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Homotopy Perturbation Method for MHD Heat and Mass Transfer Flow of Convective Fluid through a Vertical Porous Plate in the Presence of Chemical Reaction and Inclined Magnetic Field

Liberty Ebiwareme[1,*], Kubugha Wilcox Bunonyo[2] and Onengiyeofori Anthony Davies[3]

[1] Department of Mathematics, Rivers State University, Port Harcourt, Nigeria

[2] Department of Mathematics and Statistics, Federal University, Otuoke, Nigeria

[3] Department of Physics, Rivers State University, Port Harcourt, Nigeria

Abstract

The present work is devoted to study a viscous, incompressible, and electrically conducting fluid on an MHD fluid flowing past a semi-infinite porous plate in the presence of chemical reaction and inclined magnetic parameter. The governing equations are expressed in non-dimensional form and the resulting nonlinear equations are solved employing the Homotopy perturbation method for the nondimensional velocity, temperature, and concentration profiles. The effects of various controlling parameters such as Casson parameter, Hartmann number, inclined magnetic parameter, porosity parameter, Grashof number, angle of inclination, Prandtl number, Eckert number, radiation parameter, Schmidt number and thermal radiation parameters are presented graphically and discussed in detail. It was found that, velocity profile is enhanced in the presence of Casson, magnetic field and inclined angle parameters whereas it declined with positive increase in the porosity, Grashof and inclined angle numbers. Similarly, increase in Prandtl, Eckert, radiation and inclined angle numbers lead to increase in the

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*Corresponding author

temperature distribution of the fluid, while it deceased as the magnetic field parameter increased. The effect of increased thermal radiation parameter is proportional to the concentration profile, whereas it declines for increase in values of Schmidt number.

1 Introduction

Convective heat and mass transfer flows over diverse influencing parameters in several media have caught the attention of scholars for some time now. From industrial point of view, these flows have great importance especially in the metallurgical industry, meteorology, cooling towers, spray drying of milk, burning of haystacks, fluidised-bed catalysis, respiratory gas exchange in lungs, polymer production, dispersion of fog [1].

Anghel et al. [2] have investigated the combined heat and mass transfer of free convection past an inclined flat plate using analytical technique. Heat and mass transfer in magnetohydrodynamic flow of natural convection through a permeable inclined surface influenced by variable wall temperature and concentration has been explored by Chen [3]. Utilising Lie group analysis, Sivasankaran et al. [4] examined the natural convection heat and mass transfer flow in an inclined surface. Based on the Adomian decomposition method, Ebiwareme et al. [5] conducted theoretically a heat transfer analysis of an MHD flow through an infinite vertical porous plate under the influence of suction. Ebiwareme et al. [6] studied an MHD Casson fluid flow past an inclined semi-infinite porous plate in the presence of magnetic field and radiation absorption for analytical solution using Adomian decomposition method (ADM). The result in this study showed the control flow parameters have significant influence on the nondimensional velocity, temperature, and concentration gradients of the fluid. Noor et al. [7] analysed the heat and mass transfer of thermophoretic MHD flow over an inclined radiate isothermal permeable surface in the presence of heat source/sink. A numerical study of an unsteady free convective magnetohydrodynamic flow of a dissipative fluid along a vertical plate subjected to a constant heat flux have been examined by Jordán [8]. Mass transfer effects on a mixed convective flow past a heated vertical flat permeable plate with thermophoresis was investigated numerically using RKFM by Selim [9]. Sandeep and Sugunamma [10] researched on the impact of inclined magnetic field on unsteady free convective flow of dissipative fluid through a vertical plate. Investigative solution on the consequences of chemical reaction and radiation on unsteady MHD free convective flow and mass transfer through viscous incompressible fluid past a heated vertical plate immersed in porous medium in the presence of heat source has been analysed by Sharma et al. [11]. Mahaptra et al. [12] have discussed the chemical reaction impact on free convection through a porous medium bounded by a vertical surface. Combined effects on Casson MHD fluid flow over a vertical plate with heat source/sink in the presence of chemical reaction, radiation and Dufour have been studied analytically by Vedavathi et al. [13]. Ramaprasad et al. [14] have examined an unsteady MHD convective heat and mass transfer flow past an inclined moving surface with heat absorption. Similarly, Srinivas Reddy [15] have explored the effect of chemical reaction on MHD free convection heat and mass transfer from vertical surfaces in porous media considering thermal diffusion and thermo diffusion effects.

In recent times, many scientific problems encountered in science and engineering are strongly nonlinear in nature, hence are not amenable to traditional mathematical techniques for analytical solution. Due to simplicity and growing interest, semi-analytical or semi-exact methods have been developed and deployed to solve these problems for approximate analytical solution. These methods have strong appeal among the academia because of their ease of implementation and degree of accuracy even when only very few terms are needed to obtain the solution. Some of these innovative methods includes: Adomian decomposition method (ADM), variational iteration method (VIM), Differential transformation method (DTM), Differential Quadrature method (DQM), Homotopy perturbation method (HPM), Homotopy analysis method (HAM), Abkari-Ganji method (AGM), Optimal Homotopy asymptotic method (OHAM), Chebyshev Wavelet method (CWM), Hermite Wavelet method (HWM), Legendre-Wavelet method

(LWM), Petro-Galerkin method and others. These methods have been applied to a great effect in solving diverse problems [16-19]

The homotopy perturbation method (HPM) was first developed by Ji-Huan He in 1999 [20-24]. Using this method, the required solution is considered as the sum of an infinite series which converges rapidly to the accurate solution. The main advantages of homotopy perturbation method (HPM) over other semi-analytical methods is that it obtains the exact solutions with higher accuracy, minimal calculations without loss of physical verification, gives the solution by using initial conditions only and solves nonlinear problems without using Adomian polynomials as is the case with the Adomian decomposition method for nonlinear terms. This method has found application in different fields of nonlinear science and have been favorably applied to solve different problems. Many authors and researchers studied the homotopy perturbation method and used it for solving nonlinear ordinary differential equations, solved the Nonlinear ordinary differential equations with nth order, the oscillators equation with discontinuities, one dimensional nonlinear wave equation, physical models, chemical ion transport through the soil, Dengue disease model, nonlinear partial differential equation, MHD Jefferey-Hamel problem, heat transfer analysis for Squeezing flow between parallel disk and unsteady squeezing nanofluid flow problem [25-34].

In this paper, we have successfully employed the homotopy perturbation method to explore an MHD fluid flow past a semi-infinite porous plate under the influence of chemical reaction and inclined magnetic field which is an extension of earlier work by Ebiwareme et al. [6]. The dimensionless flow gradients are obtained, and the result displayed in graphical form and discussed quantitatively. The study is arranged as follows: In the next section, governing partial differential equation that characterize the flow are presented. In Section 3, solution of the problem in the form of nondimensional profiles under the influence of pertinent parameters are obtained using the solution technique. The discussion of results in graphical form and their accompanying discussion for the influence of different parameters is given in Section 4. Major findings of the study are summarized in itemized form in Section 5 and finally Section 6 draw the conclusion of the study.

2 Governing Equation

$$\frac{\partial v^*}{\partial y^*} = 0, \quad \Rightarrow v^* = -V_0 > 0 \tag{1}$$

$$v^*\frac{\partial u^*}{\partial y^*} = v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_\infty\right) - \frac{\sigma B_0^2}{\rho}\sin^2\gamma u^* - \frac{vu^*}{K^*} \tag{2}$$

$$v^*\frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_\rho}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{v}{C_\rho}\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{\sigma B_0^2}{\rho}u^{*2} + \frac{Q_0}{\rho C_\rho}\left(T^* - T_\infty\right) + \frac{R^*}{\rho C_\rho}\left(C^* - C_\infty\right) \tag{3}$$

$$v^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} - K_1\left(C^* - C_\infty\right) \tag{4}$$

The boundary conditions are as follows:

$$\begin{aligned} &u^* = 0, \quad C^* = C_w, \quad T^* = T_w \quad \text{at } y^* = 0 \\ &u^* \to 0, \quad C^* \to C_\infty, \quad T^* \to T_\infty \quad \text{as } y^* \to \infty \end{aligned} \tag{5}$$

Using the following non-dimensional numbers:

$$u = \frac{u^*}{v_0}, y = \frac{v_0 y^*}{v}, \mathrm{Pr} = \frac{v_\rho C_\rho}{\kappa}, \theta = \frac{T^* - T_\infty}{T_0 - T_\infty}, \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, Gr = \frac{vg\beta\left(T_w - T_\infty\right)}{v_0^3} \tag{6}$$

Putting Eq. (6) into Eqs. (1-4), we obtain the following simplified non-dimensional equations as follows:

$$\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M^2\sin^2\gamma + K_0\right)u + Gr\cos\alpha\theta = 0 \tag{7}$$

$$\frac{\partial^2\theta}{\partial y^2} + \mathrm{Pr}\frac{\partial\theta}{\partial t} + \mathrm{Pr}\,Ec\left(\frac{\partial u}{\partial y}\right)^2 + \mathrm{Pr}\,EcM^2\sin^2\gamma u^2 + R\varphi = 0 \tag{8}$$

$$\mathrm{Sc}\frac{\partial^2\varphi}{\partial y^2} + \frac{\partial\varphi}{\partial t} - Kr\varphi = 0 \tag{9}$$

The resulting boundary conditions are

$$\begin{aligned} &u = 0, \quad \theta = 1, \quad \varphi = 1, \text{ at } y = 0 \\ &u \to 0, \quad \theta \to 1, \quad \varphi \to 1 \text{ as } y \to \infty \end{aligned} \tag{10}$$

3 Fundamentals of Homotopy Perturbation Method (HPM)

In this section, we illustrate the basic principle of the Homotopy perturbation method as expounded by He [20-26]. For this, we consider a functional differential equation of the form

$$A(u) - f(r) = 0, \quad r \in \Omega \tag{11}$$

subject to the boundary condition

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \mathcal{T} \tag{12}$$

Here A represents a general differential operator, B is a boundary operator, $u(x,t)$ is an unknown function, $\mathcal{T}$ is the boundary of the domain Ω, $f(x,t)$ is a known analytic function and $\frac{\partial}{\partial n}$ denotes the differentiation along the normal vector drawn outwards from Ω.

Decomposing the differential operator into two parts comprising linear, (L) and nonlinear (N) respectively. Therefore, we rewrite Eq. (11) in the form

$$L(u) + N(u) - f(r) = 0 \tag{13}$$

Embedding an artificial parameter p on Eq. (13) as follows

$$L(u) + p(N(u) - f(r)) = 0, \tag{14}$$

where $p \in [0,1]$ is the embedding also called artificial parameter.

By the standard Homotopy procedure proposed by He [20-26], we construct a Homotopy of the form $H(r,p) : \Omega \times [0,1] \to \Re$ to Eq. (14) that satisfies

$$H(v,p) = (1-p)\left[L(v) - L\left(u_0\right)\right] + p[L(v) + N(v) - f(r)] = 0 \tag{15}$$

which is equivalent to

$$H(v,p) = L(v) - L\left(u_0\right) + pL\left(u_0\right) + p[N(v) - f(r)] = 0 \tag{16}$$

where $u_0(x,t)$ is the initial approximation which satisfies the boundary condition of Eq. (12)

Substituting $p=0$ and $p=1$ into Eq. (13), we obtain the following equations

$$H(v,0)=L(v)-L\left(u_0\right), H(v,1)=A(v)-f(r) \tag{17}$$

The changing process of p monotonically from zero to unity is that of $H(v,p)$ from $L(v)-L\left(u_0\right)$ to $A(v)-f(r)$. This is called deformation in topology, whereas the terms $L(v)-L\left(u_0\right)$ and $A(u)-f(r)$ are homotopic to each other.

Since $p\in[0,1]$ is a small parameter, we consider the solution of Eq. (16) as power series of p follows

$$v=\sum_{n=0}^{\infty} p^{(n)} v_n=v_0+p v_1+p^2 v_2+\cdots \tag{18}$$

The approximate solution of Eq. (18) can be obtained by setting $p=1$

$$u(x,t)=\lim_{p\rightarrow 1} v_n=v_0+v_1+v_2+\cdots \tag{19}$$

Similarly, the nonlinear term, $N(u)$ can be expressed in He's polynomial [27]

$$N(u)=\sum_{n=0}^{\infty} p^{(m)} H_m\left(v_0+v_1+\cdots+v_m\right) \tag{20}$$

where

$$H_m\left(v_0+v_1+\cdots+v_m\right)=\frac{1}{m!}\frac{\partial^m}{\partial p^m}\left[\mathcal{N}\left(\sum_{k=0}^{m} p^k v_k\right)\right]_{p=0}, m=0,1,2,\ldots \tag{21}$$

$$H_0=N\left(u_0\right)$$

$$H_1=u_1 N'\left(u_0\right)$$

$$H_2=u_2 N'\left(u_0\right)+\tfrac{1}{2} N_1^2 N''\left(u_0\right)$$

$$H_3=u_3 N'\left(u_0\right)+u_1 u_2 N''\left(u_0\right)+\tfrac{1}{6} N_1^3 N'''\left(u_0\right)$$

$$H_4=u_4 N'\left(u_0\right)+\left(\tfrac{1}{2} u_2^2+u_1 u_3\right) N''\left(u_0\right)+\tfrac{1}{2} u_1^2 u_2 N_1^3 N'''\left(u_0\right)+\tfrac{1}{24} u_4^3 N^{(iv)}\left(u_0\right)$$

4 Solution Procedure using HPM

By the so-called HPM procedure, we construct the following homotopy

$$H_1(u,p) : \phi \times [0,1] \to R$$
$$H_2(\theta,p) : v \times [0,1] \to R$$
$$H_3(\varphi,p) : \omega \times [0,1] \to R$$

Now, the above is equivalent to the expression below

$$\begin{aligned} H_1(u,p) =&(1-p)\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2}\right) \\ &+ p\left[\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M^2 \sin^2\gamma + K_0\right)u + Gr\cos\alpha\theta\right] \end{aligned} \tag{22}$$

$$\begin{aligned} H_2(\theta,p) =&(1-p)\left(\frac{\partial^2 \theta}{\partial y^2} - \frac{\partial^2 \theta_0}{\partial y^2}\right) \\ &+ p\left[\frac{\partial^2 \theta}{\partial y^2} + \Pr\frac{\partial \theta}{\partial y} + \Pr Ec\left(\frac{\partial u}{\partial y}\right)^2 + \Pr EcM^2\sin^2\gamma u^2 + R\varphi\right] \end{aligned} \tag{23}$$

$$H_3(\varphi,p) = (1-p)\left(\frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \varphi_0}{\partial y^2}\right) + p\left[\mathrm{Sc}\,\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial \varphi}{\partial t} - \mathrm{Kr}\,\varphi\right] \tag{24}$$

By letting $H_1(u,p) = H_2(u,p) = H_3(u,p) = 0$, we have the equivalent equations of the form

$$\begin{aligned} &(1-p)\left(\frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u_0}{\partial y^2}\right) \\ &+ p\left[\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \left(M^2 \sin^2\gamma + K_0\right)u + Gr\cos\alpha\theta\right] = 0 \end{aligned} \tag{25}$$

$$\begin{aligned} &(1-p)\left(\frac{\partial^2 \theta}{\partial y^2} - \frac{\partial^2 \theta_0}{\partial y^2}\right) \\ &+ p\left[\frac{\partial^2 \theta}{\partial y^2} + \Pr\frac{\partial \theta}{\partial y} + \Pr Ec\left(\frac{\partial u}{\partial y}\right)^2\right. \\ &\left. + \Pr EcM^2\sin^2\gamma u^2 + R\varphi\right] = 0 \end{aligned} \tag{26}$$

$$(1-p)\left(\frac{\partial^2\varphi}{\partial y^2}-\frac{\partial^2\varphi_0}{\partial y^2}\right)+p\left[\mathrm{Sc}\,\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial\varphi}{\partial t}-Kr\varphi\right]=0 \tag{27}$$

Assuming the solutions of Eqs. (25-27) in power series of the form, we get

$$\begin{aligned}
u(y,t)&=\sum_{p\to1}^{\infty}p^n u_n(y,t)=u_0+pu_1+p^2u_2+\cdots\\
\theta(y,t)&=\sum_{p\to1}^{\infty}p^n \theta_n(y,t)=\theta_0+p\theta_1+p^2\theta_2+\cdots\\
\varphi(y,t)&=\sum_{p\to1}^{\infty}p^n \varphi_n(y,t)=\varphi_0+p\varphi_1+p^2\varphi_2+\cdots
\end{aligned} \tag{28}$$

Putting Eq. (12) into Eqs. (12-14), we obtain the following system in the powers of the perturbation parameter.

$$p^0:\frac{\partial^2 u}{\partial y^2}=0,\quad \frac{\partial^2\theta}{\partial y^2}=0,\quad \frac{\partial^2\varphi}{\partial y^2}=0 \tag{29}$$

The corresponding boundary conditions are given as

$$\begin{aligned}
&u_0(0)=0,\quad \theta_0(0)=1,\quad \varphi_0(0)=1\\
&u_0(\infty)\to0,\quad \theta_0(\infty)\to1,\quad \varphi_0(\infty)\to1
\end{aligned} \tag{30}$$

$$p^1:\frac{\partial^2 u_1}{\partial y^2}+\frac{1}{\beta}\frac{\partial^2 u_0}{\partial y^2}+\frac{\partial u_0}{\partial y}-\left(M^2\sin^2\gamma+K_0\right)u_0+\mathrm{Gr}\cos\alpha\theta_0=0 \tag{31}$$

$$\frac{\partial^2\theta_1}{\partial y^2}+\Pr\frac{\partial\theta_0}{\partial t}+\Pr Ecu_0'+\Pr EcM^2\sin\gamma u_0'+R\varphi_0=0 \tag{32}$$

$$\frac{\partial^2\varphi_1}{\partial y^2}+\frac{\partial\varphi_0}{\partial y}-Kr\varphi_0=0 \tag{33}$$

subject to the boundary conditions

$$\begin{aligned}
&u_1(0)=0,\quad \theta_1(0)=1,\quad \varphi_1(0)=1\\
&u_1(\infty)\to0,\quad \theta_1(\infty)\to1,\quad \varphi_1(\infty)\to1
\end{aligned} \tag{34}$$

$$p^2:\frac{\partial^2 u_2}{\partial y^2}+\frac{1}{\beta}\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial u_1}{\partial y}-\left(M^2\sin^2\gamma+K_0\right)u_1+\mathrm{Gr}\cos\alpha\theta_1=0 \tag{35}$$

$$\frac{\partial^2\theta_2}{\partial y^2} + \Pr\frac{\partial\theta_1}{\partial t} + \Pr Ecu_1' + \Pr EcM^2\sin\gamma u_1' + R\varphi_1 = 0 \tag{36}$$

$$\frac{\partial^2\varphi_2}{\partial y^2} + \frac{\partial\varphi_1}{\partial y} - \mathrm{Kr}\,\varphi_1 = 0 \tag{37}$$

subject to the appropriate boundary conditions

$$\begin{aligned} &u_2(0) = 0, \quad \theta_2(0) = 1, \quad \varphi_2(0) = 1 \\ &u_2(\infty) \to 0, \quad \theta_2(\infty) \to 1, \quad \varphi_2(\infty) \to 1 \end{aligned} \tag{38}$$

Solving Eqs. (29), (31-33) and (35-37) subject the boundary conditions in Eqs. (30), (34) and (38), the approximate solutions for the different orders become

$$u_0(y) = \alpha_1 y, \quad \theta_0(y,t) = 1 + \alpha_2 y, \quad \varphi_0(y,t) = 1 + \alpha_3 y \tag{39}$$

$$u_1(y) = y^2\left(\frac{\mathrm{CosGr}\,\alpha}{2} - \frac{\alpha_1}{2}\right) + y^3\left(-\frac{\mathrm{Ko}\,\alpha_1}{3} - \frac{1}{3}M^2\,\mathrm{Sin}^2\,\gamma\alpha_1 + \frac{1}{3}\,\mathrm{CosGr}\,\alpha\alpha_2\right) \tag{40}$$

$$\theta_1(y,t) = \frac{1}{4}\mathrm{Ec}M^2\,\mathrm{PrSin}^2\,y^4\gamma\alpha_1^2 + y^2\left(-\frac{R}{2} - \frac{1}{2}\,\mathrm{EcPr}\,\alpha_1^2\right) - \frac{1}{3}Ry^3\alpha_3 \tag{41}$$

$$\varphi_1(y,t) = \frac{\mathrm{Kr}y^2}{2\mathrm{Sc}} + \frac{\mathrm{Kr}^3\alpha_3}{3\mathrm{Sc}} \tag{42}$$

$$\begin{aligned} u_2(y) = &-\frac{1}{24}\,\mathrm{CosEcGrM}\,M^2\,\mathrm{PrSin}^2\,y^6\alpha\gamma\alpha_1^2 + y^2\left(-\frac{\mathrm{CosGr}\,\alpha}{2\beta} + \frac{\alpha_1}{2\beta}\right) \\ &+ y^4\left(-\frac{1}{8}\,\mathrm{CosGrKo}\,\alpha - \frac{1}{8}\,\mathrm{CosGr}\,R\alpha - \frac{1}{8}\,\mathrm{CosGr}\,M^2\,\mathrm{Sin}^2\,\alpha\gamma + \frac{3\,\mathrm{Ko}_1}{8}\right. \\ &\quad \left. + \frac{3}{8}M^2\,\mathrm{Sin}^2\,\gamma\alpha_1 - \frac{1}{8}\,\mathrm{CosEcGrPr}\,\alpha\alpha_1^2 - \frac{1}{4}\,\mathrm{CosGr}\,\alpha\alpha_2\right) \\ &+ y^3\left(-\frac{1}{3}\,\mathrm{CosGr}\,\alpha + \frac{\alpha_1}{3} + \frac{2\,\mathrm{Ko}_1}{3\beta} + \frac{2M^2\,\mathrm{Sin}^2\,\gamma\alpha_1}{3\beta} - \frac{2\,\mathrm{Cos}\,Gr\alpha\alpha_2}{3\beta}\right) \\ &+ y^5\left(\frac{\mathrm{Ko}^2\alpha_1}{15} + \frac{2}{15}\,\mathrm{KoM}^2\,\mathrm{Sin}^2\,\gamma\alpha_1 + \frac{1}{15}M^4\,\mathrm{Sin}^4\,\gamma^2\alpha_1\right. \\ &\quad \left. - \frac{1}{15}\,\mathrm{CosGrKo}\,\alpha\alpha_2 - \frac{1}{15}\,\mathrm{CosGr}\,M^2\,\mathrm{Sin}^2\,\alpha\gamma\alpha_2 - \frac{1}{15}\,\mathrm{CosGr}\,R\alpha\alpha_3\right) \end{aligned} \tag{43}$$

$$\theta_2(y,t) = -\frac{y^4\left(\mathrm{KrR} + 2\,\mathrm{Cos}^2\,\mathrm{EcGr}^2\,\mathrm{PrSc}\,\alpha^2 + 2\mathrm{EcPrSc}\alpha_1\left(-2\,\mathrm{CosGr}\,\alpha + \alpha_1\right)\right)}{8\mathrm{Sc}}$$
$$+\frac{1}{21}\mathrm{EcM}^2\,\mathrm{PrSin}^2\,y^7\gamma\left(\mathrm{CosGr}\,\alpha - \alpha_1\right)\left((\mathrm{Ko} + M^2\,\mathrm{Sin}^2\,\gamma)\alpha_1 - \mathrm{CosGr}\,\alpha\alpha_2\right)$$
$$-\frac{1}{72}\,\mathrm{Ec}^2\,\mathrm{PrSin}^2\,y^8\gamma\left(\left(\mathrm{Ko} + M^2\,\mathrm{Sin}^2\,\gamma\right)\alpha_1 - \mathrm{CosGr}\,\alpha\alpha_2\right)^2$$
$$-\frac{1}{24}\,\mathrm{EcPr}^6\left(\left(M^2\,\mathrm{Sin}^2\,\gamma + 4\left(\mathrm{Ko} + M^2\,\mathrm{Sin}^2\,\gamma\right)^2\right)\alpha_1^2\right.$$
$$-2\,\mathrm{CosGr}\,\alpha\alpha_1\left(M^2\,\mathrm{Sin}^2\,\gamma + 4\left(\mathrm{Ko}^2 + M^2\,\mathrm{Sin}^2\,\gamma\right)\alpha_2\right) +$$
$$\left.\mathrm{Cos}^2\,\mathrm{Gr}^2\alpha^2\left(M^2\,\mathrm{Sin}^2\,\gamma + 4\alpha_2^2\right)\right)$$
$$-\frac{1}{15\mathrm{Sc}}y^5\left(6\mathrm{Ec}\,\mathrm{PrSc}\left(\mathrm{CosGr}\,\alpha - \alpha_1\right)\left(-\left(\left(\mathrm{Ko} + M^2\,\mathrm{Sin}^2\,\gamma\right)\alpha_1\right)\right.\right.$$
$$\left. + \mathrm{CosGr}\,\alpha\alpha_2\right) + \mathrm{Kr}\,R\alpha_3 \tag{44}$$

$$\varphi_2(y,t) = \frac{\mathrm{Kr}^2 y^4}{8\mathrm{Sc}^2} + \frac{\mathrm{Kr}^2 y^5\alpha_3}{15\mathrm{Sc}^2} \tag{45}$$

The terms, $u_n(y,t), \theta_n(y,t)$ and $\varphi_n(y,t)$ when $n \geq 3$ are too large to be mentioned graphically. The three-term solution of Eq. (28), when $p \to 1$ is expressed as follows:

$$\begin{aligned} u(y,t) &= u_0(y,t) + u_1(y,t) + u_2(y,t) + \cdots \\ \theta(y,t) &= \theta_0(y,t) + \theta_1(y,t) + \theta_2(y,t) + \cdots \\ \varphi(y,t) &= \varphi_0(y,t) + \varphi_1(y,t) + \varphi_2(y,t) + \cdots \end{aligned} \tag{46}$$

5 Results and Discussions

In this work, we have successfully applied the Homotopy perturbation method (HPM) to obtain approximate analytical solution to the problem of magnetohydrodynamics fluid flowing past a vertical porous plate in the presence of inclined magnetic field and chemical reaction for various values of the controlling parameters for the velocity, temperature and concentration profiles are obtained. The approximate analytical solution obtained for the dimensionless velocity, temperature and concentration using HPM are compared with those in literature using MATLAB bvp4c routine. The result showed excellent agreement. This confirmed the proposed solution technique is efficient, accurate and feasible to solve variety of nonlinear differential equations. The influence of the flow characteristics on velocity, temperature and concentration are analyzed graphically, displayed in Figures 1-13 and discussed.

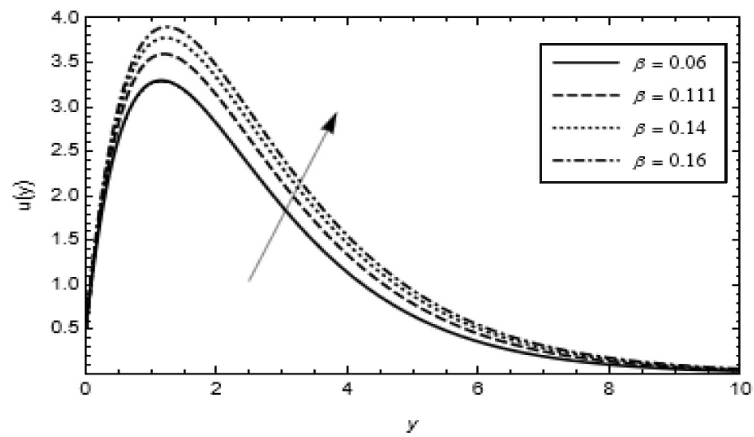

Figure 1: Velocity profile for variation in the Casson Parameter when $M = 2$, $Ko = 0.15, Gr = 5, \alpha = \gamma = \pi/3, Pr = 0.71,\ Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

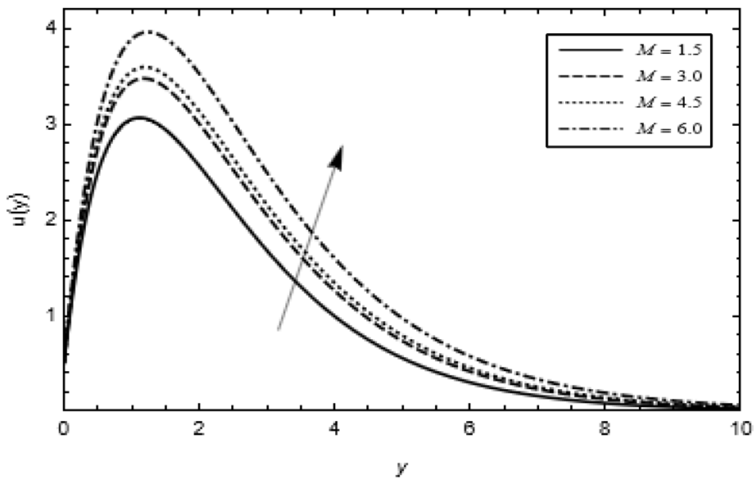

Figure 2: Velocity profile for variation Hartmann in Parameter when $Ko = 0.15$, $Gr = 5, \alpha = \gamma = \pi/3, \beta = 4.5, Pr = 0.71,\ Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

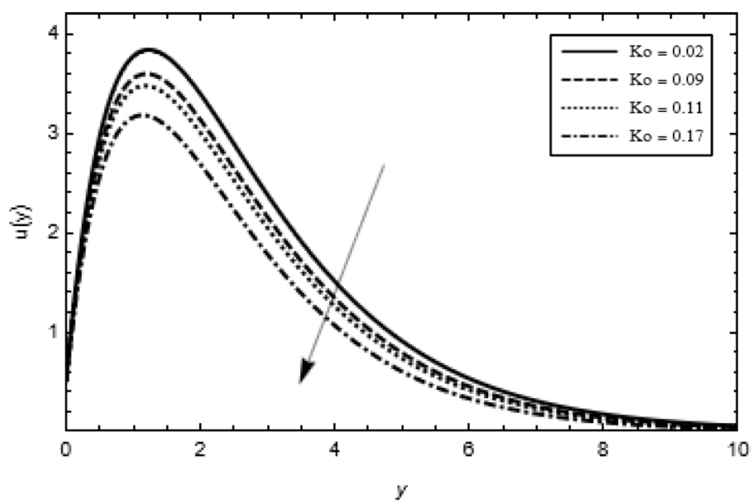

Figure 3: Velocity profile for variation in porosity Parameter when $M = 2, Gr = 5, \alpha = \gamma = \pi/3, \beta = 4.5, Pr = 0.71, Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

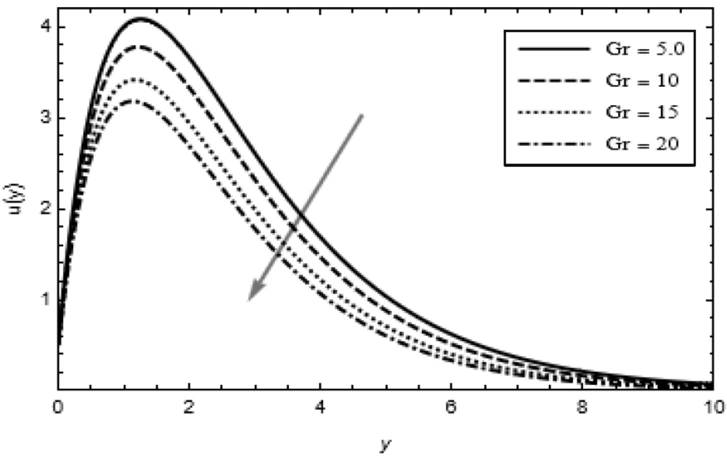

Figure 4: Velocity profile for variation in Grashof Parameter when $M = 2, Ko = 0.15, \alpha = \gamma = \pi/3, \beta = 4.5, Pr = 0.71,\ Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

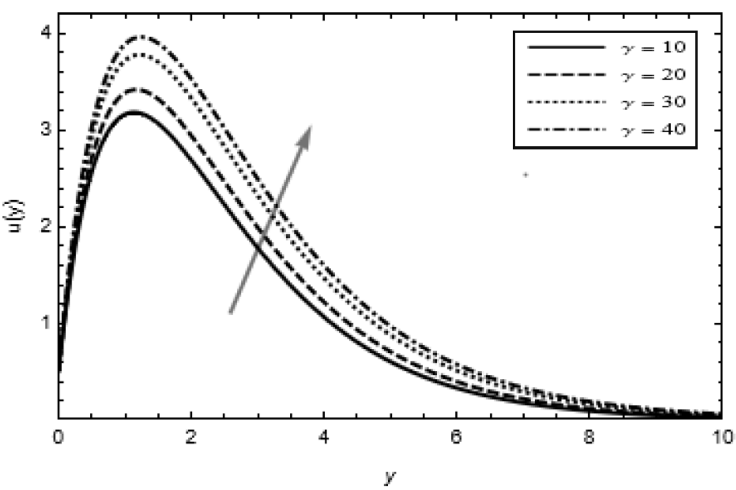

Figure 5: Velocity profile for variation in inclined magnetic Parameter when $M = 2,\ Ko = 0.15, Gr = 5, \alpha = \pi/3, \beta = 4.5,\ \Pr = 0.71, Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

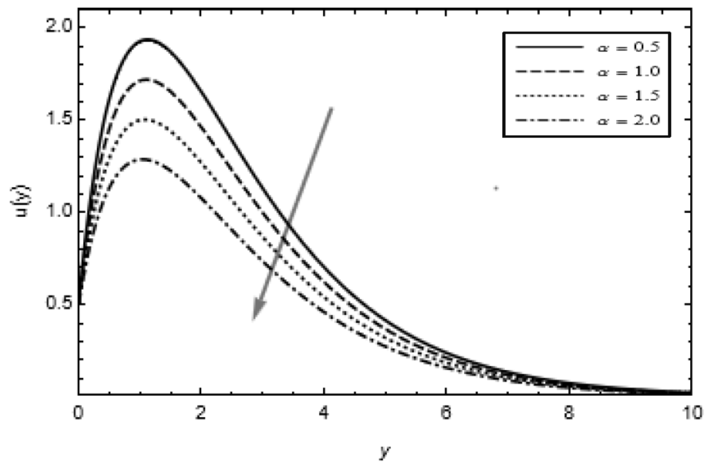

Figure 6: Velocity profile for variation in angle of inclination Parameter when $M = 2, Ko = 0.15, Gr = 5, \beta = \pi/3, \beta = 4.5,\ Pr = 0.71, Ec = 0.001, R = 0.62, M = 2, Sc = 0.6, Kr = 1.5$.

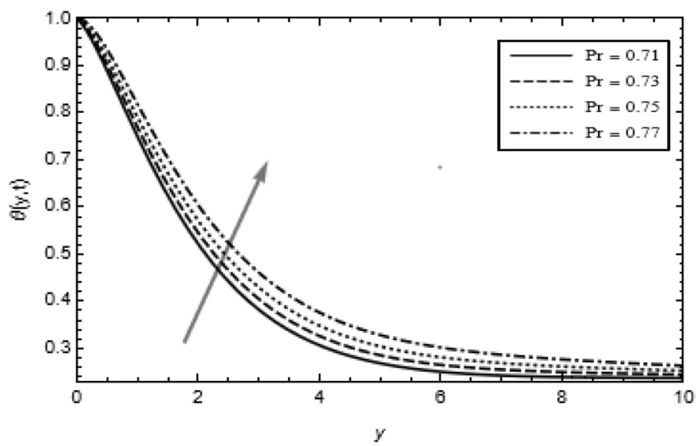

Figure 7: Temperature profile for variation in Prandtl number for fixed $Ko = 0.15$, $Gr = 5, \gamma = \pi/3, \beta = 4.5, M = 2,\ R = 0.62, Sc = 0.6, Kr = 1.5, \alpha = \pi/3, Ec = 0.001$.

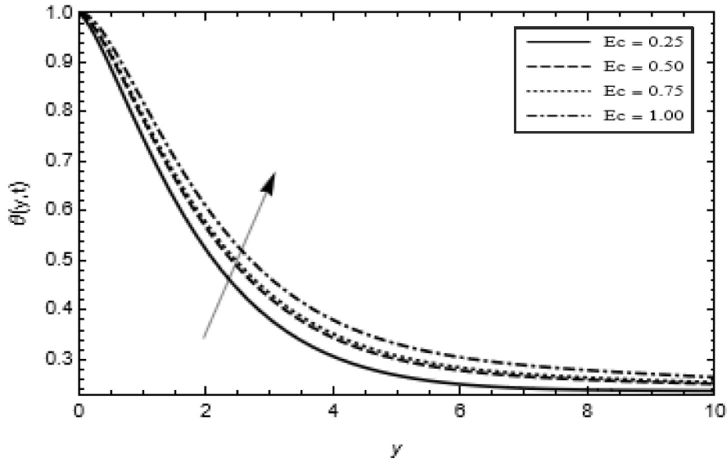

Figure 8: Temperature profile for variation in Eckert number for fixed $Ko = 0.15$, $Gr = 5, \gamma = \pi/3, \beta = 4.5, M = 2,\ R = 0.62, Sc = 0.6, Kr = 1.5, \alpha = \pi/3, \mathrm{Pr} = 0.71$.

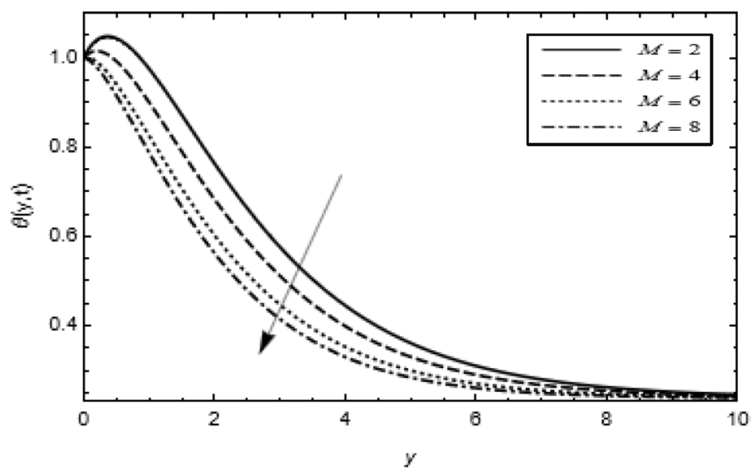

Figure 9: Temperature profile for variation in Hartmann umber for fixed $Ko = 0.15, Gr = 5, \gamma = \pi/3, \beta = 4.5, R = 0.62,\ Sc = 0.6, \mathrm{Kr} = 1.5, \alpha = \pi/3, Ec = 0.001, \mathrm{Pr} = 0.71$.

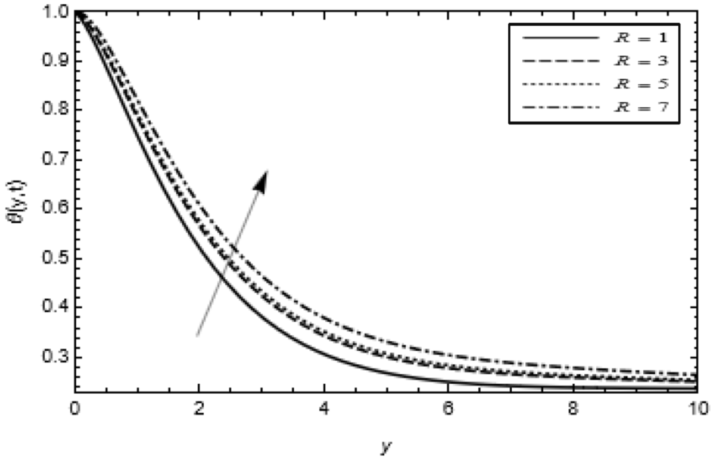

Figure 10: Temperature profile for variation in radiation parameter for fixed $Ko = 0.15, Gr = 5, \gamma = \pi/3, \beta = 4.5, M = 2, Sc = 0.6, Kr = 1.5, \alpha = \pi/3, Ec = 0.001, Pr = 0.71$.

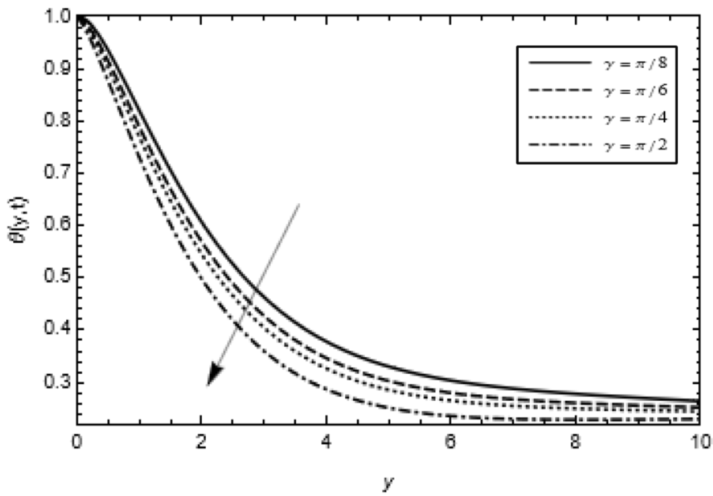

Figure 11: Temperature profile for variation in inclined angle parameter for fixed $Ko = 0.15, Gr = 5, \beta = 4.5, M = 2, R = 0.62, Sc = 0.6, Kr = 1.5, \alpha = \pi/3, Ec = 0.001, Pr = 0.71$.

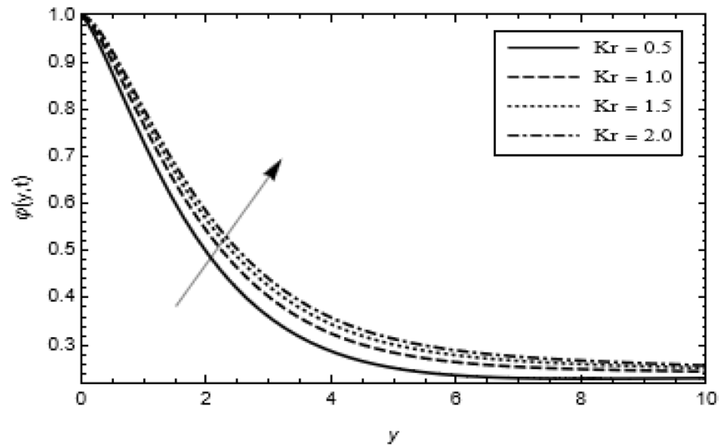

Figure 12: Concentration profile for variation in chemical reaction parameter for fixed $Ko = 0.15, Gr = 5, \gamma = \pi/3, \beta = 4.5,\ M = 2, R = 0.62, Sc = 0.6, \alpha = \pi/3, Ec = 0.001, Pr = 0.71$.

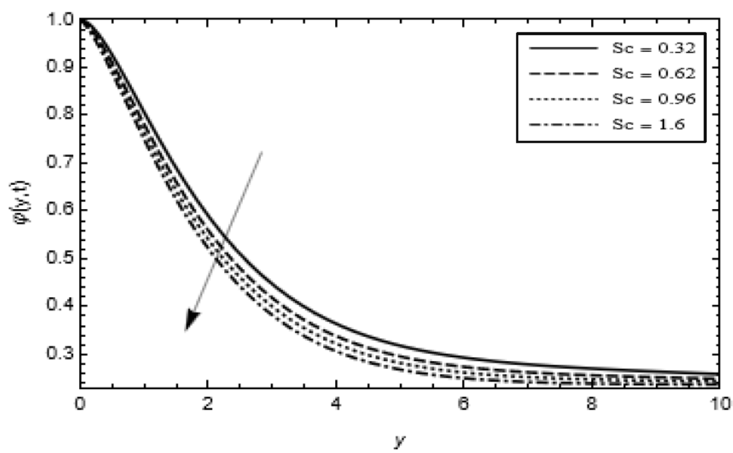

Figure 13: Concentration profile for variation in Schmidt number for fixed $Ko = 0.15, Gr = 5, \gamma = \pi/3, \beta = 4.5, M = 2,\ R = 0.62, Kr = 1.5, \alpha = \pi/3, Ec = 0.001, \mathrm{Pr} = 0.71$.

Figure 1 depicts the effect of Casson parameter for heat and mass transfer on the velocity profile of the fluid. It is observed that positive increment in the values of Casson parameter keeping other parameters fixed enhanced the velocity distribution of the fluid where the values of the velocity increases rapidly near the porous plate before decaying gradually to the free stream velocity.

The influence of magnetic field parameter (Hartmann number) on the velocity distribution is displayed in Figure 2. The result shows, the velocity profile of the fluid increased with increase in the Hartmann number. In Figure 3, the angle of inclination effects to the vertical direction of the fluid is presented. It is obtained that, the velocity profile of the fluid increase in the presence of inclined angle magnetic parameter.

Figure 4 illustrates the variations of porosity parameter for fixed values of the other parameters. It is found that the velocity decrease with increasing values of the porosity parameter.

The impact of variation in the Grashof number against the velocity distribution of the fluid for constant values of the other parameters is shown in Figure 5. The result revealed, the presence of Grashof number lead to a decrease in the velocity profile of the fluid.

In Figure 6, the angle of inclination and the velocity distribution of the fluid vary proportionally to each other. Figure 7 shows the relationship between the temperature curve and y using different values of the Prandtl number (Pr). This number showed that the temperature profile decreases as Prandtl number increases. This is because the fluid is highly conductive to the small values of the Prandtl number. The influence of the Eckert number (Ec) on the temperature profile is seen in Figure 8. From this figure, an increment of temperature is enhanced by the positive increase in the values of the Eckert number.

The effect of radiation parameter on temperature profiles against y is displayed in Figure 9. It is observed from Figure 9 that the temperature profiles increase as radiation parameter increases. The influence of inclined magnetic parameter on the temperature profile is shown in Figure 10. It is observed that the temperature profile is enhanced in the presence of positive increase in the inclined magnetic angle parameter.

In Figure 11, the relationship between the temperature distribution and increase in the magnetic field parameter (Hartmann number) is presented. The finding showed that, increase in the Hartmann number lead to a decrease in the temperature profile of the fluid.

Influence of Schmidt number on concentration profile is shown in Figure 12, from this figure it is noticed that concentration decreases with an increase in Schmidt number. This agrees with the fact that, Schmidt number as a dimensionless number defines the ratio of momentum diffusivity and mass diffusivity, as it characterizes fluid flows in which there is combined momentum and mass diffusion convection processes. Therefore, an increase in Schmidt number decreases the concentration boundary layer of the fluid. Figure 13 portrays the influence of chemical reaction effect (Kr) on the concentration profile. The concentration profile increases with positive increase in the values of the chemical reaction parameter, which leads to a thinner solutal boundary layer thickness of the fluid.

6 Conclusion

In this paper, HPM is successfully implemented to solve for the approximate analytical solution on the effects of chemical reaction on magnetohydrodynamic fluid flowing past an inclined porous plate in the presence of magnetic field. Effects of different controlling parameters on dimensionless velocity, temperature and concentration profiles are investigated. The obtained results using HPM are compared with established literature and excellent agreement is observed. The main findings of the study are summarized as follows:

i. The dimensionless velocity of the fluid increased with an increase in the

Casson parameter, magnetic field parameter, and inclined magnetic angle parameters, respectively.

ii. In the presence of the porosity parameter, Grashof number, and angle of inclination, the fluid velocity declines rapidly.

iii. Increase in Prandtl number, Eckert number, and inclined angle parameters enhanced the temperature profile of the fluid.

iv. Positive increment in the radiation parameters decreases the temperature distribution of the fluid.

v. In the existence of Schmidt number, the fluid concentration declines, whereas it rises in the presence of the thermal radiation parameter.

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