibull distribution with estimable shift parameter. *Earthline Journal of Mathematical Sciences*, *13*(1), 183-208. `https://doi.org/10.34198/ejms.13123.183208`

[20] Onyekwere, C. K., & Obulezi, O. J. (2022). Chris-Jerry distribution and its applications. *Asian Journal of Probability and Statistics*, *20*(1), 16-30. `https://doi.org/10.9734/ajpas/2022/v20i130480`

[21] Onyekwere, C. K., Okoro, C. N., Obulezi, O. J., Udofia, E. M., & Anabike, I. C. (2022). Modification of Shanker distribution using quadratic rank transmutation map. *Journal of Xidian University*, *16*(8), 179-198.

[22] Oramulu, D. O., Igbokwe, C. P., Anabike, I. C., Etaga, H. O., & Obulezi, O. J. (2023). Simulation study of the Bayesian and non-Bayesian estimation of a new lifetime distribution parameters with increasing hazard rate. *Asian Research Journal of Mathematics*, *19*(9), 183-211. `https://doi.org/10.9734/arjom/2023/v19i9711`

[23] Snow, R. W. (2015). Global malaria eradication and the importance of Plasmodium falciparum epidemiology in Africa. *BMC Medicine*, *13*(1), 1-3. `https://doi.org/10.1186/s12916-014-0254-7`

[24] Tolba, A. H., Onyekwere, C. K., El-Saeed, A. R., Alsadat, N., Alohali, H., & Obulezi, O. J. (2023). A new distribution for modeling data with increasing hazard rate: a case of COVID-19 pandemic and vinyl chloride data. *Sustainability*, *15*(17), 12782. `https://doi.org/10.3390/su151712782`

[25] Zekar, L., & Sharman, T. (2020). *Plasmodium falciparum malaria.* National Library of Medicine.