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On Necessary and Sufficient Conditions for Absolute Matrix Summability

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Abstract

This study gets a new general theorem related to necessary and sufficient conditions for $\varphi - \mid D, \beta; \delta \mid_k$ summability of the series $\sum a_n \lambda_n$ whenever the series $\sum a_n$ is summable $\varphi - \mid C, \beta; \delta \mid$, where $C = (c_{nv})$ and $D = (d_{nv})$ are two positive normal matrices, $k \geq 1$, $\delta \geq 0$ and $-\beta(\delta k + k - 1) + k > 0$.

1 Introduction

In summability theory, the topic of summability of some infinite series (IS) or factored IS attracts the attention of researchers. There are many studies that give sufficient conditions for absolute Riesz summability and absolute matrix summability of IS. Let us mention about the recent studies. Bor [1, 2, 3] proved the theorems on sufficient conditions for absolute Riesz summability of the series $\sum a_n \frac{P_n \lambda_n}{n p_n}$ by using the different class of sequences. Bor and Agarwal [4] proved a theorem gives absolute summability of an factored IS by using an almost increasing sequence. Sonker and Munjal [5], Karakaş [6], Kartal [7] obtained the sufficient conditions for absolute Riesz summability of the series $\sum a_n \lambda_n$ via an almost increasing sequence. Karakaş [8], Kartal [9, 10], Özarslan [11, 12, 13], Özarslan and Kartal [14] proved theorems on generalized absolute matrix summability methods. In [15], Özarslan and Şakar obtained the sufficient conditions for absolute Riesz summability of the series $\sum a_n \mu_n$ where (μ_n) is a (ϕ, δ) monotone sequence. Also, we can refer to some other papers about the relative strength of

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absolute summability methods, the sufficient or necessary conditions for absolute summability of IS, some equivalence theorems on summability. In [16], Özarslan and Kandefer studied on the relative strength of two absolute matrix summability methods. Özarslan and Ari [17] acquired the necessary and sufficient conditions in order that $\varphi - \mid A;\delta \mid_k$ summability of the series $\sum a_n$ implies the $\varphi - \mid B;\delta \mid_k$ summability of the same series, where $A = (a_{nv})$ and $B = (b_{nv})$ are two positive normal matrices, and (φ_n) is a sequence of positive numbers, $k \geq 1$, $\delta \geq 0$. Özarslan and Özgen [18] got a result gives necessary conditions for absolute matrix summability, and then Özgen [19] obtained the sufficiency part of this result. Sezer and Çanak [20], Bor [21] obtained the equivalence theorems about summability of the series. In the present article, the purpose is to get a more general theorem gives the necessary and sufficient conditions for absolute matrix summability than the theorems in articles some of which mentioned here.

2 Preliminaries

Let $\sum a_n$ be an IS with its partial sums (s_n). Let $C = (c_{nv})$ be a normal matrix which means a lower triangular matrix of nonzero diagonal entries. Then C defines the sequence-to-sequence transformation, mapping the sequence $s = (s_n)$ to $Cs = (C_n(s))$, where

$$C_n(s) = \sum_{v=0}^{n} c_{nv} s_v, \quad n = 0, 1, ...$$

Let $C = (c_{nv})$ be a normal matrix, then two lower semimatrices $\bar{C} = (\bar{c}_{nv})$ and $\hat{C} = (\hat{c}_{nv})$ are defined as follows:

$$\bar{c}_{nv} = \sum_{i=v}^{n} c_{ni}, \quad n, v = 0, 1, ... \tag{1}$$

$$\hat{c}_{00} = \bar{c}_{00} = c_{00}, \quad \hat{c}_{nv} = \bar{c}_{nv} - \bar{c}_{n-1,v}, \quad n = 1, 2, ... \tag{2}$$

and

$$\bar{\Delta} C_n(s) = C_n(s) - C_{n-1}(s) = \sum_{v=0}^{n} \hat{c}_{nv} a_v. \tag{3}$$

If C is a normal matrix, then $C' = (c'_{nv})$ denotes the inverse of C, and $\hat{C} = (\hat{c}_{nv})$ is a normal matrix and it has two-sided inverse $\hat{C}' = (\hat{c}'_{nv})$ which is also normal (see [22]).

Definition 1. *(l_1, l_k) denotes the set of all matrices C which map l_1 into l_k defined by $l_k := \left\{ x = (x_j) : \sum |x_j|^k < \infty \right\}$.*

Lemma 1. [23] *Let $1 \leq k < \infty$. $C = (c_{nv}) \in (l_1, l_k) \iff \sup_v \sum_{n=1}^{\infty} |c_{nv}|^k < \infty$.*

Definition 2. [24] *Let (φ_n) be a sequence of positive numbers. The series $\sum a_n$ is said to be summable $\varphi - \mid C, \beta; \delta \mid_k$, $k \geq 1$, $\delta \geq 0$ and β is a real number, if*

$$\sum_{n=1}^{\infty} \varphi_n^{\beta(\delta k + k - 1)} \mid C_n(s) - C_{n-1}(s) \mid^k < \infty$$

For $\varphi_n = n$ for all values of n, $\beta = 1$ and $\delta = 0$, we get $|C|_k$ summability method [25].

3 Known Result

The following theorem about absolute matrix summability of IS has proved in [26].

Theorem 1. *Let $k \geq 1$. Let $C = (c_{nv})$ and $D = (d_{nv})$ be two positive normal matrices satisfy*

$$c_{n-1,v} \geq c_{nv} \ for \ n \geq v + 1 \tag{4}$$

$$\bar{c}_{n0} = 1, \quad n = 0, 1, ... \tag{5}$$

In order that $\sum a_n \lambda_n$ is summable $|D|_k$ whenever $\sum a_n$ is summable $|C|$, it is necessary that

$$|\lambda_n| = O\left(n^{\frac{1}{k} - 1} \frac{c_{nn}}{d_{nn}} \right) \tag{6}$$

$$\sum_{n=v+1}^{\infty} n^{k-1} \left| \Delta_v \left(\hat{d}_{nv} \lambda_v \right) \right|^k = O\left(c_{vv}^k \right) \tag{7}$$

$$\sum_{n=v+1}^{\infty} n^{k-1} \left| \hat{d}_{n,v+1} \lambda_{v+1} \right|^k = O\left(1\right). \tag{8}$$

Also (6)-(8) and

$$\overline{d}_{n0} = 1, \quad n = 0, 1, ... \tag{9}$$

$$c_{nn} - c_{n+1,n} = O(c_{nn} c_{n+1,n+1}) \tag{10}$$

$$\sum_{v=r+2}^{\infty} |\hat{d}_{nv} \hat{c}'_{vr} \lambda_v| = O\left(\left| \hat{d}_{n,r+1} \lambda_{r+1} \right| \right) \tag{11}$$

are sufficient for the consequent to hold.

4 Main Result

The aim of the article is to generalize Theorem 1 as in the following form.

Theorem 2. *Let $C = (c_{nv})$ and $D = (d_{nv})$ be two positive normal matrices satisfy (4) and (5). Let $\varphi_n^{\beta\delta} = O(1)$. In order that $\sum a_n \lambda_n$ is summable $\varphi - \mid D, \beta; \delta \mid_k$ whenever $\sum a_n$ is summable $\varphi - \mid C, \beta; \delta \mid$, it is necessary that*

$$|\lambda_n| = O\left(\varphi_n^{\frac{-\beta(\delta k + k - 1)}{k}} \frac{c_{nn}}{d_{nn}} \right) \tag{12}$$

$$\sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k + k - 1)} \left| \Delta_v \left(\hat{d}_{nv} \lambda_v \right) \right|^k = O\left(c_{vv}^k \right) \tag{13}$$

$$\sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k + k - 1)} \left| \hat{d}_{n,v+1} \lambda_{v+1} \right|^k = O\left(1\right). \tag{14}$$

Also (9)-(11) and (12)-(14) are sufficient for the consequent to hold, where $k \geq 1$, $\delta \geq 0$ and $-\beta(\delta k + k - 1) + k > 0$.

Proof of Theorem 2

Necessity. Let (I_n) and (U_n) denote C-transform and D-transform of the series $\sum a_n$ and $\sum a_n\lambda_n$. By (3), we get

$$\bar{\Delta} I_n = \sum_{v=0}^{n} \hat{c}_{nv} a_v \quad \text{and} \quad \bar{\Delta} U_n = \sum_{v=0}^{n} \hat{d}_{nv} a_v \lambda_v. \tag{15}$$

Let us define

$$C = \left\{ a = (a_i) : \sum a_i \quad \text{is summable} \quad \varphi - \mid C, \beta; \delta \mid \right\}$$

$$D = \left\{ a\lambda = (a_i\lambda_i) : \sum a_i\lambda_i \quad \text{is summable} \quad \varphi - \mid D, \beta; \delta \mid_k \right\}.$$

If the above spaces are normed by

$$||I|| = \left\{ \sum_{n=0}^{\infty} \varphi_n^{\beta\delta} \left| \bar{\Delta} I_n \right| \right\} \quad \text{and} \quad ||U|| = \left\{ \sum_{n=0}^{\infty} \varphi_n^{\beta(\delta k + k - 1)} \left| \bar{\Delta} U_n \right|^k \right\}^{\frac{1}{k}} \tag{16}$$

then these are BK-spaces. By the hypotheses of the theorem, since the $\varphi - \mid C, \beta; \delta \mid$ summability of the series $\sum a_n$ implies the $\varphi - \mid D, \beta; \delta \mid_k$ summability of the series $\sum a_n\lambda_n$, we can write

$$||I|| < \infty \quad \Longrightarrow \quad ||U|| < \infty.$$

Let us consider the inclusion map $T : C \to D$ defined by $T(x) = x$. Since C and D are BK-spaces, this map is continuous. Thus, there exists a constant $M > 0$ so that $||U|| \leq M \, ||I||$. By writing $a_v = e_v - e_{v+1}$ in (15), we have

$$\bar{\Delta} I_n = \begin{cases} 0, & n < v \\ \hat{c}_{nv}, & n = v \\ \Delta_v \hat{c}_{nv}, & n > v \end{cases} \quad \text{and} \quad \bar{\Delta} U_n = \begin{cases} 0, & n < v \\ \hat{d}_{nv}\lambda_v, & n = v \\ \Delta_v \left(\hat{d}_{nv}\lambda_v \right), & n > v \end{cases}.$$

Then (16) implies the following equality

$$||I|| = \left\{ \varphi_v^{\beta\delta} c_{vv} + \sum_{n=v+1}^{\infty} \varphi_n^{\beta\delta} \left| \Delta_v \hat{c}_{nv} \right| \right\}$$

and

$$||U|| = \left\{ \varphi_v^{\beta(\delta k+k-1)} d_{vv}^k \left|\lambda_v\right|^k + \sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\Delta_v\left(\hat{d}_{nv}\lambda_v\right)\right|^k \right\}^{\frac{1}{k}}.$$

Here, using the fact that $||U|| \leq M\,||I||$, we get

$$\varphi_v^{\beta(\delta k+k-1)} d_{vv}^k \left|\lambda_v\right|^k + \sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\Delta_v\left(\hat{d}_{nv}\lambda_v\right)\right|^k$$
$$\leq M^k \varphi_v^{\beta(\delta k)} c_{vv}^k + M^k \left(\sum_{n=v+1}^{\infty} \varphi_n^{\beta\delta} \left|\Delta_v\left(\hat{c}_{nv}\right)\right| \right)^k.$$

By using the above inequality and the equality $\sum_{n=v+1}^{\infty} |\Delta_v(\hat{c}_{nv})| = O(c_{vv})$ by (1), (2), (4), we get

$$\varphi_v^{\beta(\delta k+k-1)} d_{vv}^k \left|\lambda_v\right|^k + \sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\Delta_v\left(\hat{d}_{nv}\lambda_v\right)\right|^k = O(c_{vv}^k)$$

which means each term on the left equals to $O(c_{vv}^k)$. For the the first one, we have

$$\varphi_v^{\beta(\delta k+k-1)} d_{vv}^k \left|\lambda_v\right|^k = O(c_{vv}^k) \quad \Longrightarrow \quad \left|\lambda_v\right| = O\left(\varphi_v^{\frac{-\beta(\delta k+k-1)}{k}} \frac{c_{vv}}{d_{vv}} \right)$$

and this gives that (12) is necessary.

For the second one, we get

$$\sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\Delta_v\left(\hat{d}_{nv}\lambda_v\right)\right|^k = O(c_{vv}^k)$$

that means (13) is necessary. Similarly, by writing $a_v = e_{v+1}$ in (15), we have

$$\bar{\Delta} I_n = \begin{cases} 0, & n \leq v \\ \hat{c}_{n,v+1}, & n > v \end{cases} \quad \text{and} \quad \bar{\Delta} U_n = \begin{cases} 0, & n \leq v \\ \hat{d}_{n,v+1}\lambda_{v+1}, & n > v \end{cases}.$$

Then again from (16), we obtain

$$||I|| = \left\{ \sum_{n=v+1}^{\infty} \varphi_n^{\beta\delta} \left|\hat{c}_{n,v+1}\right| \right\} \quad \text{and} \quad ||U|| = \left\{ \sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\hat{d}_{n,v+1}\lambda_{v+1}\right|^k \right\}^{\frac{1}{k}}.$$

Similary, we can write

$$\sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\hat{d}_{n,v+1}\lambda_{v+1}\right|^k \leq M^k \left\{\sum_{n=v+1}^{\infty} \varphi_n^{\beta\delta} \left|\hat{c}_{n,v+1}\right|\right\}^k$$

and we get $\sum_{n=v+1}^{\infty} |\hat{c}_{n,v+1}| = O(1)$ by (1), (2), (4), (5). Therefore

$$\sum_{n=v+1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} \left|\hat{d}_{n,v+1}\lambda_{v+1}\right|^k = O(1)$$

that means (14) is necessary.

Sufficiency.

By the equalities in (15), we can write $a_v = \sum_{r=0}^{v} \hat{c}'_{vr}\bar{\Delta} I_r$ and $\bar{\Delta} U_n = \sum_{v=0}^{n} \hat{d}_{nv}\lambda_v \sum_{r=0}^{v} \hat{c}'_{vr}\bar{\Delta} I_r$.

Since $\hat{d}_{n0} = \bar{d}_{n0} - \bar{d}_{n-1,0} = 0$, we have

$$\begin{aligned} \bar{\Delta} U_n &= \sum_{v=1}^{n} \hat{d}_{nv}\lambda_v \sum_{r=0}^{v} \hat{c}'_{vr}\bar{\Delta} I_r \\ &= \sum_{v=1}^{n} \hat{d}_{nv}\lambda_v \hat{c}'_{vv}\bar{\Delta} I_v + \sum_{v=1}^{n} \hat{d}_{nv}\lambda_v \hat{c}'_{v,v-1}\bar{\Delta} I_{v-1} + \sum_{v=1}^{n} \hat{d}_{nv}\lambda_v \sum_{r=0}^{v-2} \hat{c}'_{vr}\bar{\Delta} I_r \\ &= \hat{d}_{nn}\lambda_n \hat{c}'_{nn}\bar{\Delta} I_n + \sum_{v=1}^{n-1} \left(\hat{d}_{nv}\lambda_v \hat{c}'_{vv} + \hat{d}_{n,v+1}\lambda_{v+1}\hat{c}'_{v+1,v}\right) \bar{\Delta} I_v \\ &\quad + \sum_{r=0}^{n-2} \bar{\Delta} I_r \sum_{v=r+2}^{n} \hat{d}_{nv}\lambda_v \hat{c}'_{vr}. \end{aligned} \tag{17}$$

For δ_{nv}(Kronecker delta), by using the equality $\sum_{k=v}^{n} \hat{c}'_{nk}\hat{c}_{kv} = \delta_{nv}$, we get

$$\begin{aligned} \hat{d}_{nv}\lambda_v \hat{c}'_{vv} + \hat{d}_{n,v+1}\lambda_{v+1}\hat{c}'_{v+1,v} &= \frac{\hat{d}_{nv}\lambda_v}{\hat{c}_{vv}} + \hat{d}_{n,v+1}\lambda_{v+1}\left(-\frac{\hat{c}_{v+1,v}}{\hat{c}_{vv}\hat{c}_{v+1,v+1}}\right) \\ &= \frac{\hat{d}_{nv}\lambda_v}{c_{vv}} - \hat{d}_{n,v+1}\lambda_{v+1}\frac{(\bar{c}_{v+1,v} - \bar{c}_{vv})}{c_{vv}c_{v+1,v+1}} \\ &= \frac{\hat{d}_{nv}\lambda_v}{c_{vv}} - \hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{v+1,v+1} + c_{v+1,v} - c_{vv})}{c_{vv}c_{v+1,v+1}} \\ &= \frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}} + \hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv} - c_{v+1,v})}{c_{vv}c_{v+1,v+1}}. \end{aligned}$$

If we write this equality in (17), we obtain

$$\begin{aligned}\bar{\Delta}U_n &= \frac{d_{nn}\lambda_n}{c_{nn}}\bar{\Delta}I_n+\sum_{v=1}^{n-1}\frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}}\bar{\Delta}I_v+\sum_{v=1}^{n-1}\hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv}-c_{v+1,v})}{c_{vv}c_{v+1,v+1}}\bar{\Delta}I_v\\ &\quad+\sum_{r=0}^{n-2}\bar{\Delta}I_r\sum_{v=r+2}^{n}\hat{d}_{nv}\lambda_v\hat{c}'_{vr}\\ &= U_{n,1}+U_{n,2}\end{aligned}$$

so that

$$U_{n,1} = \frac{d_{nn}\lambda_n}{c_{nn}}\bar{\Delta}I_n+\sum_{v=1}^{n-1}\frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}}\bar{\Delta}I_v+\sum_{v=1}^{n-1}\hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv}-c_{v+1,v})}{c_{vv}c_{v+1,v+1}}\bar{\Delta}I_v$$

and

$$U_{n,2} = \sum_{r=0}^{n-2}\bar{\Delta}I_r\sum_{v=r+2}^{n}\hat{d}_{nv}\lambda_v\hat{c}'_{vr}.$$

Now, we show that

$$\sum_{n=1}^{\infty}\varphi_n^{\beta(\delta k+k-1)}\left|U_{n,i}\right|^k<\infty \quad \text{for} \quad i=1,2.$$

First

$$\begin{aligned}\varphi_n^{\frac{\beta(\delta k+k-1)}{k}}U_{n,1} &= \varphi_n^{\frac{\beta(\delta k+k-1)}{k}}\frac{d_{nn}\lambda_n}{c_{nn}}\bar{\Delta}I_n+\varphi_n^{\frac{\beta(\delta k+k-1)}{k}}\sum_{v=1}^{n-1}\frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}}\bar{\Delta}I_v\\ &+ \varphi_n^{\frac{\beta(\delta k+k-1)}{k}}\sum_{v=1}^{n-1}\hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv}-c_{v+1,v})}{c_{vv}c_{v+1,v+1}}\bar{\Delta}I_v\\ &= \sum_{v=1}^{\infty}h_{nv}\bar{\Delta}I_v\end{aligned}$$

such that

$$h_{nv}=\begin{cases}\varphi_n^{\frac{\beta(\delta k+k-1)}{k}}\left\{\frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}}+\hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv}-c_{v+1,v})}{c_{vv}c_{v+1,v+1}}\right\}, & 1\le v\le n-1\\ \varphi_n^{\frac{\beta(\delta k+k-1)}{k}}\frac{d_{nn}\lambda_n}{c_{nn}}, & v=n\\ 0, & v>n\end{cases}.$$

By Lemma 1, $\sum \left|\varphi_n^{\frac{\beta(\delta k+k-1)}{k}} U_{n,1}\right|^k < \infty$ whenever $\sum \left|\bar{\Delta} I_n\right| < \infty$ is equivalently $\sup_v \sum_{n=1}^{\infty} |h_{nv}|^k < \infty$ and by using (10), (12), (13), (14), we have

$$\begin{aligned}\sum_{n=1}^{\infty} |h_{nv}|^k &= O(1)\left\{\varphi_n^{\beta(\delta k+k-1)}\left|\frac{d_{nn}\lambda_n}{c_{nn}}\right|^k\right.\\ &\quad \left.+\sum_{n=v+1}^{\infty}\varphi_n^{\beta(\delta k+k-1)}\left|\frac{\Delta_v(\hat{d}_{nv}\lambda_v)}{c_{vv}}+\hat{d}_{n,v+1}\lambda_{v+1}\frac{(c_{vv}-c_{v+1,v})}{c_{vv}c_{v+1,v+1}}\right|^k\right\}\\ &= O(1) \quad \text{as} \quad v\to\infty.\end{aligned}$$

Now

$$\varphi_n^{\frac{\beta(\delta k+k-1)}{k}} U_{n,2} = \varphi_n^{\frac{\beta(\delta k+k-1)}{k}} \sum_{r=0}^{n-2} \bar{\Delta} I_r \sum_{v=r+2}^{n} \hat{d}_{nv}\lambda_v \hat{c}'_{vr} = \sum_{r=0}^{\infty} p_{nr}\bar{\Delta} I_r$$

so that

$$p_{nr} = \begin{cases} \varphi_n^{\frac{\beta(\delta k+k-1)}{k}} \sum_{v=r+2}^{n} \hat{d}_{nv}\lambda_v \hat{c}'_{vr}, & 0 \le r \le n-2 \\ 0, & r > n-2 \end{cases}.$$

Again by Lemma 1, $\sum \left|\varphi_n^{\frac{\beta(\delta k+k-1)}{k}} U_{n,2}\right|^k < \infty$ whenever $\sum \left|\bar{\Delta} I_n\right| < \infty$ is equivalently $\sup_r \sum_{n=1}^{\infty} |d_{nr}|^k < \infty$ and by using (11), (14), we have

$$\begin{aligned}\sum_{n=1}^{\infty} |d_{nr}|^k = \sum_{n=r+2}^{\infty} |d_{nr}|^k &= O(1)\sum_{n=r+2}^{\infty}\varphi_n^{\beta(\delta k+k-1)}\left\{\sum_{v=r+2}^{\infty}\left|\hat{d}_{nv}\lambda_v\hat{c}'_{vr}\right|\right\}^k\\ &= O(1)\sum_{n=r+2}^{\infty}\varphi_n^{\beta(\delta k+k-1)}\left|\hat{d}_{n,r+1}\lambda_{r+1}\right|^k\\ &= O(1) \quad \text{as} \quad r\to\infty.\end{aligned}$$

Thus, we obtain that

$$\sum_{n=1}^{\infty} \varphi_n^{\beta(\delta k+k-1)} |U_{n,i}|^k < \infty \quad \text{for} \quad i = 1, 2.$$

5 Conclusions

In this paper, a new general theorem related to necessary and sufficient conditions for absolute matrix summability of IS is proved. In case of $\varphi_n = n$ for all values of n, $\beta = 1$ and $\delta = 0$, the equalities (12), (13) and (14) reduce to the equalities (6), (7) and (8) respectively, thence Theorem 2 reduces to Theorem 1. Theorem 2 may be the basis for further studies, the result can be generalized for different summability methods.

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