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On a Certain Subclass for Multivalent Analytic Functions with a Fixed Point Involving Linear Operator

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Abstract

In the present paper, we study a subclass for multivalent analytic functions with a fixed point w defined in the unit disk U involving linear operator. Also, we obtain coefficient estimates, extreme points, integral representation and radii of starlikeness and convexity.

1. Introduction

Denote by $\mathcal{A}(p, w)$ the class of functions f of the form:

$$f(z) = (z-w)^p + \sum_{n=1}^{\infty} a_{n+p}(z-w)^{n+p} \qquad (p \in \mathbb{N}), \tag{1.1}$$

which are analytic in the unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ and w is a fixed point in U.

Let $S(p, w)$ denote subclass of $\mathcal{A}(p, w)$ containing of functions of the form:

$$f(z) = (z-w)^p - \sum_{n=1}^{\infty} a_{n+p}(z-w)^{n+p} \qquad (a_{n+p} \geq 0,\ p \in \mathbb{N}). \tag{1.2}$$

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For the functions $f \in S(p, w)$ given by (1.2) and $g \in S(p, w)$ defined by

$$g(z) = (z - w)^p - \sum_{n=1}^{\infty} b_{n+p}(z - w)^{n+p} \qquad (b_{n+p} \geq 0,\ p \in \mathbb{N}),$$

we define the Hadamard product of *f* and *g* by

$$(f * g)(z) = (z - w)^p - \sum_{n=1}^{\infty} a_{n+p} b_{n+p}(z - w)^{n+p}.$$

For $a \in \mathbb{R}, c \in \mathbb{R}\backslash\mathbb{Z}_0^-$, with $\mathbb{Z}_0^- = \{0, -1, -2, \ldots\}$, $0 \leq \delta < 1$, $p \in \mathbb{N}$, $\tau > -p$, $\alpha, \beta \in \mathbb{R}$ with $\alpha + \beta - p < 1$ and $f \in S(p, w)$. The linear operator $\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c)$: $S(p, w) \to S(p, w)$ (see [3]) is defined by

$$\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z) = (z - w)^p + \sum_{n=1}^{\infty} \varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}(z - w)^{n+p}, \quad (1.3)$$

where

$$\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) = \frac{(c)_n (p + 1 - \alpha)_n (p + 1 - \delta + \beta)_n (\tau + p)_n}{(a)_n (p + 1)_n (p + 1 - \alpha + \beta)_n n!}. \quad (1.4)$$

Now, we define the class $S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$ consisting the functions $f \in S(p, w)$ such that

$$\left| \frac{(z - w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''' - (p - 2)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''}{\lambda(z - w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''' + (\eta - \mu)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''} \right| < 1, \quad (1.5)$$

where $0 \leq \lambda < 1$, $0 < \eta \leq 1$, $0 \leq \mu < 1$, $p \in \mathbb{N}$ and $p > 2$.

We note other studies of various other classes with different results, like, Ghanim and Darus [2], Najafzadeh and Rahimi [4], Shenan [5], Atshan and Wanas [1] and Wanas [6].

2. Main Results

In the first theorem, we find sharp coefficient estimates for the class $S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$.

Theorem 2.1. *Let* $f \in S(p, w)$. *Then* $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$ *if and only if*

$$\sum_{n=1}^{\infty}(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}$$

$$\leq p(p-1)(\eta-\mu+\lambda(p-2)), \tag{2.1}$$

where $0 \leq \lambda < 1,\ 0 < \eta \leq 1,\ 0 \leq \mu < 1$ *and* $\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)$ *is given by* (1.4).

The result is sharp for the function f given by

$$f(z) = (z-w)^p - \frac{p(p-1)(\eta-\mu+\lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}(z-w)^{n+p} \quad (n \geq 1). \tag{2.2}$$

Proof. Suppose that the inequality (2.1) holds true and $(z-w) \in \partial U$, where ∂U denotes the boundary of U. Then, we find from (1.5) that

$$|(z-w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c)f(z))''' - (p-2)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c)f(z))''|$$

$$-|\lambda(z-w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c)f(z))''' + (\eta-\mu)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c)f(z))''|$$

$$= \left| -\sum_{n=1}^{\infty} n(n+p)(n+p-1)\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}(z-w)^{n+p-2} \right|$$

$$-\Big| p(p-1)(\eta-\mu+\lambda(p-2))(z-w)^{p-2} - \sum_{n=1}^{\infty}(n+p)(n+p-1)$$

$$\times(\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}(z-w)^{n+p-2}\Big|$$

$$\leq \sum_{n=1}^{\infty} n(n+p)(n+p-1)\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}(z-w)^{n+p-2}$$

$$- p(p-1)(\eta - \mu + \lambda(p-2))|z-w|^{p-2}$$

$$+ \sum_{n=1}^{\infty} (n+p)(n+p-1)$$

$$\times (\eta - \mu + \lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}|z-w|^{n+p-2}$$

$$= \sum_{n=1}^{\infty} (n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}$$

$$- p(p-1)(\eta - \mu + \lambda(p-2)) \leq 0.$$

Hence, by maximum modulus theorem, we conclude $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$.

Conversely, suppose that $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$. Then from (1.3), we have

$$\left| \frac{(z-w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''' - (p-2)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''}{\lambda(z-w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''' + (\eta-\mu)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z))''} \right|$$

$$= \left| \frac{\sum_{n=1}^{\infty} n(n+p)(n+p-1)\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}(z-w)^{n+p-2}}{\begin{array}{l} p(p-1)(\eta-\mu+\lambda(p-2))(z-w)^{p-2} - \sum_{n=1}^{\infty}(n+p)(n+p-1) \\ \times (\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p) a_{n+p}(z-w)^{n+p-2} \end{array}} \right|$$

$$< 1.$$

So, we obtain

$$\operatorname{Re}\left\{\frac{\sum_{n=1}^{\infty} n(n+p)(n+p-1)\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}(z-w)^{n+p-2}}{\begin{array}{l} p(p-1)(\eta-\mu+\lambda(p-2))(z-w)^{p-2}-\sum_{n=1}^{\infty}(n+p)(n+p-1) \\ \times(\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}(z-w)^{n+p-2} \end{array}}\right\} < 1.$$

By letting $(z-w) \to 1^{-}$, through real values, we have

$$\sum_{n=1}^{\infty}(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)a_{n+p}$$
$$\leq p(p-1)(\eta-\mu+\lambda(p-2)).$$

Corollary 2.1. *Let* $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$. *Then*

$$a_{n+p} \leq \frac{p(p-1)(\eta-\mu+\lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} \quad (n \geq 1).$$

In the next result, we discuss extreme points for the class $S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$.

Theorem 2.2. *Let* $f_p(z) = (z-w)^p$ *and*

$$f_{n+p}(z) = (z-w)^p - \frac{p(p-1)(\eta-\mu+\lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}(z-w)^{n+p} \quad (n \geq 1).$$

Then $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$ *if and only if it can be expressed in the form*

$$f(z) = \sum_{n=0}^{\infty} \gamma_{n+p} f_{n+p}(z), \tag{2.3}$$

where $\gamma_{n+p} \geq 0,\ \sum_{n=0}^{\infty} \gamma_{n+p} = 1.$

Proof. Let the f of the form (2.3). Then

$$f(z) = \gamma_p f_p(z) + \sum_{n=1}^{\infty} \gamma_{n+p} \Bigg((z-w)^p - \frac{p(p-1)(\eta - \mu + \lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} (z-w)^{n+p} \Bigg)$$

$$= (z-w)^p - \sum_{n=1}^{\infty} \frac{p(p-1)(\eta - \mu + \lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} \times \gamma_{n+p}(z-w)^{n+p}.$$

Now,

$$\sum_{n=1}^{\infty} \frac{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(\eta-\mu+\lambda(p-2))} \times \frac{p(p-1)(\eta - \mu + \lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} \gamma_{n+p}$$

$$= \sum_{n=1}^{\infty} \gamma_{n+p} = 1 - \gamma_p \leq 1.$$

Thus $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$.

Conversely, let $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$. It follows from Corollary 2.1 that

$$a_{n+p} \leq \frac{p(p-1)(\eta - \mu + \lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} \ (n \geq 1).$$

Setting

$$\gamma_{n+p} = \frac{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(\eta-\mu+\lambda(p-2))} a_{n+p} \quad (n \geq 1)$$

and $\gamma_p = 1 - \sum_{n=1}^{\infty} \gamma_{n+p}$, we have

$$f(z) = (z-w)^p - \sum_{n=1}^{\infty} a_{n+p}(z-w)^{n+p}$$

$$= (z-w)^p - \sum_{n=1}^{\infty} \frac{p(p-1)(\eta-\mu+\lambda(p-2))}{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)} \times \gamma_{n+p}(z-w)^{n+p}$$

$$= (z-w)^p - \sum_{n=1}^{\infty} ((z-w)^p - f_{n+p}(z))\gamma_{n+p}$$

$$= \left(1 - \sum_{n=1}^{\infty} \gamma_{n+p}\right)(z-w)^p + \sum_{n=1}^{\infty} \gamma_{n+p} f_{n+p}(z)$$

$$= \gamma_p f_p(z) + \sum_{n=1}^{\infty} \gamma_{n+p} f_{n+p}(z) = \sum_{n=0}^{\infty} \gamma_{n+p} f_{n+p}(z),$$

that is the required representation.

In the following theorem, we establish integral representation for functions belongs to the class $S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$.

Theorem 2.3. *Let* $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$. *Then*

$$\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a, c) f(z) = \int_0^z \int_0^z \exp\left[\int_0^z \frac{(\eta-\mu)\psi(t_1) + p - 2}{(t_1 - w)(1 - \lambda\psi(t_1))} dt_1\right] dt_2 dt_3,$$

where $|\psi(z)| < 1$, $z \in U$.

Proof. By putting $\dfrac{(z-w)(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))'''}{(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))''} = Q(z)$ in (1.5), we have

$$\left|\frac{Q(z)-p+2}{\lambda Q(z)+\eta-\mu}\right| < 1,$$

or equivalently

$$\frac{Q(z)-p+2}{\lambda Q(z)+\eta-\mu} = \psi(z), \ \ (|\psi(z)| < 1, \ z \in U).$$

So

$$\frac{(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))'''}{(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))''} = \frac{(\eta-\mu)\psi(z)+p-2}{(z-w)(1-\lambda\psi(z))},$$

after integration, we get

$$\log((\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))'') = \int_0^z \frac{(\eta-\mu)\psi(t_1)+p-2}{(t_1-w)(1-\lambda\psi(t_1))}\,dt_1.$$

Therefore

$$(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))'' = \exp\left[\int_0^z \frac{(\eta-\mu)\psi(t_1)+p-2}{(t_1-w)(1-\lambda\psi(t_1))}\,dt_1\right].$$

By integration once again, we have

$$(\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z))' = \int_0^z \exp\left[\int_0^z \frac{(\eta-\mu)\psi(t_1)+p-2}{(t_1-w)(1-\lambda\psi(t_1))}\,dt_1\right]dt_2.$$

Also, after integration, we conclude that

$$\mathcal{L}_{\alpha,\beta,\delta}^{p,w,\tau}(a,c)f(z) = \int_0^z\int_0^z \exp\left[\int_0^z \frac{(\eta-\mu)\psi(t_1)+p-2}{(t_1-w)(1-\lambda\psi(t_1))}\,dt_1\right]dt_2\,dt_3$$

and this the required result.

Theorem 2.4. *If* $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$, *then* f *is starlike of order* θ

$(0 \le \theta < p)$ *in the disk* $|z - w| < r_1$, *where*

$$r_1 = \inf_n \left\{ \frac{(n+p)(p-\theta)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(n+p-\theta)(\eta-\mu+\lambda(p-2))} \right\}^{\frac{1}{n}}.$$

The result is sharp for the function f given by (2.2).

Proof. It is sufficient to show that

$$\left| \frac{(z-w)f'(z)}{f(z)} - p \right| \le p - \theta \quad \text{for } |z - w| < r_1. \tag{2.4}$$

But

$$\left| \frac{(z-w)f'(z)}{f(z)} - p \right| = \left| \frac{-\sum_{n=1}^{\infty} n a_{n+p}(z-w)^{n+p}}{(z-w)^p - \sum_{n=1}^{\infty} a_{n+p}(z-w)^{n+p}} \right| \le \frac{\sum_{n=1}^{\infty} n a_{n+p}|z-w|^n}{1 - \sum_{n=1}^{\infty} a_{n+p}|z-w|^n}.$$

Thus (2.4) will be satisfied if

$$\frac{\sum_{n=1}^{\infty} n a_{n+p}|z-w|^n}{1 - \sum_{n=1}^{\infty} a_{n+p}|z-w|^n} \le p - \theta,$$

or if

$$\sum_{n=1}^{\infty} \frac{(n+p-\theta)}{(p-\theta)} a_{n+p}|z-w|^n \le 1, \tag{2.5}$$

with the aid of (2.1), (2.5) is true if

$$\frac{(n+p-\theta)}{(p-\theta)}|z-w|^n$$

$$\le \frac{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(\eta-\mu+\lambda(p-2))},$$

or equivalently

$$|z-w| \le \left\{\frac{(n+p)(p-\theta)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(n+p-\theta)(\eta-\mu+\lambda(p-2))}\right\}^{\frac{1}{n}} \quad (n \ge 1),$$

which follows the result.

Theorem 2.5. *If* $f \in S_p^w(\lambda, \eta, \mu, \tau, \alpha, \beta, \delta)$, *then* f *is convex of order* θ $(0 \le \theta < p)$ *in the disk* $|z-w| < r_2$, *where*

$$r_2 = \inf_n \left\{\frac{(p-\theta)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{(p-1)(n+p-\theta)(\eta-\mu+\lambda(p-2))}\right\}^{\frac{1}{n}}.$$

The result is sharp for the function f given by (2.2).

Proof. It is sufficient to show that

$$\left|\frac{(z-w)f''(z)}{f'(z)} + 1 - p\right| \le p - \theta \quad \text{for } |z-w| < r_2. \tag{2.6}$$

But

$$\left|\frac{(z-w)f''(z)}{f'(z)} + 1 - p\right| = \left|\frac{-\sum_{n=1}^{\infty} n(n+p)a_{n+p}(z-w)^{n+p-1}}{p(z-w)^{p-1} - \sum_{n=1}^{\infty}(n+p)a_{n+p}(z-w)^{n+p-1}}\right|$$

$$\le \frac{\sum_{n=1}^{\infty} n(n+p)a_{n+p}|z-w|^n}{p - \sum_{n=1}^{\infty}(n+p)a_{n+p}|z-w|^n}.$$

Thus (2.6) will be satisfied if

$$\frac{\sum_{n=1}^{\infty} n(n+p)a_{n+p}|z-w|^n}{p-\sum_{n=1}^{\infty}(n+p)a_{n+p}|z-w|^n} \leq p-\theta,$$

or if

$$\sum_{n=1}^{\infty} \frac{(n+p)(n+p-\theta)}{p(p-\theta)} a_{n+p}|z-w|^n \leq 1, \tag{2.7}$$

with the aid of (2.1), (2.7) is true if

$$\frac{(n+p)(n+p-\theta)}{p(p-\theta)}|z-w|^n$$

$$\leq \frac{(n+p)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{p(p-1)(\eta-\mu+\lambda(p-2))},$$

or equivalently

$$|z-w| \leq \left\{\frac{(p-\theta)(n+p-1)(n+\eta-\mu+\lambda(n+p-2))\varphi(a, c, \alpha, \beta, \delta, \tau, n, p)}{(p-1)(n+p-\theta)(\eta-\mu+\lambda(p-2))}\right\}^{\frac{1}{n}}$$

$$(n \geq 1),$$

which follows the result.

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