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A New Modification of Shanker Distribution with Applications to Increasing Failure Rate Data

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Abstract

In this paper, a new distribution is proposed by mixing the exponential distribution and the Shanker distribution with a mixing proportion being the same as those that yielded the Shanker distribution. The proposed distribution is referred to as the XShanker distribution. The distributional properties of the XShanker distribution namely, quantile function, moments and their associated measures,the mode, moment generating function, characteristic function, distribution of order statistics, and entropy are derived and studied. The reliability analysis shows that the failure rate is a strictly increasing function. The parameter of the model was estimated using the maximum likelihood function. We illustrated the usefulness of XShanker distribution using data on waiting times of 100 bank customers and vinyl chloride from clean ungradient ground-water monitoring wells in (g/L).

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1 Introduction

Since [2] proposed a two-component distribution called Lindley distribution using an exponential distribution with scale parameter θ and a gamma distribution having shape parameter 2 and scale parameter θ with mixing proportion $p = \frac{\theta}{\theta+1}$, the statistical literature has evolved with so many researches of this nature. [3] introduced a one-parameter distribution named Chris-Jerry distribution from a two-component mixture of the exponential distribution having a scale parameter θ and gamma distribution with shape parameter 3 and scale parameter θ with mixing proportion $p = \frac{\theta}{\theta+2}$. [1] derived a one-parameter distribution called Pranav distribution from two distributions namely exponential distribution with scale parameter θ and gamma distribution having shape parameter 4 and scale parameter θ. [11] introduced a two-parameter lifetime distribution named, 'Shukla distribution' which includes several one-parameter lifetime distributions. A new one-parameter lifetime distribution named Sujatha Distribution with an increasing hazard rate for modeling lifetime data was suggested by [10]. [9] studied a one-parameter lifetime distribution named Ishita distribution based on a two-component mixture of an Exponential distribution having a shape parameter θ and a Gamma distribution having a shape parameter 3 and scale parameter θ with mixing proportion $\frac{\theta^3}{\theta^3+2}$. [4] studied a one-parameter lifetime distribution named Akash distribution based on a two-component mixture of an exponential distribution having a shape parameter θ and a Gamma distribution having a shape parameter 2 and scale parameter θ with mixing proportion $\frac{\theta}{\theta+1}$. [8] studied a one-parameter lifetime distribution named Rani distribution based on a two-component mixture of an exponential distribution having a shape parameter θ and a gamma distribution having a shape parameter 5 and scale parameter θ with mixing proportion $\frac{\theta^5}{\theta^5+24}$. [7] studied a one-parameter lifetime distribution named Rama distribution based on a two-component mixture of an exponential distribution having a shape parameter θ and a gamma distribution having a shape parameter 4 and scale parameter θ with mixing proportion $\frac{\theta^3}{\theta^3+6}$. [6] studied a one-parameter lifetime distribution named Aradhana distribution based on a two-component mixture of an exponential distribution having a shape parameter θ and a gamma distribution having a shape parameter 2 and scale parameter θ with mixing proportion $\frac{1}{\theta+1}$. [5] studied a one-parameter lifetime distribution named Shanker based on a two-component mixture of an exponential distribution having a shape parameter θ and a gamma distribution having a shape parameter 2 and scale parameter θ with mixing proportion $\frac{\theta^2}{\theta^2+1}$.

In this paper, we suggest a one-parameter lifetime distribution and call it the XShanker distribution. The pdf is a mixture of two distributions namely exponential distribution and Shanker distribution having a scale parameter θ with a mixing proportion $p = \frac{\theta^2}{\theta^2+1}$. The mixture is of the form $f_{XShanker}(x,\theta) = pExp(x,\theta) + (1-p)Shanker(x,\theta)$.

Let $X \sim$ XShanker(θ), then the pdf and cdf are respectively

$$f(x) = \frac{\theta^2}{(\theta^2+1)^2}\left[\theta^3 + 2\theta + x\right] e^{-\theta x}; \quad x > 0, \quad \theta > 0 \tag{1}$$

and

$$F(x) = 1 - \left[1 + \frac{\theta x}{(\theta^2+1)^2}\right] e^{-\theta x} \tag{2}$$

The behaviour of the distribution is such that $\lim_{x\to 0} f(x) = \frac{\theta^5+2\theta^3}{\theta^4+2\theta^2+1}$, which is monotonic increasing for $\theta \geq 1$ and $\lim_{x\to\infty} f(x) = 0$ which is an extinction point of mortality rate.

The survival and hazard rate functions are respectively

$$S(x) = \left[1 + \frac{\theta x}{(\theta^2+1)^2}\right] e^{-\theta x} \tag{3}$$

and

$$hrf(x) = \frac{\theta^2\left(\theta^3 + 2\theta + x\right)}{\theta^4 + 2\theta^2 + \theta x + 1} \tag{4}$$

The limiting values of the XShanker hazard function are

$$\lim_{x\to 0} hrf(x) = \frac{\theta^5 + 2\theta^3}{\theta^4 + 2\theta^2 + 1} \quad \text{and} \quad \lim_{x\to\infty} hrf(x) = \theta.$$

Hence, the hazard function is monotonic increasing as $x \to 0$ for $\theta \geq 1$ and there is stability as $x \to \infty$.

The plots are displayed in the Figures 1, 2, 3, and 4 below

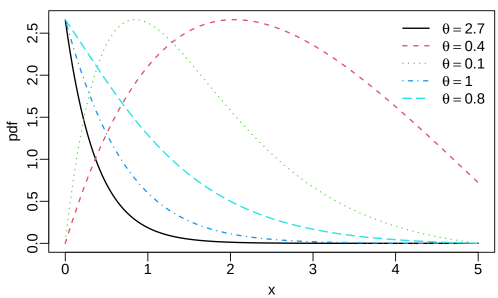

Figure 1: pdf of XShanker distribution.

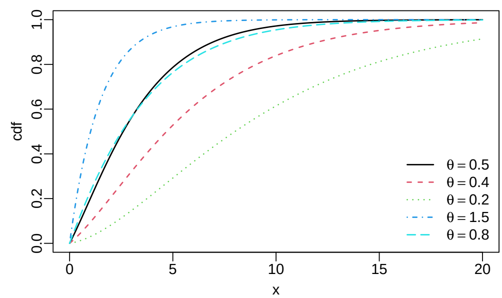

Figure 2: cdf of XShanker distribution.

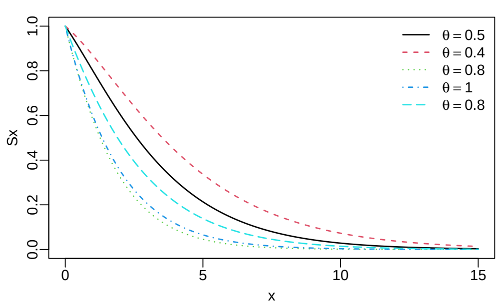

Figure 3: survival function of XShanker distribution.

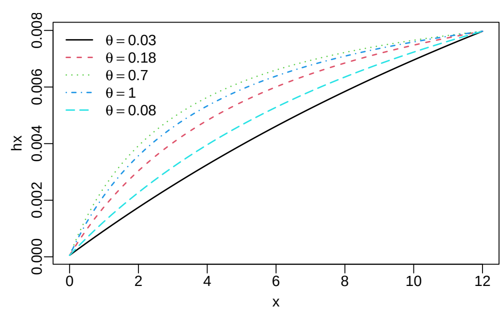

Figure 4: hazard function of XShanker distribution.

The XShanker distribution has a unique mode $X_{mode} = \frac{1-2\theta^2-\theta^4}{\theta}$

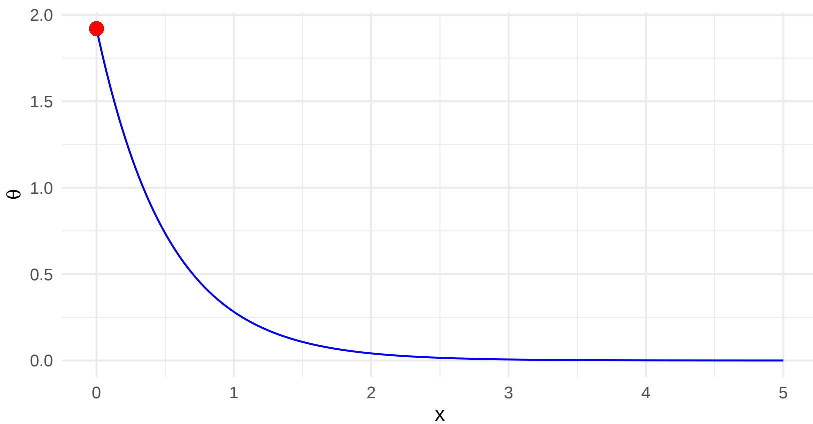

Figure 5: The graph of the Mode of XShanker distribution.

The remaining part of this article is in the following sequence; Section 2, discusses the basic distributional properties of the proposed distribution. Section 3 is on the maximum likelihood estimation of the parameter. In Section 4, application to real-life data is executed and the article is concluded in Section 5.

2 Distributional Properties of XShanker Distribution

In this section, we derive the basic distributional properties of the proposed model.

2.1 Quantile function

Let $X \sim$ XShanker (θ), suppose $u \sim U(0,1)$, we obtain by inverting the cdf of the XShanker distribution in eq 2 the quantile function as follows

$$\ln\left\{\frac{1-u}{(\theta^2+1)^2}\right\} = \ln\left(\theta^4+2\theta^2+1+\theta x_q\right) - \theta x_q \tag{5}$$

which is analytically not tractable and hence numerically implemented.

2.2 Moment

Let $X \sim$ XShanker (θ) with pdf in eq 1, the r^{th} crude moment is obtained as follows

$$\begin{aligned} \mu_r^{'} = E(X^r) = \int_0^\infty x^r f(x)dx &= \int_0^\infty x^r \left[\frac{\theta^2}{\left(\theta^2+1\right)^2}\left[\theta^3+2\theta+x\right]e^{-\theta x}\right]dx \\ &= \frac{\theta^2}{\left(\theta^2+1\right)^2}\left\{\left(\theta^3+2\theta\right)\int_0^\infty x^r e^{-\theta x}dx + \int_0^\infty x^{(r+1)}e^{-\theta x}dx\right\} \end{aligned} \tag{6}$$

Recall that Gamma function is defined as

$$\int_0^\infty x^{(r-1)}e^{-\lambda x}dx = \left(\frac{1}{\lambda}\right)^\alpha \Gamma_\alpha$$

hence

$$\begin{aligned} \mu_r^{'} &= \frac{\theta^2}{\left(\theta^3+1\right)^2}\left\{\left(\theta^3+2\theta\right)\int_0^\infty x^{(r+1)-1}e^{-\theta x}dx + \int_0^\infty x^{(r+2)-1}e^{-\theta x}dx\right\} \\ &= \frac{\theta^2}{\left(\theta^3+1\right)^2}\left\{\left(\theta^3+2\theta\right)\frac{1}{\theta^{(r+1)}}\Gamma_{(r+1)} + \frac{1}{\theta^{(r+2)}}\Gamma_{(r+2)}\right\} \end{aligned} \tag{7}$$

The first crude moment which is the mean is obtained by substituting $r = 1$ in eq 7. That is;

$$\mu = \frac{\left(\theta^4+2\theta^2+2\right)}{\theta\left(\theta^2+1\right)^2} \tag{8}$$

The second, third, and fourth crude moments are obtained by substituting $r = 2, 3 \quad \text{and} \quad 4$ in eq 7 which are respectively

$$\mu_2^{'} = \frac{\left(2\theta^3+4\theta^2+6\right)}{\theta^2\left(\theta^2+1\right)^2}; \quad \mu_3^{'} = \frac{\left(6\theta^4+12\theta^2+24\right)}{\theta^3\left(\theta^2+1\right)^2} \quad \text{and} \quad \mu_4^{'} = \frac{\left(24\theta^4+48\theta^2+120\right)}{\theta^4\left(\theta^2+1\right)^2} \tag{9}$$

To obtain the variance, recall from the basic statistical theory that $Var(x) = E(X^2) - (EX)^2$, but

$$E(X) = \frac{\left(\theta^4 + 2\theta^2 + 2\right)}{\theta\left(\theta^2+1\right)^2} \quad \text{and} \quad E(X^2) = \frac{\left(2\theta^3 + 4\theta^2 + 6\right)}{\theta^2\left(\theta^2+1\right)^2}$$
$$\text{therefore} \quad Var(x) = \frac{2\theta^7 + 4\theta^5 + 6\theta^4 + 2\theta^3 + 4\theta^2 + 2 - \theta^8}{\theta^2\left(\theta^2+1\right)^4} \tag{10}$$

Table 1: Mean (μ), Variance (σ^2), Coefficient of Variation (CV), Skewness and Kurtosis at various parameters (θ) and sample sizes (n).

n	θ	μ	σ^2	CV	Skewness	Kurtosis
25	0.1	82.73493	7214.78264	1.02665	2.01822	7.39351
	0.5	4.27394	19.25314	1.02665	2.01822	7.39351
	2	3.13102	10.33276	1.02665	2.01822	7.39351
	2.5	4.17576	18.37880	1.02665	2.01822	7.39351
50	0.1	106.10849	12744.83837	1.06394	2.97375	14.64380
	0.5	5.48137	34.01047	1.06394	2.97375	14.64380
	2	4.01556	18.25270	1.06394	2.97375	14.64380
	2.5	5.35546	32.46595	1.06394	2.97375	14.64380
75	0.1	99.70269	10108.38523	1.00840	2.95429	16.21497
	0.5	5.15046	26.97491	1.00840	2.95429	16.21497
	2	3.77314	14.47687	1.00840	2.95429	16.21497
	2.5	5.03215	25.74990	1.00840	2.95429	16.21497
100	0.1	98.16215	9505.39882	0.99321	2.79002	15.11934
	0.5	5.07088	25.36580	0.99321	2.79002	15.11934
	2	3.71484	13.61329	0.99321	2.79002	15.11934
	2.5	4.95440	24.21387	0.99321	2.79002	15.11934

2.3 Moment generating function

Let $X \sim$ XShanker (θ) with pdf in eq 1, the moment generating function is

$$\begin{aligned} M_X(t) = E(e^{xt}) &= \int_0^\infty e^{xt} f(x)dx \\ &= \frac{\theta^6 - \theta^5 t + 2\theta^4 - 2\theta^3 t + \theta^2}{\theta^6 - 2\theta^5 t + \theta^4 t^2 + 2\theta^4 - 4\theta^3 t + 2\theta^2 t^2 + \theta^2 - 2\theta t + t^2} \end{aligned} \tag{11}$$

2.4 Characteristic function

Let $X \sim$ XShanker (θ) with pdf in eq 1, the characteristic function is

$$\begin{aligned} \Phi_X(it) = E(e^{itx}) &= \int_0^\infty e^{itx} f(x)dx \\ &= \frac{\theta^6 - \theta^5 it + 2\theta^4 - 2\theta^3 it + \theta^2}{\theta^6 - 2\theta^5 it - \theta^4 t^2 + 2\theta^4 - 4\theta^3 it - 2\theta^2 t^2 + \theta^2 - 2\theta it - t^2} \end{aligned} \tag{12}$$

2.5 Entropy of XShanker distribution

Entropy is a measure of disorder in a system and the most popular is the Reny entropy which is defined and derived as follows

$$I_\omega(x;\theta) = \frac{1}{1-\omega} \log \int_0^\infty f^\omega(x)dx \tag{13}$$

Where $f(.)$ is defined in eq 1. hence;

$$\begin{aligned} I_\omega(x;\theta) &= \frac{1}{1-\omega} \log \frac{\theta^{2\omega}}{(\theta^2+1)^{2\omega}} \left[(\theta^3 + 2\theta + x)^\omega e^{-\theta\omega x} dx \right] \\ &= \frac{1}{1-\omega} \log \left\{ \frac{\theta^{2\omega}}{(\theta^2+1)^{2\omega}} \sum_{j=0}^{\omega} \binom{\omega}{j} (\theta^3 + 2\theta)^j \frac{\Gamma(\omega - j + 1)}{(\theta\omega)^{\omega - j + 1}} \right\} \end{aligned} \tag{14}$$

2.6 Distribution of the Order Statistics

Let $X_1, X_2, ..., X_n$ be a random sample of $X_{(r)}$; $\quad (r = 1, 2, ..n)$ and the $r^{(th)}$ order statistics obtained by arranging X_r in ascending order of magnitude from the XShanker distribution is

$$\begin{aligned} f_{r:n}(x;\theta) &= \binom{n}{r} f(x) \{F(x)\}^{r-1} \{1 - F(x)\}^{n-r} \\ &= \binom{n}{r} \frac{\theta^2}{(\theta^2+1)^2}(\theta^3 + 2\theta + x)e^{-\theta x} \left\{1 - \left[1 + \frac{\theta x}{(\theta^2+1)^2}\right] e^{-\theta x}\right\}^{r-1} \\ &\times \left\{1 + \left[\frac{\theta x}{(\theta^2+1)^2}\right] e^{-\theta x}\right\}^{n-r} \end{aligned} \tag{15}$$

The pdf of the largest order statistics is obtained when $r = n$ and that is;

$$f_{n:n}(x;\theta) = \frac{n\theta^2}{(\theta^2+1)^2}(\theta^3 + 2\theta + x)e^{-\theta x} \left\{1 - \left[1 + \frac{\theta x}{(\theta^2+1)^2}\right] e^{-\theta x}\right\}^{n-1} \tag{16}$$

The pdf of the smallest order statistics is obtained when $r = 1$ and it is

$$f_{1:n}(x;\theta) = \frac{n\theta^2}{(\theta^2+1)^2}(\theta^3 + 2\theta + x) \left\{1 + \frac{\theta x}{(\theta^2+1)^2}\right\}^{n-1} e^{-\theta n x} \tag{17}$$

3 Maximum Likelihood Estimation

Let $x_1, x_2, ..., x_n$ random samples of size n independently drawn from an XShanker distribution. The likelihood function is defined as

$$L(x_i;\theta) = \prod_{i=1}^{n} f(x_i, \theta) = \frac{\theta^{2n} e^{-\theta \sum_{i=1}^{n} x_i}}{(\theta^2+1)^{2n}} \prod_{i=1}^{n} (3 + 2\theta + x) \tag{18}$$

where $f(.)$ is defined in eq 1 The log-likelihood function becomes

$$\log L(x_i;\theta) = \phi = 2n \log\theta - 2n \log(\theta^2 + 1) - \theta \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \log(3 + 2\theta + x) \tag{19}$$

Differentiating with respect to θ gives the following non-linear equation to be implemented in R using **optim()** function

$$\frac{d\phi}{d\theta} = \frac{2n}{\theta} - \frac{4\theta n}{\theta^2+1} - \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{3\theta^2+2}{\theta^3+2\theta+x} \tag{20}$$

4 Applications

The first real-life data is on the waiting times of 100 bank customers studied by [12] in Table 2.

Table 2: Waiting time of 100 bank customers.

0.8	0.8	1.3	1.5	1.8	1.9	1.9	2.1	2.6	2.7	2.9	3.1
3.2	3.3	3.5	3.6	4	4.1	4.2	4.2	4.3	4.3	4.4	4.4
4.6	4.7	4.7	4.8	4.9	4.9	5	5.3	5.5	5.7	5.7	6.1
6.2	6.2	6.2	6.3	6.7	6.9	7.1	7.1	7.1	7.1	7.4	7.6
7.7	8	8.2	8.6	8.6	8.6	8.8	8.8	8.9	8.9	9.5	9.6
9.7	9.8	10.7	10.9	11	11	11.1	11.2	11.2	11.5	11.9	12.4
12.5	12.9	13	13.1	13.3	13.6	13.7	13.9	14.1	15.4	15.4	17.3
17.3	18.1	18.2	18.4	18.9	19	19.9	20.6	21.3	21.4	21.9	23
27	31.6	33.1	38.5								

The log-likelihood profile of the two data sets are

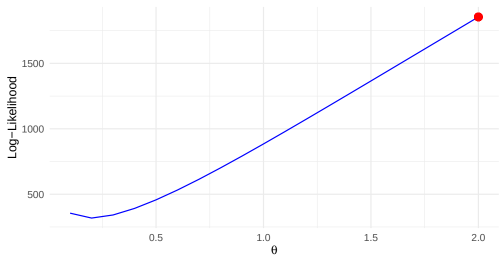

Figure 6: Log-likelihood Profile for the waiting time data.

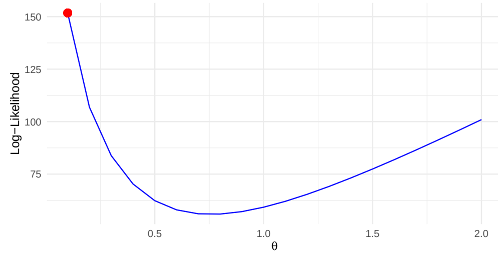

Figure 7: Log-likelihood Profile for the Vinyl Chloride data.

From Figure 6, the maximum value occurred at $\theta = 2.0$ for the waiting time data while in the case of the vinyl Chloride data in Figure 7, the maximum is way below $\theta = 0.5$. The analytical measures computed for the fitted distribution are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), HannanâQuinn information criterion (HQIC), Cramer von Mises (W^*), Anderson Darling (A^*), Kolmogorov-Smirnov (K-S) statistic and the p-value. Whereas the rest of the measures represent the model performance the K-S statistic and the p-value determine the fitness of the distribution to the data.

Table 3: Analytical Measures of Fitness and Performance for waiting time data.

Method	NLL	AIC	CAIC	BIC	HQIC	W*	A*	K-S	P-value	θ	Std Error
XShanker	317.98	637.963	638.004	640.569	639.018	0.035	0.226	0.049	0.970	0.195	0.013
Lindley	319.04	640.078	640.116	642.680	641.129	0.042	0.267	0.068	0.749	0.187	0.013
Ishita	321.85	645.700	645.740	648.305	646.754	0.036	0.235	0.109	0.186	0.302	0.017
Akash	320.96	643.929	643.970	646.534	644.984	0.053	0.341	0.100	0.267	0.296	0.017
Pranav	332.96	667.915	667.956	670.520	668.970	0.037	0.249	0.160	0.012	0.405	0.020
Chris-Jerry	320.34	642.686	642.727	645.291	643.741	0.089	0.570	0.075	0.628	0.279	0.017
Rama	331.61	665.224	665.265	667.829	666.279	0.049	0.331	0.158	0.014	0.402	0.020
XLindley	320.76	643.523	643.564	646.129	644.578	0.047	0.297	0.090	0.386	0.174	0.013

From Table 3 results, the proposed XShanker distribution is the best fit for the data on the waiting time of 100 bank customers. This is because the K-S statistic is the minimum while the p-value is the largest. Similarly, for the performance metrics namely NLL, AIC, CAIC, BIC, and HQIC, the values are minimum for the proposed distribution compared to those of other competing distributions.

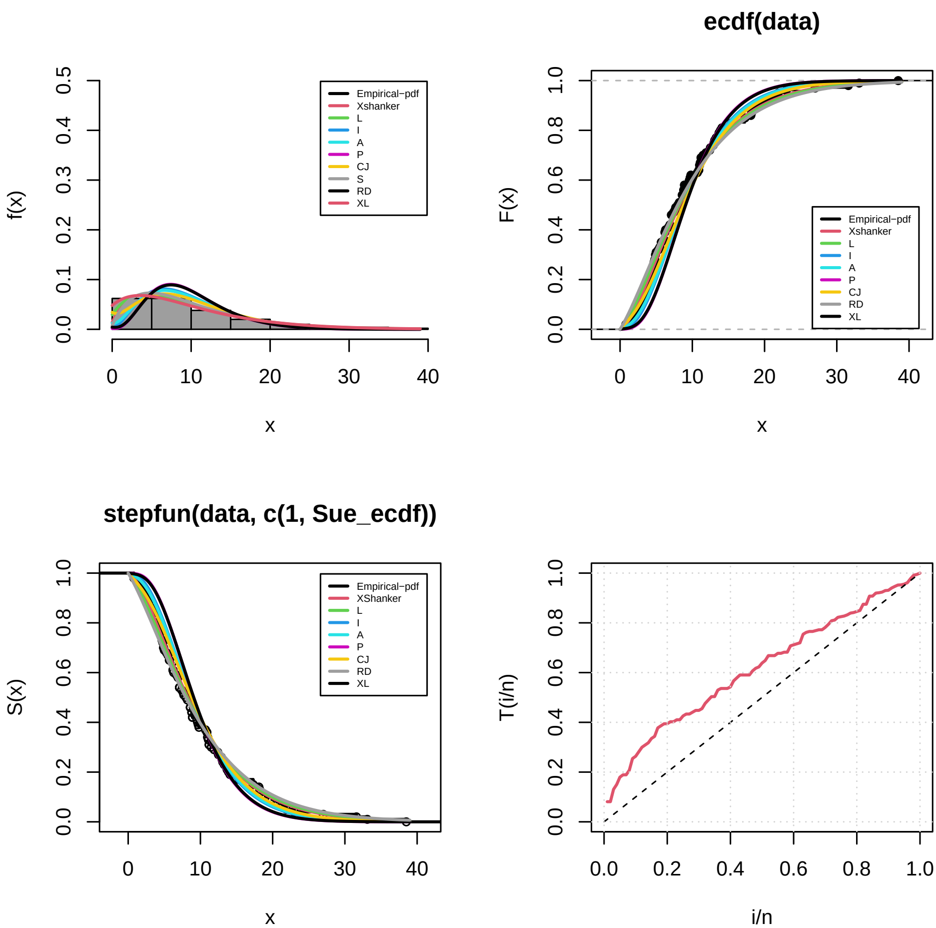

Figure 8: The density, CDF, empirical reliability, and TTT plots for the waiting time of 100 bank customer data.

Figure 8 reveals also that XShanker is best for modeling the waiting time of 100 bank customers' data.

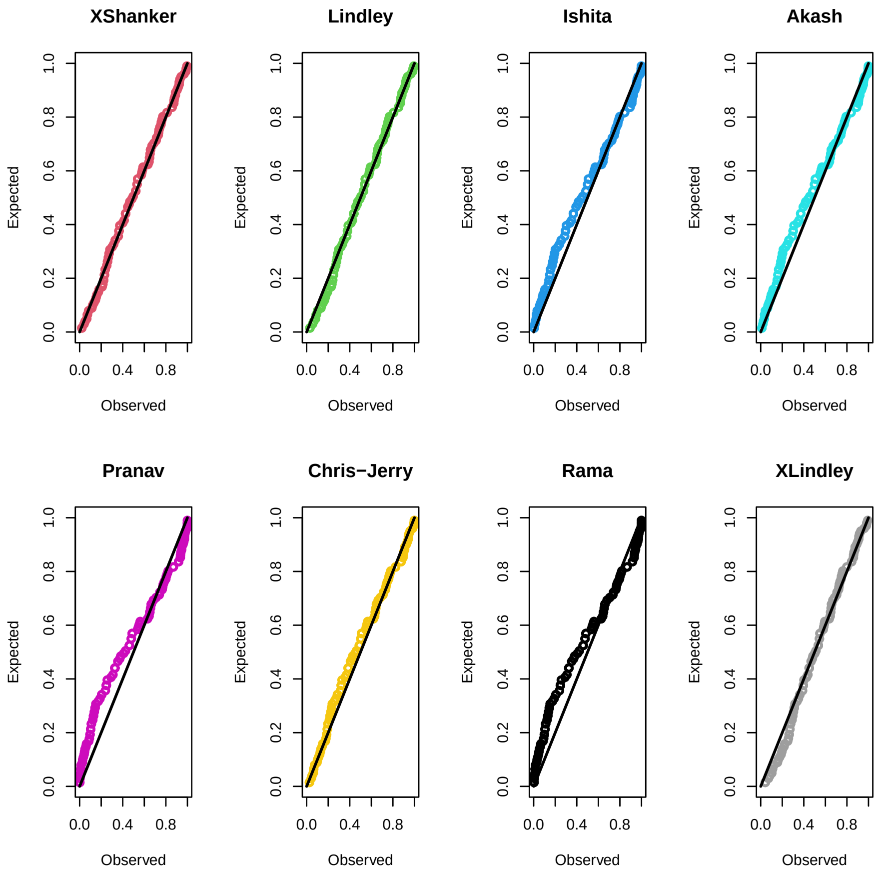

Figure 9: PP plots for the waiting time of 100 bank customer data.

The second data is on Vinyl chloride data from clean upgradient ground-water monitoring wells in (g/L) studied by [13] in Table 4. Again, the measures for fitness used are the negative Log-Likelihood (NLL), Akaike Information Criterion (AIC), Corrected AIC (CAIC), Bayesian Information Criterion (BIC), HannanâQuinn information criterion (HQIC), Cramer von mises (W^*), Anderson Darling (A^*), Kolmogorov-Smirnov (K-S) statistic and the p-value. Whereas the rest of the measures represent the model performance the K-S statistic and the p-value determine the fitness of the distribution to the data.

Table 4: Vinyl chloride data from clean upgradient ground-water monitoring wells in (g/L).

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8.0	0.8	0.4	0.6	0.9	0.4	2.0
0.5	5.3	3.2	2.7	2.9	2.5	2.3	1.0	0.2	0.1	0.1	1.8	0.9	2.0	4.0
6.8	1.2	0.4	0.2											

Table 5: Analytical Measures of Fitness and Performance for the Vinyl Chloride data.

Method	NLL	AIC	CAIC	BIC	HQIC	W*	A*	K-S	P-value	θ	Std Error
XShanker	55.85	113.693	113.818	115.219	114.213	0.057	0.369	0.111	0.798	0.757	0.083
Lindley	56.3	114.607	114.732	116.134	115.128	0.063	0.405	0.133	0.588	0.824	0.105
Ishita	57.3	116.606	116.731	118.132	117.126	0.095	0.604	0.141	0.514	1.157	0.096
Akash	57.57	117.149	117.274	118.676	117.670	0.099	0.630	0156	0.376	1.166	0.113
Pranav	58.34	118.672	118.797	120.198	119.192	0.136	0.847	0.146	0.461	1.466	0.098
Chris-Jerry	57.93	117.854	117.979	119.380	118.374	0.103	0.655	0.178	0.230	1.164	0.132
Shanker	56.46	114.913	115.038	116.440	115.433	0.064	0.413	0.131	0.607	0.853	0.095
Rama	59.34	120.683	120.808	122.210	121.204	0.154	0.952	0.177	0.238	1.531	0.118

From Table 5 results, the proposed XShanker distribution is the best fit for the data on Vinyl chloride data from clean upgradient ground-water monitoring wells in (g/L). This is because the K-S statistic is the minimum while the p-value is the largest. Similarly, for the performance metrics namely NLL, AIC, CAIC, BIC, and HQIC, the values are minimum for the proposed distribution compared to those of other competing distributions.

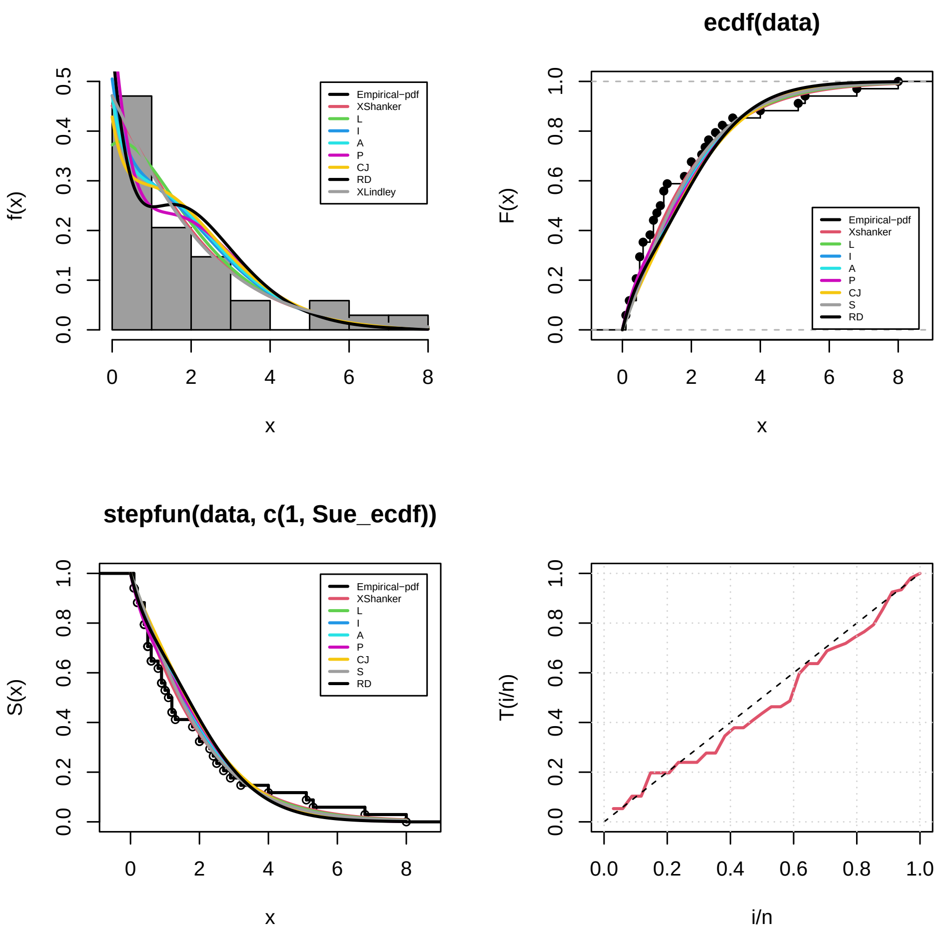

Figure 10: The density, CDF, empirical reliability, and TTT plots for the Vinyl Chloride data.

Figure 10 reveals also that XShanker is best for modeling the Vinyl chloride data from clean upgradient ground-water monitoring wells in (g/L) data.

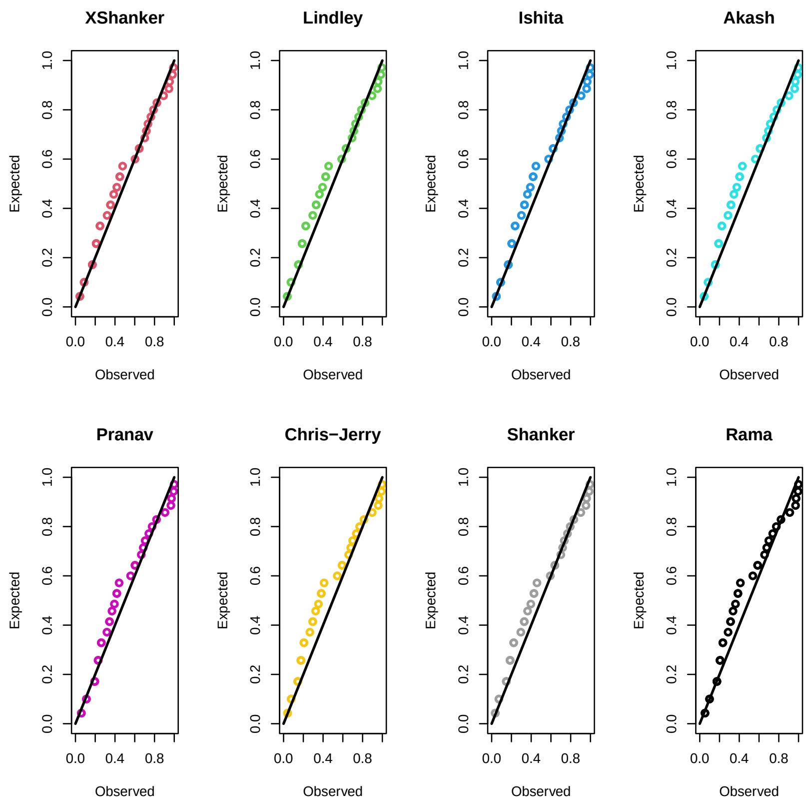

Figure 11: PP plots for the Vinyl Chloride data.

The P-P plots in Figure 11 show that the proposed XShanker distribution is best suited for the modeling of Vinyl chloride data from clean upgradient ground-water monitoring wells in (g/L).

5 Conclusion

In this article, a one-parameter XShanker distribution was proposed. The proposed distribution is better than the parent distribution Shanker in some scenarios. We derived some basic distributional properties which include the quantile function, moments and related measures, the variance, the mode, the moment generating function, the characteristic function, the distribution of order statistics, and reny entropy. The parameter of the model was estimated using the maximum likelihood estimation procedure. Two real data sets were used to explain the usability of the proposed distribution in the face of competition.

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