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Mathematical Model for the Control of Unemployment on Ghana's Labour Market

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Abstract

Unemployment is one of the major socioeconomic issues across the globe in which Ghana is no exception. The unavailability of jobs and its creation as being searched by persons belonging to the labour force actively looking for jobs makes the problem escalate rapidly in a growing economy. In this seven state model, we analysed into the three main economic sectors of Ghana to investigate how unemployment, employment, and newly created vacancy creation behave at equilibrium on the three economic sector levels. Moreover, we analysed how in a specific sector, the dynamics of the state variables control unemployment. Further analyses on the parameters indicated that, an increase in the rate of newly created vacancies results in a decrease in the number of unemployed persons and an increase in the number of employed. We assumed that a jobless person who is available for work but fails to make an effort to seek work is not part of the unemployed class among others. It was established that the model has one nonnegative equilibrium point. Lastly, we analyse the impact of perturbation of some parameters on the number of the unemployed and employed persons at equilibrium.

1. Introduction

In recent decades, computers have taken the place of a number of jobs, including bookkeepers, cashiers, operators and etc., Bresnahan [1]. According to Manyika et al., [2], there has been an overturn of events with the introduction of technologies. Inversely, industrial robots have taken on physically difficult, dangerous, or dirty jobs, such as welding, spray painting, manufacturing, aviation among others, Manyika et al., [2]. It is not surprising that new technologies make certain forms of human labour unnecessary or

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economically uncompetitive and create demand for new skills, Manyika et al., [2]. The impact of technological progress on unemployment requires an examination to which susceptible jobs are likely to be computerised. More recently, weak labour market performance in advanced economies has heightened the discussion on sectoral and industrial unemployment among economies. Ghana's economic drive depends on sector indicators for employment growth on job creation. The country's economic sectors are grouped into three main sectors thus, agriculture, industry and services according to GSS [3]. Due to limited sectoral employment opportunities within the economy, employment responses to growth have been weak and unemployment continue to rise, Aryeetey and Baah-Boateng [4]. In 2021, youth unemployment rate stood at 19.7% [5]. By 2035, the youth will overtake all other age groups in Ghana's population, a situation that presents opportunities and challenges [5]. The need to have a global target on closing the unemployment gap, gave rise to the United Nations Sustainable Development Goals 8, part of the 17 global goals, which included achieving decent work and economic growth by 2030 [6]. The responsiveness to unemployment crisis and the long term effect it carries cannot be undermined with the ongoing economic upheaval, Brand [7]. In the sub Saharan African countries, employment dynamics have witnessed new twist with the evolution of technology and inventions, pandemics, population growth, infrastructure expansion, agriculture, economic performance and other important indicators, Bloom and Freeman [8]. The problem of unemployment is very evident and severe in the West African region, Kazeem et al. [9]. There exist wide variation between labour force (skilled and unskilled) and available jobs in Ghana making unemployment a serious economic crisis that needs to be tackled, [10]. Due to this socioeconomic emergency, attempts have been made by many researchers to find solutions to the crisis in different areas some of which includes the use of econometric models, mathematical models and other stochastic models, Aryeetey and Baah-Boateng [4]. By multidisciplinary publications, several models have been proposed to study this phenomenon in different countries. Pathan and Bhathawala [11] approached the unemployment from a different aspect, developed a nonlinear dynamic model of four variables for unemployment, namely: number of unemployed persons, the number of present jobs in the market, the number of employed persons and newly created vacancies. Due to this economic gab, this research seeks to address the issue in a developing country Ghana, with limited financial resources. The number of unemployed persons in the Agriculture Sector represented by U_A, number of unemployed persons in the Industry Sector denoted by U_I, number of unemployed persons in the Service Sector U_S, Number of employed persons in the

Agriculture Sector be E_A, Number of employed persons in the Industry Sector E_I, Number of employed persons in the Service Sector is E_S, and Newly created vacancies by all three sectors C.

2. Mathematical Model

In the process of making the model, we assume that an increase in the number of unemployed is different across all three sectors. The rate of death is same for unemployed and employed populations. Vacancies can be created due to death, resignation or retirement of persons across the sectors. A jobless person who is available for work but fails to make an effort to seek work is not part of the unemployed. As some of the unemployed persons become employed, the rate of movement of unemployed persons joining the Agriculture Sector is denoted by a_2, the rate of movement of unemployed persons joining an Industry Sector denoted by a_3, rate of movement of unemployed persons joining Services Sector denoted by a_4. Where a_5 is a positive constant which represents the rate of retirement and death of employed persons from a sector. a_6 denotes the rate of employed persons getting unemployed and the rate of resignation of employed persons out of a sector is denoted by a_7. Lastly, the creation of vacancies is highly dependent on the number of unemployed and employed persons in the given sectors. αandδ are the rate of newly created vacancies by the government and private sectors in relation to the unemployed and employed persons respectively. β represents the diminution of newly created vacancies by all sectors at time t.

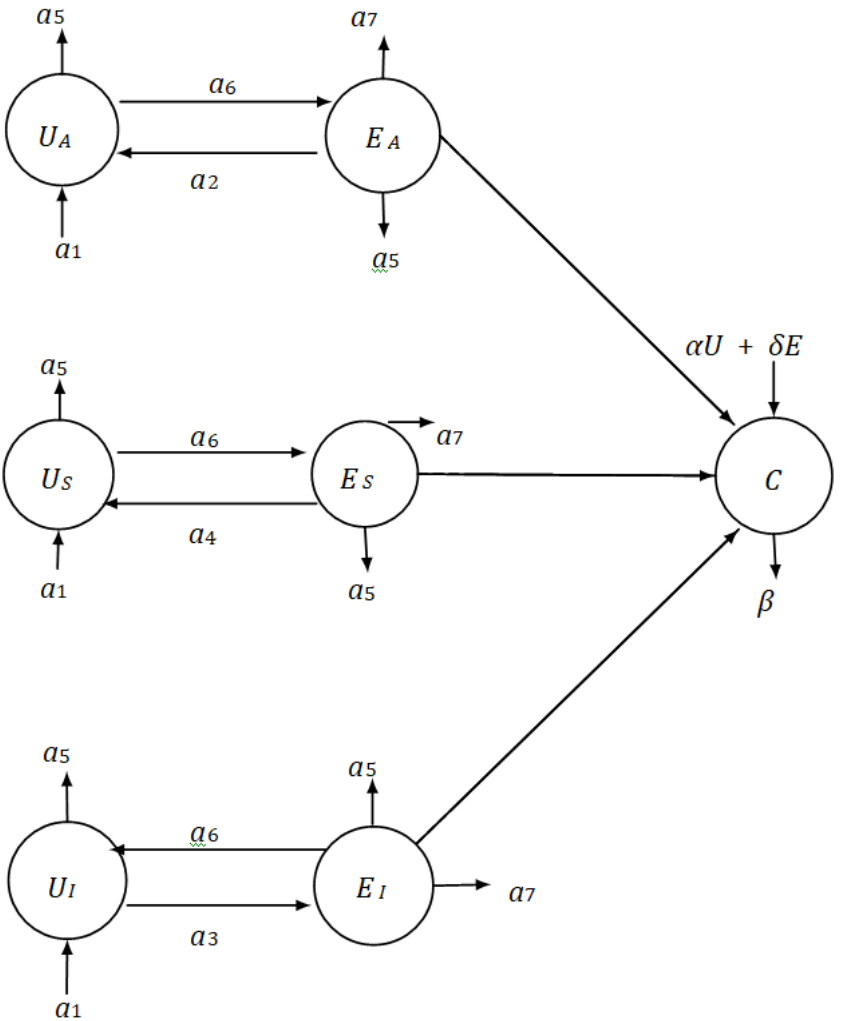

Figure 1. Schematic diagram of the model.

2.1. Positivity and boundedness of solutions

We ascertain boundedness and positivity of the system. Under suitable conditions, all solutions of system (1) are nonnegative.

Theorem 1. *Let all parameters in system* (1) *be positive and* Ω *be the region in* $\mathbb{R}_+^7$ *defined by*

$$\Omega = \left\{(U_A, U_I, U_S, E_A, E_I, E_S, C) \in \mathbb{R}_+^7 \mid 0 \leq U_A + U_I + U_S + E_A + E_I + E_S + C \leq \frac{3a_1}{\gamma}\right\}$$

where $\gamma = min\{(a_5), (a_5 + a_7 - \delta), \beta\}$ *is a region of attraction for the system* (1) *and it attracts all solutions initiating in the interior of the positive octant. Therefore, the solution is positive and bounded.*

2.2. Existence of equilibrium point

The system (1) has a non-negative equilibrium point as

$$U_A^1 = \frac{a_1a_2\delta + \left(Aa_2a_5 - a_1a_2 + a_5^2 + a_5a_6 + (Aa_2 + a_5)a_7\right)\beta + \sqrt{X_1}}{2(a_2a_5\beta - a_2a_5\delta + (a_2a_5 + a_2a_7)\alpha)}$$

$$U_I^1 = \frac{a_1a_3\delta + \left(Ia_3a_5 - a_1a_3 + a_5^2 + a_5a_6 + (Ia_3 + a_5)a_7\right)\beta + \sqrt{X_2}}{2(a_3a_5\beta - a_3a_5\delta + (a_3a_5 + a_3a_7)\alpha)}$$

$$U_S^1 = \frac{a_1a_4\delta + \left(Sa_4a_5 - a_1a_4 + a_5^2 + a_5a_6 + (Sa_4 + a_5)a_7\right)\beta + \sqrt{X_3}}{2(a_4a_5\beta - a_4a_5\delta + (a_4a_5 + a_4a_7)\alpha)}$$

$$E_A^1 = \frac{a_1a_2a_5\delta - 2(a_1a_2a_5 + a_1a_2a_7)\alpha - \left(Aa_2a_5^2 + a_1a_2a_5 + a_5^3 + a_5^2a_6 + (Aa_2a_5 + a_5^2)a_7\right)\beta - \sqrt{X_4}a_5}{2\left((a_2a_5^2 + 2a_2a_5a_7 + a_2a_7^2)\alpha + (a_2a_5^2 + a_2a_5a_7)\beta - (a_2a_5^2 + a_2a_5a_7)\delta\right)}$$

$$E_I^1 = \frac{a_1a_3a_5\delta - 2(a_1a_3a_5 + a_1a_3a_7)\alpha - \left(Ia_3a_5^2 + a_1a_3a_5 + a_5^3 + a_5^2a_6 + (Ia_3a_5 + a_5^2)a_7\right)\beta - \sqrt{X_5}a_5}{2\left((a_3a_5^2 + 2a_3a_5a_7 + a_3a_7^2)\alpha + (a_3a_5^2 + a_3a_5a_7)\beta - (a_3a_5^2 + a_3a_5a_7)\delta\right)}$$

$$E_S^1 = \frac{a_1a_4a_5\delta - 2(a_1a_4a_5 + a_1a_4a_7)\alpha - \left(Sa_4a_5^2 + a_1a_4a_5 + a_5^3 + a_5^2a_6 + (Sa_4a_5 + a_5^2)a_7\right)\beta - \sqrt{X_6}a_5}{2\left((a_4a_5^2 + 2a_4a_5a_7 + a_4a_7^2)\alpha + (a_4a_5^2 + a_4a_5a_7)\beta - (a_4a_5^2 + a_4a_5a_7)\delta\right)}$$

$$C^1 = \frac{X_7 + \sqrt{X_8}\left((a_5 + a_7)\alpha - a_5\delta\right)}{2\left((a_2a_5^2 + 2a_2a_5a_7 + a_2a_7^2)\alpha\beta + (a_2a_5^2 + a_2a_5a_7)\beta^2 - (a_2a_5^2 + a_2a_5a_7)\beta\delta\right)}$$

was obtained by solving the following set of algebraic equations;

$$F_1 = a_1 - a_2U_A(A + C - E_A) - a_5U_A + a_6E_A = 0$$

$$F_2 = a_1 - a_3U_I(I + C - E_I) - a_5U_I + a_6E_I = 0$$

$$F_3 = a_1 - a_4U_S(S + C - E_S) - a_5U_S + a_6E_S = 0$$

$$F_4 = a_2U_A(A + C - E_A) - a_5E_A - a_6E_A - a_7E_A = 0$$

$$F_5 = a_3U_I(I + C - E_I) - a_5E_I - a_6E_I - a_7E_I = 0$$

$$F_6 = a_4U_S(S + C - E_S) - a_5E_S - a_6E_S - a_7E_S = 0$$

$$F_7 = \alpha(U_A + U_I + U_S) + \delta(E_A + E_I + E_S) - \beta C = 0. \quad (1)$$

3. Results

3.1. Characteristics of equilibrium values

Here, we analyse the impact of perturbation of the values of some parameters on the number of the unemployed and employed persons at equilibrium. The parameters to be considered are:

i. The rate of movement of the unemployed class to employed class in Agriculture, Industry and Services respectively, thus: $\boldsymbol{a_2}$, $\boldsymbol{a_3}$ and $\boldsymbol{a_4}$.

ii. The rate of newly created vacancies in relation to the unemployed person α.

iii. The rate of newly created vacancies in relation to the employed persons δ.

3.2. Characteristics of rate of movement of unemployed persons joining the employed in agriculture sector ($\boldsymbol{a_2}$) at equilibrium

The characteristic of the equilibrium values of unemployed persons and employed persons with respect to a_2 (on rate of movement of unemployed persons joining the employed in Agriculture sector) is analysed to investigate it behaviour on the system.

$$F_1(U_A^1, U_I^1, U_S^1, E_A^1, E_I^1, E_S^1, C^1) = a_1 - a_2 U_A^1 (A + C^1 - E_A^1) - a_5 U_A^1 + a_6 E_A^1 \quad (2)$$

$$F_4(U_A^1, U_I^1, U_S^1, E_A^1, E_I^1, E_S^1, C^1) = a_2 U_A^1 (A + C^1 - E_A^1) - a_5 E_A^1 - a_6 E_A^1 - a_7 E_A^1 \quad (3)$$

$$\frac{dU_A^1}{da_2} = \frac{\begin{vmatrix} \frac{\partial F_1}{\partial E_A^1} & \frac{\partial F_1}{\partial a_2} \\ \frac{\partial F_4^1}{\partial E_A^1} & \frac{\partial F_A}{\partial a_2} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1}{\partial U_A^1} & \frac{\partial F_1}{\partial E_A} \\ \frac{\partial E_4^1}{\partial U_A^1} & \frac{\partial F_4}{\partial E_A^1} \end{vmatrix}} = \frac{\frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_A}{\partial a_2} - \frac{\partial F_1}{\partial a_2} \cdot \frac{\partial F_4}{\partial E_A^1}}{\frac{\partial U_A^1}{\partial E_A^1} - \frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_4}{\partial U_A^4}}$$

$$\frac{\partial F_1}{\partial U_A^1} = -a_2\left(A + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right) - a_5$$
$$\frac{\partial F_1}{\partial E_A^1} = a_2 U_A^1 + a_6 - \frac{a_2\delta}{\beta}U_A^1$$
$$\frac{\partial F_1}{\partial a_2} = -U_A^1\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$\frac{\partial F_4}{\partial U_A^1} = a_2\left(A + \frac{\alpha\left(U_A^1, U_J^1, U_S^1\right) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$\frac{\partial F_4}{\partial E_A^1} = -a_2 U_A^1 + \frac{a_2\delta}{\beta}U_A^1 - (a_5 + a_6 + a_7)$$
$$\frac{\partial F_4}{\partial a_2} = U_A^1\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$\frac{\partial F_1}{\partial E_A^1}\cdot\frac{\partial F_4}{\partial a_2} - \frac{\partial F_1}{\partial a_2}\cdot\frac{\partial F_4}{\partial E_A^1} = -(a_5 U_A^1 + a_7 U_A^1)\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta\left(E_A^1, E_I^1, E_S^1\right)}{\beta}\right)$$
$$< 0$$
$$\frac{\partial F_1}{\partial U_A^1}\cdot\frac{\partial F_4}{\partial E_A^1} - \frac{\partial F_1}{\partial E_A^1}\cdot\frac{\partial F_4}{\partial U_A^1} = (a_5 + a_7)\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$+a_2 a_5 U_A^1 + a_5^2 + a_5 a_7$$
$$> 0$$
$$\frac{dU_A^1}{da_2} < 0$$

Thus, a_2 increases, then U_A^1 decreases.

Now,

$$\Rightarrow \frac{\partial F_1}{\partial a_2}\cdot\frac{\partial F_4}{\partial U_A^1} - \frac{\partial F_1}{\partial U_A^1}\cdot\frac{\partial F_4}{\partial a_2} = a_5 U_A^1\left(A - E_A^1 + \frac{\alpha\left(U_A^1, U_J^1, U_S^1\right) + \delta\left(E_A^1, E_J^1, E_S^1\right)}{\beta}\right)$$
$$> 0$$
$$\frac{\partial F_1}{\partial U_A^1}\cdot\frac{\partial F_4}{\partial E_A^1} - \frac{\partial F_1}{\partial E_A^1}\cdot\frac{\partial F_4}{\partial U_A^1} = (a_5 + a_7)\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$+a_2 a_5 U_A^1 + a_5^2 + a_5 a_7$$
$$> 0$$
$$\frac{dE_A^1}{da_2} > 0$$

Thus, a_2 increases, then E_A^1 increases.

3.3. Characteristics of rate of movement of unemployed persons joining the employed in industry sector (a_3) at equilibrium.

The characteristic of the equilibrium values of unemployed persons and employed persons with respect to a_3 (the rate of movement of unemployed persons joining the employed in industry sector) is analysed to investigate it behaviour on the system.

$$F_2\left(U_A^1, U_l^1, U_S^1, E_A^1, E_l^1, E_S^1, C^1\right) = a_1 - a_3 U_l^1 (I + C^1 - E_A^1) - a_5 U_A^1 + a_6 E_l^1 \tag{4}$$

$$F_5\left(\left(U_A^1, U_l^1, U_S^1, E_A^1, E_l^1, E_S^1, C^1\right)\right) = a_3 U_l^1 \left(I + C^1 - E_l^1\right) - a_5 E_l^1 - a_6 E_l^1 - a_7 E_l^1 \tag{5}$$

$$\frac{\partial F_2}{\partial U_I^1} = -a_3\left(I + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right) - a_5$$

$$\frac{\partial F_2}{\partial E_I^1} = a_3 U_l^1 + a_6 - \frac{a_2 \delta}{\beta} U_I^1$$

$$\frac{\partial F_2}{\partial a_3} = -U_I^1\left(I - E_I^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$\frac{\partial F_5}{\partial U_I^1} = a_3\left(I + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$\frac{\partial F_5}{\partial E_I^1} = -a_3 U_I^1 + \frac{a_3 \delta}{\beta} U_I^1 - (a_5 + a_6 + a_7)$$

$$\frac{\partial F_5}{\partial a_3} = U_A^1\left(A - E_I^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta\left(E_A^1, E_l^1, E_S^1\right)}{\beta}\right)$$

$$\frac{\partial F_2}{\partial E_I^1} \cdot \frac{\partial F_5}{\partial a_3} - \frac{\partial F_2}{\partial a_3} \cdot \frac{\partial F_5}{\partial E_I^1} = -(a_5 U_I^1 + a_7 U_I^1)\left(I - E_I^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$< 0$$

$$\frac{\partial F_2}{\partial U_I^1} \cdot \frac{\partial F_5}{\partial E_I^1} - \frac{\partial F_2}{\partial E_I^1} \cdot \frac{\partial F_5}{\partial U_I^1} = (a_5 + a_7)\left(A - E_A^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$+ a_2 a_5 U_I^1 + a_5^2 + a_5 a_7$$

$$> 0$$

$$\frac{dU_i^1}{da_3} < 0$$

Thus, a_3 increases, then U_I^1 decreases.

Now,

$$\Rightarrow \frac{\partial F_5}{\partial a_3}\cdot\frac{\partial F_5}{\partial U_l^1}-\frac{\partial F_5}{\partial U_I^1}\cdot\frac{\partial F_5}{\partial a_3}=\alpha_5 U_I^1\left(A-E_A^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_l^1,E_S^1\right)}{\beta}\right)$$
$$>0$$
$$\frac{\partial F_5}{\partial U_I^1}\cdot\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_5}{\partial E_I^1}\cdot\frac{\partial F_5}{\partial U_I^1}=(a_5+a_7)\left(A-E_I^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_I^1,E_I^1,E_S^1\right)}{\beta}\right)$$
$$+a_3a_5U_I^1+a_5^2+a_5a_7$$
$$>0$$

$$\frac{dE_I^1}{da_3}>0$$

Thus, a_3 increases, then E_I^1 increases.

3.4. Characteristics of rate of movement of unemployed persons joining the employed in services sector at equilibrium

The characteristic of the equilibrium values of unemployed persons and employed persons with respect to a_4 (on rate of movement of unemployed persons joining the employed in services sector) is analysed to investigate it behaviour on the system.

$$F_3\left(U_A^1,U_l^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1\right)=a_1-a_4U_S^1(S+C^1-E_S^1)-a_5U_S^1+a_6E_A^1 \tag{6}$$

$$F_6\left(\left(U_A^1,U_I^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1\right)\right)=a_4U_S^1(S+C^1-E_S^1)-a_5E_S^1-a_6E_S^1-a_7E_S^1 \tag{7}$$

$$\frac{\partial F_3}{\partial U_S^1}=-a_4\left(S+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)-a_5$$
$$\frac{\partial F_3}{\partial E_S^1}=a_4U_S^1+a_6-\frac{a_4\delta}{\beta}U_S^1$$
$$\frac{\partial F_3}{\partial a_4}=-U_S^1\left(S-E_S^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$\frac{\partial F_6}{\partial U_S^1}=a_4\left(S+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$\frac{\partial F_6}{\partial E_S^1}=-a_4U_S^1+\frac{a_4\delta}{\beta}U_S^1-(a_5+a_6+a_7)$$
$$\frac{\partial F_6}{\partial a_4}=U_S^1\left(S-E_S^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_l^1,E_S^1\right)}{\beta}\right)$$

$$\frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial a_4}-\frac{\partial F_3}{\partial a_4}\cdot\frac{\partial F_6}{\partial E_S^1}=-(a_5U_S^1+a_7U_S^1)\left(S-E_S^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)$$
$$<0$$
$$\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial U_S^1}=(a_5+a_7)\left(A-E_S^1+\frac{\alpha\left(U_A^1,U_I^1,U_S^1\right)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$+a_4a_5U_S^1+a_5^2+a_5a_7$$
$$>0$$

$$\frac{dU_S^1}{da_4}<0$$

Thus, a_4 increases, then $U_S{}^1$ decreases.

Now,

$$\Rightarrow\frac{\partial F_3}{\partial a_4}\cdot\frac{\partial F_6}{\partial U_S^1}-\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial a_4}=a_5U_S^1\left(S-E_S^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)$$
$$>0$$
$$\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial U_S^1}=(a_5+a_7)\left(S-E_S^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_I^1,E_S^1\right)}{\beta}\right)$$
$$+a_4a_5U_S^1+a_5^2+a_5a_7$$
$$>0$$

$$\frac{dE_S^1}{da_4}>0$$

Thus, a_4 increases, then E_S^1 increases.

3.5. Characteristics of the equilibrium value of unemployed persons with respect to α

Characteristics of the equilibrium value of unemployed persons with respect to α (rate of newly created vacancies in relation to the unemployed persons) is analysed to investigate it behaviour on the system.

$$F_1\left(U_A^1,U_l^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1,\alpha\right)=a_1-a_2U_A^1(A+C^1-E_A^1)-a_5U_A^1+a_6E_A^1 \quad (8)$$

$$F_4\left(\left(U_A^1,U_l^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1\right),\alpha\right)=a_2U_A^1(A+C^1-E_A^1)-a_5E_A^1-a_6E_A^1-a_7E_A^1 \quad (9)$$

$$\frac{dU_A^1}{d\alpha} = \frac{\begin{vmatrix} \frac{\partial F_1}{\partial E_A^1} & \frac{\partial F_1}{\partial \alpha} \\ \frac{\partial F_A^1}{\partial E_A^1} & \frac{\partial F_A}{\partial \alpha} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_1^1}{\partial U_A^A} & \frac{\partial F_1}{\partial E_A^1} \\ \frac{\partial U_A^1}{\partial} & \frac{\partial F_A}{\partial E_A^1} \end{vmatrix}} = \frac{\frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_A}{\partial \alpha} - \frac{\partial F_1}{\partial \alpha} \cdot \frac{\partial F_A}{\partial E_A^1}}{\frac{\partial F_1}{\partial U_A^1} \cdot \frac{\partial F_A}{\partial E_A} - \frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_A}{\partial U_A^1}}$$

$$\frac{\partial F_1}{\partial U_A^1} = -a_2\left(A + \frac{\alpha\left(U_A^1, U_l^1, U_S^1\right) + \delta\left(E_A^1, E_l^1, E_S^1\right)}{\beta}\right) - a_5$$

$$\frac{\partial F_1}{\partial E_A^1} = a_2 U_A^1 + a_6 - \frac{a_2\delta}{\beta} U_A^1$$

$$\frac{\partial F_1}{\partial \alpha} = -a_2 U_A^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$\frac{\partial F_4}{\partial U_A^1} = a_2\left(A + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$\frac{\partial F_4}{\partial E_A^1} = -a_2 U_A^1 + \frac{a_2\delta}{\beta} U_A^1 - (a_5 + a_6 + a_7)$$

$$\frac{\partial F_4}{\partial \alpha} = a_2 U_A^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$\frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_4}{\partial \alpha} - \frac{\partial F_1}{\partial \alpha} \cdot \frac{\partial F_4}{\partial E_A^1} = (a_2 a_5 + a_2 a_7) U_A^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$2 - \partial F_4 = \partial F_1\, \partial F_4$$

$$\frac{\partial F_1}{\partial U_A^1} \cdot \frac{\partial F_4}{\partial E_A^1} - \frac{\partial F_1}{\partial E_A^1} \cdot \frac{\partial F_4}{\partial U_A^1} = (a_5 + a_7)\left(A - E_A^1 + \frac{\alpha\left(U_A^1, U_l^1, U_S^1\right) + \delta\left(E_A^1, E_l^1, E_S^1\right)}{\beta}\right) + a_2 a_5 U_A^1 + a_5^2 + a_5 a_7 > 0$$

Then, by further simplification, we get

$$\frac{dU_A^1}{d\alpha} < 0$$

Thus, as α increases, U_A^1 decreases.

3.6. Characteristics of the equilibrium values on employed persons with δ

The characteristic of the equilibrium values on employed person vacancies with respect to δ (rate of newly created vacancies in relation to the employed persons) is analysed to investigate it behaviour on the system.

Now from equations (8) and (9),

$$\Rightarrow \frac{\partial F_1}{\partial \delta}\cdot\frac{\partial F_4}{\partial U_A^1}-\frac{\partial F_1}{\partial U_A^1}\cdot\frac{\partial F_4}{\partial \delta}=a_5U_A^1\left(A-E_A^1+\frac{\alpha(U_A^1,U_S^1,U_S^1)+\delta\left(E_A^1,E_J^1,E_S^1\right)}{\beta}\right)$$

$$>0$$

$$\frac{\partial F_1}{\partial U_A^1}\cdot\frac{\partial F_4}{\partial E_A^1}-\frac{\partial F_1}{\partial E_A^1}\cdot\frac{\partial F_4}{\partial U_A^1}=(a_5+a_7)\left(A-E_A^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_A^1,E_l^1,E_S^1\right)}{\beta}\right)$$

$$+a_2a_5U_A^1+a_5^2+a_5a_7$$

$$>0$$

$$\frac{dE_A^1}{d\delta}>0$$

Thus, as α increases, E_A^1 increases.

3.7. Characteristic of the equilibrium values on unemployed persons with respect to α

The characteristic of the equilibrium values on unemployed person with respect to α (rate of newly created vacancies in relation to the unemployed persons) is analysed to investigate it behaviour on the system.

$$F_2(U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1)=a_1-a_3U_I^1(I+C^1-E_A^1)-a_5U_A^1+a_6E_I^1 \tag{10}$$

$$F_5\left((U_A^1,U_I^1,U_S^1,E_A^1,E_I^1,E_S^1,C^1)\right)=a_3U_I^1(I+C^1-E_I^1)-a_5E_I^1-a_6E_I^1-a_7E_I^1 \tag{11}$$

$$\frac{\partial F_2}{\partial U_I^1}=-a_3\left(I+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)-a_5$$

$$\frac{\partial F_2}{\partial E_I^1}=a_3U_I^1+a_6-\frac{a_2\delta}{\beta}U_I^1$$

$$\frac{\partial F_2}{\partial \alpha}=-a_3U_I^1\left(\frac{(U_A^1,U_I^1,U_S^1)}{\beta}\right)$$

$$\frac{\partial F_5}{\partial U_I^1}=a_3\left(I+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$

$$\frac{\partial F_5}{\partial E_I^1}=-a_3U_I^1+\frac{a_3\delta}{\beta}U_I^1-(a_5+a_6+a_7)$$

$$\frac{\partial F_5}{\partial \alpha}=a_3U_I^1\left(\frac{(U_A^1,U_I^1,U_S^1)}{\beta}\right)$$

$$\frac{\partial F_2}{\partial E_I^1}\cdot\frac{\partial F_5}{\partial \alpha}-\frac{\partial F_2}{\partial \alpha}\cdot\frac{\partial F_5}{\partial E_I^1}=-(a_5U_I^1+a_7U_I^1)\left(I-E_I^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$<0$$
$$\frac{\partial F_2}{\partial U_I^1}\cdot\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_2}{\partial E_I^1}\cdot\frac{\partial F_5}{\partial U_I^1}=(a_5+a_7)\left(A-E_A^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$+a_2a_5U_I^1+a_5^2+a_5a_7$$
$$>0$$

$$\frac{dU_I^1}{d\alpha}<0$$

Thus, α increases, then U_I^1 decreases.

3.8. Characteristic of the equilibrium values on employed persons with respect to δ

The characteristic of the equilibrium values on employed persons with respect to δ (rate of newly created vacancies in relation to the employed persons) is analysed to investigate it behaviour on the system.

Now from equations (10) and (11),

$$\frac{\partial F_5}{\partial \delta}\cdot\frac{\partial F_5}{\partial U_I^1}-\frac{\partial F_5}{\partial U_I^1}\cdot\frac{\partial F_5}{\partial \delta}=a_5U_I^1\left(A-E_A^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta(E_A^1,E_I^1,E_S^1)}{\beta}\right)$$
$$\frac{\partial F_5}{\partial U_I^1}\cdot\frac{\partial F_5}{\partial E_I^1}-\frac{\partial F_5}{\partial E_I^1}\cdot\frac{\partial F_5}{\partial U_I^1}=(a_5+a_7)\left(A-E_I^1+\frac{\alpha(U_A^1,U_I^1,U_S^1)+\delta\left(E_l^1,E_I^1,E_S^1\right)}{\beta}\right)$$
$$+a_3a_5U_I^1+a_5^2+a_5a_7$$
$$>0$$

$$\frac{dE_I^1}{d\delta}>0$$

Thus, δ increases, then E_I^1 increases.

3.9. Characteristic of the equilibrium values on unemployed persons with respect to α

The characteristic of the equilibrium values on unemployed persons with respect to α (rate of newly created vacancies in relation to the unemployed persons) is analysed to investigate it behaviour on the system.

$$F_3\left(U_A^1,U_l^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1\right)=a_1-a_4U_S^1(S+C^1-E_S^1)-a_5U_S^1+a_6E_A^1 \quad (12)$$

$$F_6\left(\left(U_A^1,U_l^1,U_S^1,E_A^1,E_l^1,E_S^1,C^1\right)\right)=a_4U_S^1(S+C^1-E_S^1)-a_5E_S^1-a_6E_S^1-a_7E_S^1 \quad (13)$$

$$\frac{dU_S^1}{d\alpha} = \frac{\begin{vmatrix} \frac{\partial F_3}{\partial E_S} & \frac{\partial F_3}{\partial \alpha} \\ \frac{\partial F_6}{\partial E_S^1} & \frac{\partial F_6}{\partial \alpha} \end{vmatrix}}{\begin{vmatrix} \frac{\partial F_3}{\partial U_S^1} & \frac{\partial F_9}{\partial E_S^t} \\ \frac{\partial F_4}{\partial U_S^1} & \frac{\partial F_6}{\partial E_S^t} \end{vmatrix}} = \frac{\frac{\partial F_3}{\partial E_S^1} \cdot \frac{\partial F_6}{\partial \alpha} - \frac{\partial F_3}{\partial \alpha} \cdot \frac{\partial F_6}{\partial E_S^1}}{\frac{\partial F_9}{\partial U_S^1} \cdot \frac{\partial F_6}{\partial E_S^1} - \frac{\partial F_3}{\partial E_S^1} \cdot \frac{\partial F_9}{\partial U_S^t}}$$

$$\frac{\partial F_3}{\partial U_S^1} = -a_4\left(S + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right) - a_5$$

$$\frac{\partial F_3}{\partial E_S^1} = a_4 U_S^1 + a_6 - \frac{a_4\delta}{\beta} U_S^1$$

$$\frac{\partial F_3}{\partial \alpha} = -a_4 U_S^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$\frac{\partial F_6}{\partial U_S^1} = a_4\left(S + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta\left(E_A^1, E_J^1, E_S^1\right)}{\beta}\right)$$

$$\frac{\partial F_6}{\partial E_S^1} = -a_4 U_S^1 + \frac{a_4\delta}{\beta} U_S^1 - (a_5 + a_6 + a_7)$$

$$\frac{\partial F_6}{\partial \alpha} = a_4 U_S^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$\frac{\partial F_3}{\partial E_S^1} \cdot \frac{\partial F_6}{\partial \alpha} - \frac{\partial F_3}{\partial \alpha} \cdot \frac{\partial F_6}{\partial E_S^1} = -(a_5 U_S^1 + a_7 U_S^1)\left(S - E_S^1 + \frac{\alpha\left(U_A^1, U_l^1, U_S^1\right) + \delta\left(E_A^1, E_l^1, E_S^1\right)}{\beta}\right)$$

$$< 0$$

$$\frac{\partial F_3}{\partial U_S^1} \cdot \frac{\partial F_6}{\partial E_S^1} - \frac{\partial F_3}{\partial E_S^1} \cdot \frac{\partial F_6}{\partial U_S^1} = (a_5 + a_7)\left(A - E_S^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$

$$+a_4 a_5 U_S^1 + a_5^2 + a_5 a_7$$

$$> 0$$

$$\frac{dU_S^1}{d\alpha} < 0$$

$$\frac{\partial F_3}{\partial U_S^1} = -a_4\left(S + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right) - a_5$$
$$\frac{\partial F_3}{\partial E_S^1} = a_4 U_S^1 + a_6 - \frac{a_4\delta}{\beta} U_S^1$$
$$\frac{\partial F_3}{\partial \alpha} = -a_4 U_S^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$
$$\frac{\partial F_6}{\partial U_S^1} = a_4\left(S + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta\left(E_A^1, E_l^1, E_S^1\right)}{\beta}\right)$$
$$\frac{\partial F_6}{\partial E_S^1} = -a_4 U_S^1 + \frac{a_4\delta}{\beta} U_S^1 - (a_5 + a_6 + a_7)$$
$$\frac{\partial F_6}{\partial \alpha} = a_4 U_S^1\left(\frac{(U_A^1, U_I^1, U_S^1)}{\beta}\right)$$

$$\frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial \alpha} - \frac{\partial F_3}{\partial \alpha}\cdot\frac{\partial F_6}{\partial E_S^1} = -(a_5 U_S^1 + a_7 U_S^1)\left(S - E_S^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$< 0$$
$$\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial E_S^1} - \frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial U_S^1} = (a_5 + a_7)\left(A - E_S^1 + \frac{\alpha(U_A^1, U_I^1, U_S^1) + \delta(E_A^1, E_I^1, E_S^1)}{\beta}\right)$$
$$+a_4 a_5 U_S^1 + a_5^2 + a_5 a_7$$
$$> 0$$
$$\frac{dU_S^1}{d\alpha} < 0$$

Thus, α increases, then U_S^1 decreases.

3.10. Characteristic of the equilibrium values on employed persons with respect to δ

The characteristic of the equilibrium values on employed persons with respect to δ (rate of newly created vacancies in relation to the employed persons) is analysed to investigate its behaviour on the system.

Now from equations (12) and (13),

$$\frac{\partial F_3}{\partial \delta}\cdot\frac{\partial F_6}{\partial U_S^1}-\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial \delta}=a_5U_S^1\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_l^1,U_S^1\right)+\delta\left(E_A^1,E_l^1,E_S^1\right)}{\beta}\right)$$
$$>0$$
$$\frac{\partial F_3}{\partial U_S^1}\cdot\frac{\partial F_6}{\partial E_S^1}-\frac{\partial F_3}{\partial E_S^1}\cdot\frac{\partial F_6}{\partial U_S^1}=(a_5+a_7)\left(S-E_S^1+\frac{\alpha\left(U_A^1,U_l^1,U_S^1\right)+\delta\left(E_A^1,E_l^1,E_S^1\right)}{\beta}\right)$$
$$+a_4a_5U_S^1+a_5^2+a_5a_7$$
$$>0$$
$$\frac{dE_S^1}{d\delta}>0$$

Thus, δ increases, then E_S^1 increases.

From (1),

$$C^1=\frac{\alpha(U_A^1+U_I^1+U_S^1)+\delta(E_A^1+E_I^1+E_S^1)}{\beta}$$

Thus, the its derivative with respect to α gives

$$\frac{dC^1}{d\alpha}=\frac{U_A^1+U_I^1+U_S^1}{\beta}$$
$$>0$$

Therefore, as α increases, C^1 also increases.

4. Conclusions

It can be deduced that at the equilibrium, the number of unemployed persons decrease as the rate of movement from the unemployed class to employed a_2, a_3, a_4 increases across the three sectors, agriculture, services and industry. Thus, at equilibrium level, rate of movement of unemployed person to employed class grows with a decrease in the number of unemployed persons. For the number of employed persons, we observe an increase when the rate of movement from the unemployed class to employed a_2, a_3, a_4 increases across the three sectors. This is as expected since an increase in employed population is as a result of a decrease in the population of the unemployed. Further analyses on the parameter α indicates that, an increase in the rate of newly created vacancies results in a decrease in the number of unemployed persons and an increase in the number of employed persons in the agriculture, services and industry sectors. As more vacancies are created, it is expected to see a decrease in the unemployed class since they will be employed by the government or private sector into agriculture, industry or service sectors respectively.

Conflicts of Interest

There is no conflict of interest.

Authors' Contributions

The authors declare that the work was realized in collaboration with the same responsibility. All authors read and approved the final manuscript.

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