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Some New Kulli-Basava Topological Indices

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Abstract

Recently, Kulli-Basava indices were introduced and studied their mathematical and chemical properties which have good response with mean isomer degeneracy. In this paper, we introduce the modified first and second Kulli-Basava indices, F_1-Kulli-Basava index, square Kulli-Basava index of a graph, and compute exact formulas for regular graphs, wheels, gear graphs and helm graphs.

1. Introduction

Throughout this paper G is a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. Let $|V(G)| = n$ and $|E(G)| = m$. The degree of an edge $e = uv$ in G is defined by $d_G(e) = d_G(u) + d_G(v) - 2$. Let $S_e(v)$ denote the sum of degrees of all edges incident to a vertex v. We refer to [1] for undefined term and notation.

Recently, the first and second Kulli-Basava indices were introduced in [2], defined as

$$KB_1(G) = \sum_{uv \in E(G)} [S_e(u) + S_e(v)], \qquad KB_2(G) = \sum_{uv \in E(G)} S_e(u) S_e(v).$$

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We introduce the modified first and second Kulli-Basava indices, defined as

$$ {}^{m}KB_1(G) = \sum_{uv \in E(G)} \frac{1}{S_e(u) + S_e(v)}, \qquad {}^{m}KB_2(G) = \sum_{uv \in E(G)} \frac{1}{S_e(u) S_e(v)}. $$

In [3], Furtula and Gutman studied the F-index, defined as

$$ F(G) = \sum_{uv \in E(G)} [S_e(u)^2 + S_e(v)^2]. $$

Recently, the square *ve*-degree index was introduced by Kulli [4], defined as

$$ Q_{ve}(G) = \sum_{uv \in E(G)} [d_{ve}(u) - d_{ve}(v)]^2. $$

We now propose the F_1-Kulli-Basava and square Kulli-Basava indices, defined as

$$ F_1KB(G) = \sum_{uv \in E(G)} [S_e(u)^2 + S_e(v)^2], $$

$$ QKB(G) = \sum_{uv \in E(G)} [S_e(u) - S_e(v)]^2. $$

Recently, some F-indices were studied, for example, in [5, 6, 7, 8, 9, 10, 11] and also some square indices were studied, for example, in [12, 13, 14, 15, 16].

We introduce the F_1-Kulli-Basava polynomial and square Kulli-Basava polynomial of a graph, defined as

$$ F_1KB(G, x) = \sum_{uv \in E(G)} x^{S_e(u)^2 + S_e(v)^2}, $$

$$ QKB(G, x) = \sum_{uv \in E(G)} x^{[S_e(u) - S_e(v)]^2}. $$

In this paper, we establish explicit formulas for the modified first and second Kulli-Basava indices, F_1-Kulli-Basava and square Kulli-Basava indices of some graphs. Also the F_1-Kulli-Basava and square Kulli-Basava polynomials of some graphs are obtained.

2. Regular Graphs

Theorem 1. *Let G be an r-regular graph with n vertices and m edges. Then*

(i) $^{m}KB_1(G) = \frac{m}{4r(r-1)}$. (ii) $^{m}KB_2(G) = \frac{m}{4r^2(r-1)^2}$.

(iii) $F_1KB(G) = 8mr^2(r-1)^2$. (iv) $QKB(G) = 0$.

Proof. Let G be an r-regular graph with n vertices. Then $S_e(u) = 2r(r-1)$ for any vertex u in G.

Thus

(i) $$^{m}KB_1(G) = \sum_{uv\in E(G)} \frac{1}{S_e(u)+S_e(v)} = \frac{m}{2r(r-1)+2r(r-1)} = \frac{m}{4r(r-1)}.$$

(ii) $$^{m}KB_2(G) = \sum_{uv\in E(G)} \frac{1}{S_e(u)S_e(v)} = \frac{m}{2r(r-1)2r(r-1)} = \frac{m}{4r^2(r-1)^2}.$$

(iii) $$F_1KB(G) = \sum_{uv\in E(G)} [S_e(u)^2 + S_e(v)^2] = m[(2r(r-1))^2 + (2r(r-1))^2]$$
$$= 8mr^2(r-1)^2.$$

(iv) $$QKB(G) = \sum_{uv\in E(G)} (S_e(u)-S_e(v))^2 = 0.$$

Corollary 1.1. *If C_n is a cycle with n vertices, then*

(i) $^{m}KB_1(C_n) = \frac{n}{8}$. (ii) $^{m}KB_2(C_n) = \frac{n}{16}$.

(iii) $F_1KB(C_n) = 32n$. (iv) $QKB(C_n) = 0$.

Corollary 1.2. *If K_n is a complete graph with n vertices, then*

(i) $^{m}KB_1(K_n) = \frac{n}{8(n-2)}$. (ii) $^{m}KB_2(K_n) = \frac{n}{8(n-1)(n-2)^2}$.

(iii) $F_1KB(K_n) = 4n(n-1)^3(n-2)^2$. (iv) $QKB(K_n) = 0$.

Theorem 2. *If G is an r-regular graph with n vertices and m edges, then*

(i) $F_1KB(G, x) = mx^{8r^2(r-1)^2}$. (ii) $QKB(G, x) = mx^0$.

Proof. Let G be an r-regular graph with n vertices and m edges. Then $S_e(u) = 2r(r-1)$ for $u \in V(G)$. Thus

(i) $$F_1KB(G, x) = \sum_{uv \in E(G)} x^{S_e(u)^2 + S_e(v)^2} = mx^{(2r(r-1))^2 + (2r(r-1))^2} = mx^{8r^2(r-1)^2}.$$

(ii) $$QKB(G, x) = \sum_{uv \in E(G)} x^{[S_e(u) - S_e(v)]^2} = mx^0.$$

Corollary 2.1. *If* C_n *is a cycle with n vertices, then*

(i) $F_1KB(C_n, x) = nx^{32}$. (ii) $QKB(C_n, x) = nx^0$.

Corollary 2.2. *If* K_n *is a complete graph with n vertices, then*

(i) $F_1KB(K_n, x) = \frac{n(n-1)}{2} x^{8(n-1)^2(n-2)^2}$. (ii) $QKB(K_n, x) = \frac{n(n-1)}{2} x^0$.

3. Wheel Graphs

A wheel W_n is the join of K_1 and C_n. Clearly W_n has $n+1$ vertices and $2n$ edges. A wheel W_n is presented in Figure 1. The vertices of C_n are called *rim vertices* and the vertex of K_1 is called *apex*.

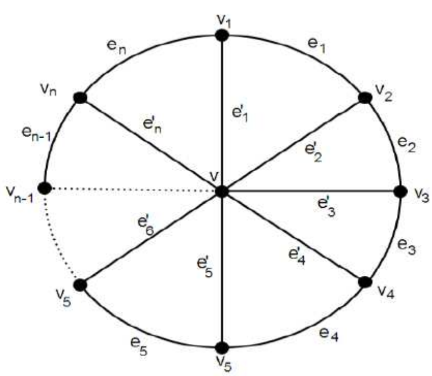

Figure 1. Wheel W_n.

Lemma 1. *Let* W_n *be a wheel with* $n+1$ *vertices and* $2n$ *edges,* $n \geq 3$. *Then*

$$E_1 = \{uv \in E(W_n) | S_e(u) = n+9, (S_e(v) = n(n+1))\}, \qquad |E_1| = n$$

$$E_2 = \{uv \in E(W_n) | S_e(u) = n+9, (S_e(v) = n+9)\}, \qquad |E_2| = n.$$

Theorem 3. *Let* W_n *be a wheel with* $n+1$ *vertices and* $2n$ *edges,* $n \geq 3$. *Then*

(i) $^m KB_1(W_n) = \dfrac{n}{n^2+2n+9} + \dfrac{n}{2n+18}.$

(ii) $^m KB_2(W_n) = \dfrac{1}{(n+9)(n+1)} + \dfrac{n}{(n+9)^2}.$

(iii) $F_1KB(W_n) = (n^3 + 5n^2 + 55n + 243).$

(iv) $QKB(W_n) = n(n^2 - 9)^2.$

Proof. By using definitions and Lemma 1, we derive

(i) $$^m KB_1(W_n) = \sum_{uv \in E(W_n)} \frac{1}{S_e(u) + S_e(v)}$$

$$= |E_1| \left(\frac{1}{n+9+n(n+1)} \right) + |E_2| \left(\frac{1}{n+9+n+9} \right)$$

$$= \frac{n}{n^2+2n+9} + \frac{n}{2n+18}.$$

(ii) $$^m KB_2(W_n) = \sum_{uv \in E(W_n)} \frac{1}{S_e(u) S_e(v)}$$

$$= |E_1| \left(\frac{1}{(n+9) \times n(n+1)} \right) + |E_2| \left(\frac{1}{(n+9)(n+9)} \right)$$

$$= \frac{1}{(n+9)(n+1)} + \frac{n}{(n+9)^2}.$$

(iii) $$F_1KB(W_n) = \sum_{uv \in E(W_n)} [S_e(u)^2 + S_e(v)^2]$$

$$= |E_1|[(n+9)^2 + (n(n+1))^2] + |E_2|[(n+9)^2 + (n+9)^2]$$

$$= n(n^3 + 5n^2 + 55n + 243).$$

(iv) $$QKB(W_n) = \sum_{uv \in E(W_n)} [S_e(u) - S_e(v)]^2$$

$$= |E_1|[(n+9) - (n(n+1))]^2 + |E_2|[(n+9) - (n+9)]^2$$

$$= n(n^2 - 9)^2.$$

Theorem 4. *Let W_n be a wheel with $n+1$ vertices and 2n edges, $n \geq 3$. Then*

(i) $F_1KB(W_n, x) = nx^{n^3+3n^2+19n+81} + nx^{2(n+9)^2}$.

(ii) $QKB(W_n, x) = nx^{(n^2-9)^2} + nx^0$.

Proof. By using definitions and Lemma 1, we deduce

(i) $$F_1KB(W_n, x) = \sum_{uv \in E(W_n)} x^{[S_e(u)^2 + S_e(v)^2]}$$

$$= |E_1| x^{(n+9)^2 + n^2(n+1)^2} + |E_2| x^{(n+9)^2 + (n+9)^2}$$

$$= nx^{n^3+3n^2+19n+81} + nx^{2(n+9)^2}.$$

(ii) $$QKB(W_n, x) = \sum_{uv \in E(W_n)} x^{[S_e(u) - S_e(v)]^2}$$

$$= |E_1| x^{[n+9-n(n+1)]^2} + |E_2| x^{[(n+9)-(n+9)]^2}$$

$$= nx^{(n^2-9)^2} + nx^0.$$

4. Gear Graphs

A graph is a gear graph obtained from W_n by adding a vertex between each pair of adjacent rim vertices and it is denoted by G_n. Clearly G_n has $2n+1$ vertices and 3n edges. A graph G_n is depicted in Figure 2.

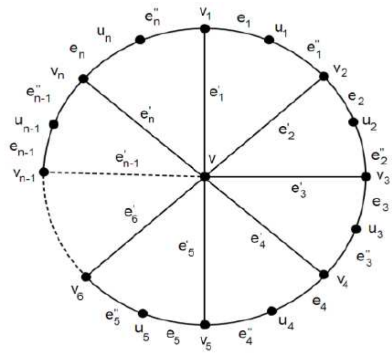

Figure 2. Gear graph G_n.

Lemma 2. *Let* G_n *be a gear graph with* $3n$ *edges. Then* G_n *has two types of edges as follows*:

$$E_1 = \{uv \in E(G_n) \mid S_e(u) = n(n+1),\ S_e(v) = n+7\}, \qquad |E_1| = n.$$

$$E_2 = \{uv \in E(G_n) \mid S_e(u) = n+7,\ S_e(v) = 6\}, \qquad |E_2| = 2n.$$

Theorem 5. *If* G_n *is a gear graph with* $2n+1$ *vertices and* $3n$ *edges*, *then*

(i) $^{m}KB_1(G_n) = \dfrac{n}{n^2+2n+7} + \dfrac{2n}{n+13}$.

(ii) $^{m}KB_2(G_n) = \dfrac{1}{(n+1)(n+7)} + \dfrac{n}{3(n+7)}$.

(iii) $F_1KB(G_n) = n(n^4 + 2n^3 + 4n^2 + 42n + 219)$.

(iv) $QKB(G_n) = n(n^4 - 12n^2 + 4n + 51)$.

Proof. By using definitions and Lemma 2, we deduce

(i) $$^{m}KB_1(G_n) = \sum_{uv \in E(G_n)} \frac{1}{S_e(u)+S_e(v)}$$

$$= |E_1|\left(\frac{1}{n(n+1)+(n+7)}\right) + |E_2|\left(\frac{1}{n+7+6}\right)$$

$$= \frac{n}{n^2+2n+7} + \frac{2n}{n+13}.$$

(ii) $$^{m}KB_2(G_n) = \sum_{uv \in E(G_n)} \frac{1}{S_e(u)S_e(v)}$$

$$= |E_1| \left(\frac{1}{n(n+1)(n+7)} \right) + |E_2| \left(\frac{1}{(n+7)6} \right)$$

$$= \frac{1}{(n+1)(n+7)} + \frac{n}{3(n+7)}.$$

(iii) $$F_1KB(G_n) = \sum_{uv \in E(G_n)} [S_e(u)^2 + S_e(v)^2]$$

$$= |E_1| [(n^2+n)^2 + (n+7)^2] + |E_2| [(n+7)^2 + 6^2]$$

$$= n(n^4 + 2n^3 + 4n^2 + 42n + 219).$$

(iv) $$QKB(G_n) = \sum_{uv \in E(G_n)} [S_e(u) - S_e(v)]^2$$

$$= |E_1| (n^2 + n - n - 7)^2 + |E_2| (n + 7 - 6)^2$$

$$= n(n^4 - 12n^2 + 4n + 51).$$

Theorem 6. *Let G_n be a gear graph with $2n+1$ vertices and $3n$ edges, $n \geq 3$. Then*

(i) $$F_1KB(G_n, x) = nx^{n^3+3n^2+15n+49} + 2nx^{n^2+14n+85}.$$

(ii) $$QKB(G_n, x) = nx^{(n^2-7)^2} + 2nx^{(n+1)^2}.$$

Proof. By using definitions and Lemma 2, we obtain

(i) $$F_1KB(G_n, x) = \sum_{uv \in E(G_n)} x^{[S_e(u)^2 + S_e(v)^2]}$$

$$= |E_1| x^{(n^2+n)^2+(n+7)^2} + |E_2| x^{(n+7)^2+6^2}$$

$$= nx^{n^3+3n^2+15n+49} + 2nx^{n^2+14n+85}.$$

(ii) $$QKB(G_n, x) = \sum_{uv \in E(G_n)} x^{[S_e(u) - S_e(v)]^2}$$

$$= |E_1| x^{(n^2+n-n-7)^2} + |E_2| x^{(n+7-6)^2}$$

$$= nx^{(n^2-7)^2} + 2nx^{(n+1)^2}.$$

5. Helm Graphs

A helm graph H_n is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly H_n has $2n+1$ vertices and $3n$ edges. A graph H_n is shown in Figure 3.

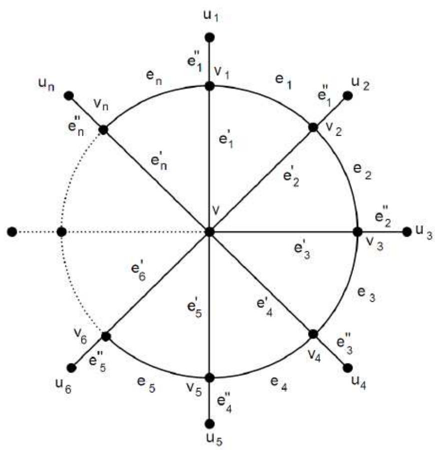

Figure 3. Helm graph H_n.

Lemma 3. *Let H_n be a helm graph with $3n$ edges. Then H_n has three types of edges as given below*:

$E_1 = \{uv \in E(H_n) | S_e(u) = n(n+2), S_e(v) = n+17\}$, $|E_1| = n$.

$E_2 = \{uv \in E(H_n) | S_e(u) = S_e(v) = n+17\}$, $|E_2| = n$.

$E_3 = \{uv \in E(H_n) | S_e(u) = n+17, S_e(v) = 3\}$, $|E_3| = n$.

Theorem 7. L*et H_n be a helm graph with $2n+1$ vertices and $3n$ edges. Then*

(i) $$^{m}KB_1(H_n) = \frac{n}{n^2+3n+17} + \frac{n}{2n+34} + \frac{n}{n+20}.$$

(ii) $^{m}KB_2(H_n) = \frac{1}{(n+2)(n+17)} + \frac{n}{(n+17)^2} + \frac{n}{3(n+17)}.$

(iii) $FKB(H_n) = n[n^2(n+2)^2 + (n+17)^2] + 2n(n+17)^2 + n[(n+17)^2 + 9].$

(iv) $QKB(H_n) = n(n^2 + n - 17)^2 + n(n+14)^2.$

Proof. By using definitions and Lemma 3, we deduce

(i) $$^{m}KB_1(H_n) = \sum_{uv \in E(H_n)} \frac{1}{S_e(u) + S_e(v)}$$

$$= |E_1| \left(\frac{1}{n(n+2) + n + 17} \right) + |E_2| \left(\frac{1}{n + 17 + n + 17} \right)$$

$$+ |E_3| \left(\frac{1}{n + 17 + 3} \right)$$

$$= \frac{n}{n^2 + 3n + 17} + \frac{n}{2n + 34} + \frac{n}{n + 20}.$$

(ii) $$^{m}KB_2(H_n) = \sum_{uv \in E(H_n)} \frac{1}{S_e(u) S_e(v)}$$

$$= |E_1| \left(\frac{1}{n(n+2)(n+17)} \right) + |E_2| \left(\frac{1}{(n+17)(n+17)} \right)$$

$$+ |E_3| \left(\frac{1}{(n+17)3} \right)$$

$$= \frac{1}{(n+2)(n+17)} + \frac{n}{(n+17)^2} + \frac{n}{3(n+17)}.$$

(iii) $$F_1KB(H_n) = \sum_{uv \in E(H_n)} [S_e(u)^2 + S_e(v)^2]$$

$$= |E_1| [n^2(n+2)^2 + (n+17)^2] + |E_2| [(n+17)^2 + (n+17)^2]$$

$$+ |E_3| [(n+17)^2 + 3^2]$$

$$= n[n^2(n+2)^2 + (n+17)^2] + 2n(n+17)^2 + n[(n+17)^2 + 9]$$

(iv) $QKB(H_n) = \sum_{uv \in E(H_n)} [S_e(u) - S_e(v)]^2$

$$= |E_1|(n^2 + 2n - n - 17)^2 + |E_2|(n + 17 - n - 17)^2$$

$$+ |E_3|(n + 17 - 3)^2$$

$$= n(n^2 + n - 17) + n(n + 14)^2.$$

Theorem 8. *Let* H_n *be a helm graph with* $2n + 1$ *vertices and* $3n$ *edges. Then*

(i) $F_1KB(H_n, x) = nx^{n^2(n+2)^2+(n+17)^2} + nx^{2(n+17)^2} + nx^{(n+17)^2+9}$.

(ii) $QKB(H_n, x) = nx^{(n^2+n-17)^2} + nx^0 + nx^{(n+14)^2}$.

Proof. By using definitions and Lemma 3, we obtain

(i) $F_1KB(H_n, x) = \sum_{uv \in E(H_n)} x^{[S_e(u)^2 + S_e(v)^2]}$

$$= |E_1| x^{n^2(n+2)^2+(n+17)^2} + |E_2| x^{(n+17)^2+(n+17)^2}$$

$$+ |E_3| x^{(n+17)^2+3^2}$$

$$= nx^{n^2(n+2)^2+(n+17)^2} + nx^{2(n+17)^2} + nx^{(n+17)^2+9}.$$

(ii) $QKB(H_n, x) = \sum_{uv \in E(H_n)} x^{[S_e(u) - S_e(v)]^2}$

$$= |E_1| x^{(n^2+2n-n-17)^2} + |E_2| x^{(n+17-n-17)^2} + |E_3| x^{(n+17-3)^2}$$

$$= nx^{(n^2+n-17)^2} + nx^0 + nx^{(n+14)^2}.$$

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