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An Analysis of the Exchange Rate Volatility in Poland using the GARCH, GJR-GARCH and EGARCH Models

Nneka Karen Enumah[1,*] and Hezekiah Seun Adewinbi[2]

[1] Wrołcaw University of Science and Technology, Poland
e-mail: karenenumah@gmail.com

[2] Kent State University, Kent, OH, United States
e-mail: hadewinb@kent.edu

Abstract

This paper employs the symmetric GARCH and the asymmetric GJR-GARCH(1,1) and E-GARCH(1,1) models to explain the dynamics of the PLN/EUR and PLN/USD exchange rates in Poland for the periods of January 2015 to July 2022. The result of our study shows that the USD rate is more susceptible to market fluctuations and events than the EUR rate. Additionally, both rates' volatility persists after a market crisis for a while, with the EUR rate taking longer until volatility subsides. Using the Akaike information criterion and Bayesian information criterion, we find the E-GARCH model to be the best model out of all three models considered.

1 Introduction

Poland's economic development has been rather steady. The average growth rate from 2016 to 2019 was 4.5 percent, compared to the European Union's average rate of 2.1 percent. Poland did not escape the covid-19 epidemic, just like other nations. However, the pandemic caused a 2.7 percent reduction in the Polish GDP, which was still less than the EU's average GDP decline of 6 percent [1].

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*Corresponding author

With imports from the United States and exports from nations like Germany and France, international trade also plays a part in the Polish economy. We are thus driven to investigate the Polish zloty's volatility about the Euro and the US dollar by using the GARCH models.

Our choice of the symmetric and asymmetric GARCH models stems from a knowledge of its practicality in analyzing time series data. The plain vanilla GARCH model, known as the Generalized Autoregressive Conditional Heteroscedasticity model, is a generalized form of the ARCH model, which Robert Engle first proposed in his paper in 1982 [2]. The GARCH model was proposed in 1986 by Tim Bollerslev. Over time, there have been different variations of the GARCH models. Typically, this variation is a result of the modification of the conditional variance equation. Thus we take a look at these other variations of the GARCH model, namely the Exponential-Generalized Autoregressive Conditional Heteroscedasticity (E-GARCH) model, which was proposed by Daniel Nelson in his paper in 1991 [3] and the Glosten-Jagannathan-Runkle-Generalized Autoregressive Conditional Heteroscedasticity (GJR-GARCH) model proposed by Glosten et al. in 1993 [4].

The remainder of this paper is structured as follows. An overview of earlier research in this field is given in the next section, and Section 3 describes our data collection. We outline the methods used in Section 4. Our findings are presented in Section 5, and our conclusions are made in Section 6.

2 Literature Review

Since the creation of pricing models, the topic of volatility has received a lot of attention from researchers and academicians as they attempt to accurately forecast volatility into the future. Numerous scholars have attempted to investigate the connection between exchange rate volatility and other economic issues particularly because it cannot be overemphasized how much a country's exchange rate affects the total condition of its economy, even more so since the elimination of the Brent Woods system, giving rise to the floating exchange rate system.

A considerable body of research analyzes the impact of exchange rate volatility on international trade vis-a-vis exportation and importation in different continents. In west African countries such as Nigeria, exchange rate volatility significantly influences the exportation of crude oil [5]. More generally, the instability in crude oil prices can be used to predict exchange rate dynamics [6]. In the middle east, China's exchange rate volatility has more impact on imports than on the exports industries, with US exporters benefiting from the increase in exchange rate volatility of the dollar-yuan. The Sino-US EPU ratio positively impacts the long-term volatility of the Chinese exchange rate [7]. In Turkey, while low and high volatility causes an increase in Turkey's exportation in the short-term, there is a decrease in the long-term [8]. Using the linear and non-linear autoregressive distributed lag model, the authors in [9] showed that the Indonesian exports to the US, South Korea and India are negatively affected by exchange rate fluctuations.

In [10], Eichler et al. showed that a country's exchange rate volatility declines when its central bank is transparent. However, this effect is more obvious in nations with less central bank conservatism and more interest rate sensitivity. In [11], the authors examined the effect of exchange rate volatility on employment growth. Their result showed that an increase in real exchange rate volatility had a negative impact on job growth, particularly for manufacturing businesses. This impact has shown to be substantially stronger in businesses with large export percentages of output.

Ulm et al. investigated the relationship between interest rate differentials (IRD) and exchange rate volatility. Their findings revealed that although a rise in the low-interest rate had a favorable impact on volatility, a rise in the high-interest rate had the opposite impact [12]. Using the GARCH and TARCH model to examine exchange rate volatility in five countries, Fidrmuc et al. showed that volatility is more prominent during exchange rate appreciation [13]. In [14], Bahmani-Oskooee et al. studied the impact of exchange rate volatility on US bilateral trades with African nations using the non-linear ARDL approach. Their findings revealed that higher exchange rate volatility has a negative impact on US exports to nations like Kenya, Conga, Liberia, and Uganda, among others, and a positive impact on nations like Nigeria, Ghana, Tanzania, and Mozambique, among others.

Doğan et al. showed that an increase in total oil prices has a negative effect on real exchange rates, as shown in the case of Turkey [15]. In [16] using the GARCH(1,1) and GJR-GARCH(1,1) models, the authors examined the relationship between historical volatility of exchange rates and present volatility of exchange rates for the bureaux de change, the official exchange rate, and the international foreign exchange market in Nigeria. The volatilities of these three distinct exchange rates between the periods 1995 to 2014 were compared. Their findings indicated that compared to their equivalent interbank rates, volatility clustering was more prevalent in the bureau de change exchange rates [15].

In research from [17], the authors examined the relationship between the returns of exporting and domestic firms and the currency fluctuation against the USD and EUR for Czech, Hungary, Poland, and Russia. Their findings showed that each country is highly affected by exchange rate exposure, albeit in a different direction. The direction and extent of each country's exposure depend significantly on each country's export market structure. While Czech, Poland, and Hungary have negative exposure to changes in the exchange rates against the euro and USD, with exposure to EUR being much greater, Russia shows a positive exposure. A negative exposure means that an appreciation of the domestic currency will cause an increase in the share price of firms.

While most research focuses on European countries in general, only a few focus on Poland as a single entity. Thus our motivation for current study. We augment the existing literature by carrying out an analysis of the dynamics of the Polish złoty to the Euro and dollar by using the plain vanilla GARCH model, GJR-GARCH, and E-GARCH model and taking into account recent exchange rate data from the central bank of Poland, namely National Bank of Poland (Narodowy Bank Polski).

3 Data

Our data set contains the Polish Złoty's exchange rate against the Euro and the US dollars from January 2015 to July 2022. The data is obtained from the yahoo finance website and consists of a total of 91 observations. The choice of duration of our dataset was chosen abritraily. The average USD exchange rate in our dataset, which spans seven years, is 3.85 PLN/USD and 4.36 PLN/EUR, with the highest exchange rates being 4.68 PLN/USD and 4.77 PLN/EUR. Similarly, the lowest exchange rate is 3.37 PLN/USD and 4.03 PLN/EUR. Below is a table of the summary statistics and plot of the exchange rate move of the Polish złoty.

	Mean	Variance	Skewness	Kurtosis
PLN/USD	3.8524	0.04596	0.8349	2.1315
PLN/EUR	4.3636	0.02424	0.5396	-0.3533

Table 1: Summary statistics of monthly exchange rate returns

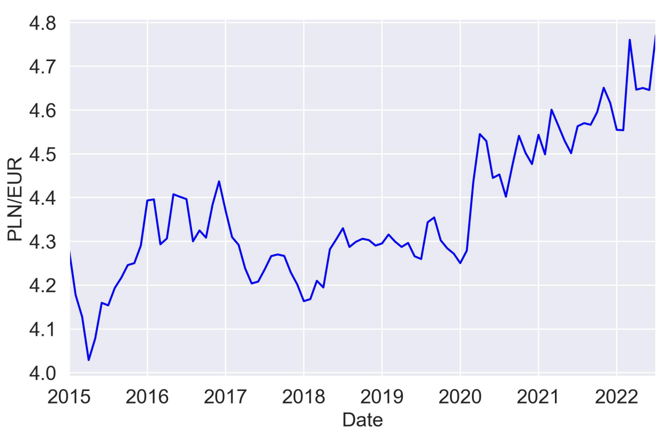

Figure 1: PLN to EUR exchange rate

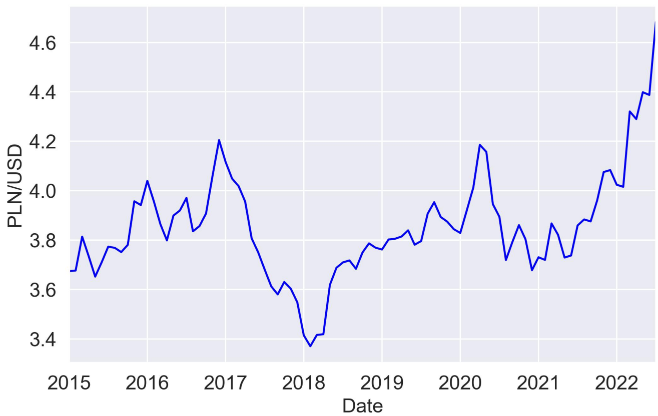

Figure 2: PLN to USD exchange rate

The two exchange rates' monthly growth rates are depicted below by a plot of their returns which is calculated as follows:

$$X_t = \frac{R_t - R_{t-1}}{R_{t-1}}$$

where R_t the monthly exchange rate of the PLN to the EUR and USD at time t.

A positive change in returns suggests a depreciation in the value of the PLN relative to the EUR and USD. As a result, more PLN will be exchanged for each unit of the USD and EUR. Our plot shows little volatility clustering; this can be attributed to the frequency of our monthly data. The volatility clustering effect tends to arise in high-frequency data, such as intraday and daily data and disappears in low-frequency data, such as monthly data.

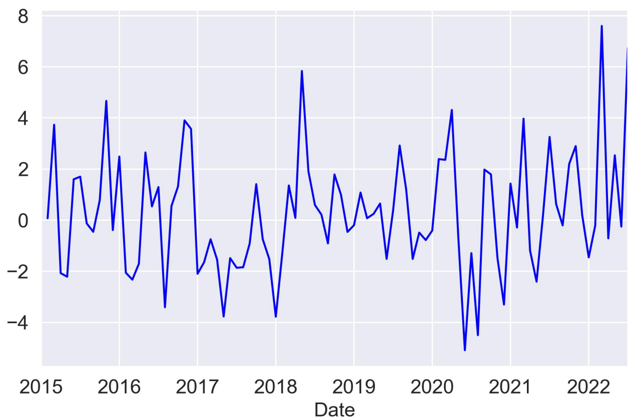

Figure 3: Returns PLN/USD monthly exchange rate

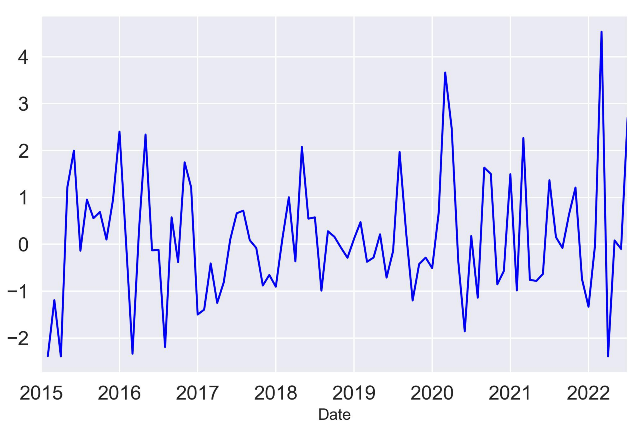

Figure 4: Returns PLN/EUR monthly exchange rate

As shown in Figure 5, the PLN/USD and the PLN/EUR returns are right-tailed. They both have thinner tails than the normal distribution as implied by their kurtosis which is less than three. They also show unconditional skewness with both returns being right-tailed. The lighter tails than the normal distribution can be attributed to the fact that our returns are measured at a lower frequency than daily returns.

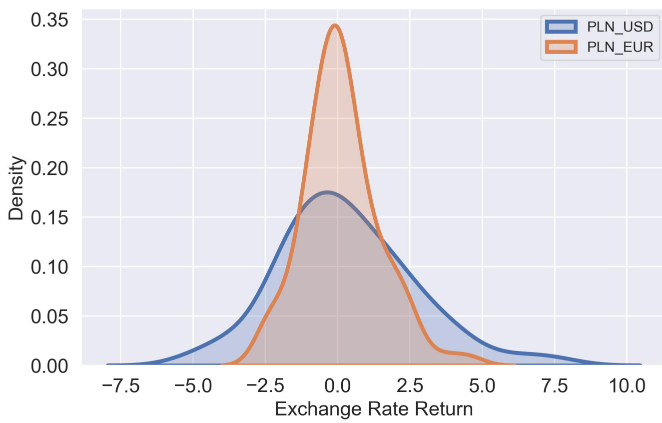

Figure 5: Distribution of exchange rate returns

Using the Dickey-Fuler test, we test for the presence of a unit root in our exchnage rate return time series. Our results are summarized in Table 2.

	PLN/USD	PLN/EUR
t-statistics	-7.768	-5.6852
p-value	9.08e-12	8.31e-07
1% critical value	-3.5060	-3.5097
5% critical value	-2.8946	-2.8961
10% critical value	-2.5844	-2.5852

Table 2: Dickey-Fuller Test

The results of our Dickey-Fuller(DF) test show no presence of a unit root in our returns time series as our t-statistic is lower than the 1%, 5%, and 10% critical values. We thus reject the null hypothesis of the DF test. The return time series is stationary.

3.1 Heteroscedasticity Test

We test for the presence of ARCH effect using the Engle's Langrange Multiplier test. The null hypothesis, H_0 of the ARCH test is the absence of heteroscedasticity, while the alternative hypothesis H_1 indicates the presence of the ARCH effect. The results of our ARCH-LM Test is shown in Table 3. The test provides evidence to reject the null hypothesis, this implies the presence of ARCH effect.

	PLN/USD	PLN/EUR
χ^2	66.9830	69.4770
Df	12.0000	12.0000
p-value	1.17e-9	4.01e-10

Table 3: ARCH-LM Test

4 Methodology

4.1 GARCH Model

Our GARCH models are heteroscedastic, which means that unlike homoskedasticity models, they do not assume constant variance. They are autoregressive, which means that they depend on past observations to account for present ones.. The plain-vanilla GARCH model, also known as the symmetric GARCH model, is given by the following equations:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}, \tag{1}$$

where α, β, and γ are parameters to be estimated. α measures the level of sensitivity of volatility to market shocks and β measures the persistence of volatility, ϵ_t is given by $(y_t - \bar{y})$, $\bar{y}$ is the sample mean and y_t is the monthly return at time t [18]. ω is the models constant paramter.We have the following constraints on each parameters $\alpha \geq 0$, $\beta \geq 0$, and $\omega > 0$

4.2 GJR-GARCH Model

The GJR-GARCH model is given by the following equations:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma\mathbf{I}_{\epsilon_{t-1}<0}\epsilon_{t-1}^2 + \beta\sigma_t \tag{2}$$

$$\mathbf{I}_{\epsilon_{t-1}<0} = \begin{cases} 1 & \text{if } \epsilon_t < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The parameter γ captures the leveraging effect and $\mathbf{I}_{\epsilon_{t-1}<0}$ is an indicative function which takes the values as shown in equation 3 above.

4.3 E-GARCH Model

The E-GARCH guarantees the conditional variance is positive by defining the variance in terms of its log rather than by placing restrictions on our parameter values. The variance equation in this model is given as :

$$\ln(\sigma_t^2) = \omega + f(x_{t-1}) + \beta\ln(\sigma_{t-1}^2), \tag{4}$$

where $X_t \sim N(0,1)$, $E(X_t) = \sqrt{2/\pi}$, x_t is a realization of X_t and $f(x_t)$ is the asymmetric response function

5 Results

We estimate the parameters for the basic GARCH model for the PLN/USD and PLN/EUR exchange rates using monthly data from 2015 to 2022 and the Maximum Log-Likelihood estimation technique. The long-term unconditional variance is also derived. Tables 3 and 4 show the results from our model estimation for the US dollars and Euro respectively.

	GARCH(1,1)			GJR-GARCH(1,1)			E-GARCH(1,1)		
	estimate	t-stat	p-value	estimate	t-stat	p-value	estimate	t-stat	p-value
ω	0.0219	7.336e-02	0.942	0.0219	7.200e-02	0.943	0.1334	1.640e+08	0.0000
α	0.0000	0.0000	1.0000	0.0000	0.0000	1.0000	-0.5017	-6205.26	0.0000
β	1.0000	28.533	4.6e-179	1.0000	29.232	7.568e-188	0.9119	1.750e+09	0.0000
γ	-	-	-	6.8561e-12	8.521e-11	1.0000	-0.0998	-43.784	0.0000

Table 4: USD

	GARCH(1,1)			GJR-GARCH(1,1)			E-GARCH(1,1)		
	estimate	t-stat	p-value	estimate	t-stat	p-value	estimate	t-stat	p-value
ω	0.3437	0.719	0.472	0.0822	1.043	0.297	0.0276	3.652e+06	0.000
α	0.0583	0.804	0.421	0.0614	1.218	0.223	-0.2789	-1.351e+07	0.0000
β	0.7294	2.590	9.610e-03	0.9171	23.008	3.89e-117	0.9986	2.427e+11	0.0000
γ	-	-	-	-0.0614	-1.248	0.212	0.0437	7.251e+04	0.0000

Table 5: EUR

Both the volatility model and the constant mean model yield the following results for our GARCH(1,1) model of the PLN/USD exchange rate return:

$$\sigma_t^2 = 0.0219 + \sigma_{t-1}^2 \tag{5}$$

$$\mu = 0.2704 \tag{6}$$

We can see from the estimated parameters that there is no sensitivity of the USD exchange rate to market occurrences. Because of this, the GARCH(1,1) model's applicability to this dataset is called into question. Additionally, the estimate demonstrates that the USD exchange rate is extremely persistent and stays turbulent for a longer duration after a market crisis.

The EUR exhange rate return volatility with a constant mean model is estimated by the GARCH(1,1) model as follows:

$$\sigma_t^2 = 0.3437 + 0.0583\epsilon_{t-1}^2 + 0.7294\sigma_{t-1}^2 \quad (7)$$

$$\mu = 0.1518 \quad (8)$$

According to the model parameter estimate, the volatility of exchange rate returns for the euro is more susceptible to shifting market conditions than the dollar rate. It does, however, have a relatively low sensitivity.

Compared to the PLN/USD rate, the volatility of the PLN/EUR dies out sooner after a market shock. Hence the PLN/EUR exchange rate is more likely to recover faster from a market crisis than the PLN/USD.

For both rates, the long term unconditional variance, can be calculated as

$$\frac{\omega}{1-(\alpha+\beta)} \quad (9)$$

The parameters estimates show our long-term unconditional variance to be infinite and 0.10 for the USD and EUR rates respectively. The unconditional variance is infinite for the USD exchange rate return; this, however, defies the parameter constraints on the GARCH models, which state that $\omega > 0$ and $\alpha + \beta < 1$.

This confirms that the GARCH(1,1) model is not appropriate for our USD dataset, and other models would be better. The unconditional variance of the EUR of 0.10 shows that the long-term volatility of the PLN to the EUR is relatively low.

The GJR-GARCH model for the USD rate is given as

$$\sigma_t^2 = 0.0219 + 6.856 \times 10^{-12}\mathbf{I}_{\epsilon_{t-1}<0}\epsilon_{t-1}^2 + \sigma_{t-1}^2 \quad (10)$$

$$\mu = 0.2706 \quad (11)$$

This model considers the leverage effect in our dataset. However, this effect is minimal as shown by the γ parameter estimate. For the EUR rate, the volatility and constant mean model as estimated by the GJR-model is given as:

$$\sigma_t^2 = 0.0822 + 0.0614\epsilon_{t-1}^2 + -0.0614 \times 10^{-12}\mathbf{I}_{\epsilon_{t-1}<0}\epsilon_{t-1}^2 + 0.9171\sigma_{t-1}^2 \quad (12)$$

$$\mu = 0.1636 \quad (13)$$

The value of the γ parameter estimate suggests that the EUR rate volatility is more likely to increase following a positive market shock. This model suggests that while the long-term unconditional variance of the EUR is relatively higher than that of the USD, they both have low long-term volatility.

The E-GARCH volatility model for the US dollar and the Euro is given by equations 12 and 13, respectively.

$$ln(\sigma_t^2) = 0.1334 + f(x_{t-1}) + 0.9119\sigma_{t-1}^2 \tag{14}$$

$$ln(\sigma_t^2) = 0.0276 + f(x_{t-1}) + 0.9869\sigma_{t-1}^2 \tag{15}$$

The long term unconditional variance for this model is given as:

$$\exp\left(\frac{\omega}{1-\beta}\right) \tag{16}$$

Using the Akaike information criterion and Bayesian information criterion, we find the best model for our data-set. The result are shown in tables below:

	GARCH(1,1)	GJR-GARCH(1,1)	E-GARCH(1,1)
AIC	414.546	416.546	397.685
BIC	424.545	429.045	410.184
Log-likelihood	-203.273	-203.273	-193.842

Table 6: USD

	GARCH(1,1)	GJR-GARCH(1,1)	E-GARCH(1,1)
AIC	308.015	309.218	288.505
BIC	318.014	321.717	301.004
Log-likelihood	-150.007	-149.609	-139.252

Table 7: EUR

A comparison of the Akaike information criterion (AIC) and the Bayesian information criteria (BIC) shows that the E-GARCH model provides the best fit for our sample data set among the three models tested for both the EUR and the USD exchange rates returns.Furthermore, a residual diagonistic test shows that the models do not contain any heteroscedasticity effect as the p-values are greater than 0.05.

6 Conclusions

This paper examines the dynamics of the exchange rate volatility of the Polish Zloty against the EUR and USD. The findings indicate that the PLN/USD rate is more vulnerable to market changes and outside factors than the PLN/EUR rate. The volatility of both rates also lasts for some time following a market crisis, with the EUR rate taking longer to stabilize. The long-term unconditional variance result, provided by the formula in equation 16, indicates that the PLN/USD rate is substantially more volatile over the long run than the EUR rate.

In the long term, Polish investors in the USD market would likely experience more risk owing to exchange rate fluctuations in the USD market than in the EUR market. In order to handle risk effectively, Polish investors would need to incorporate effective hedging strategies into their portfolio management style. Furthermore, non-polish investors who trade in any currency pairs taken into account in this study should consider including efficient hedging instruments such as derivatives in their investment portfolios to efficiently hedge against foreign currency risks.

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