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P^*-Skew-Bi-Normal Operator on Hilbert Space

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Abstract

In this paper we introduce an operator on Hilbert space $\mathcal{H}$ called P^*-skew-bi-normal operator. An operator $\mathcal{L}$ is called P^*-skew-bi-normal operator if and only if $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$, where P is a nonnegative integer. New theorems and properties are given on Hilbert space $\mathcal{H}$.

Introduction

Let $\mathcal{B}(\mathcal{H})$ be an algebra of every bounded linear operator on Hilbert space $\mathcal{H}$. The operator $\mathcal{L}$ is called *normal* iff $\mathcal{L}^*\mathcal{L} = \mathcal{L}\mathcal{L}^*$. In 2018, Meenambika et al. introduced skew normal operator and defined as $(\mathcal{L}^*\mathcal{L})\mathcal{L} = \mathcal{L}(\mathcal{L}\mathcal{L}^*)$. In [3] Campbell introduced the class of binormal operators which is defined as $\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^* = \mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L}$.

In this paper we introduced new type of operators on Hilbert space $\mathcal{H}$ called P^*-skew-bi-normal operator. An operator $\mathcal{L}$ is called P^*-*skew-bi-normal operator* if and only if $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$, where P is a nonnegative integer. New theorems are given on some properties on Hilbert space.

1. Main Results

Definition 1.1. Let $\mathcal{L}$ be a bounded linear operator on Hilbert space $\mathcal{H}$. Then $\mathcal{L}$ is said to be P^*-*skew-bi-normal operator* if and only if $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$, where P is a nonnegative integer.

Theorem 1.2. *If $\mathcal{L}$ is a normal operator on Hilbert space $\mathcal{H}$, then $\mathcal{L}$ is P^*-skew-bi-*

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normal operator.

Proof. Suppose $\mathcal{L}$ is a normal operator. We need to prove that $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

$$\begin{aligned}(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P &= \mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L}(\mathcal{L}^*)^P \\ &= \mathcal{L}\mathcal{L}^*\mathcal{L}^*(\mathcal{L}^*)^P\mathcal{L} \\ &= \mathcal{L}\mathcal{L}^*(\mathcal{L}^*)^P\mathcal{L}^*\mathcal{L} \\ &= \mathcal{L}(\mathcal{L}^*)^P\mathcal{L}^*\mathcal{L}^*\mathcal{L} \\ &= (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L}).\end{aligned}$$

Hence, $\mathcal{L}$ is P^*-skew-bi-normal operator.

Theorem 1.3. *The set $\Omega(\mathcal{H})$ of all P^*-skew-bi-normal operators on Hilbert space $\mathcal{H}$ is a closed subset of $\mathcal{B}(\mathcal{H})$ under scalar multiplication.*

Proof. Suppose

$\Omega(\mathcal{H}) = \{\mathcal{L} \in \mathcal{B}(\mathcal{H})\colon \mathcal{L}$ is P^*-skew-bi-normal operators on $\mathcal{H}$ for some nonnegative integer $P\}$.

Let $\mathcal{L} \in \Omega(\mathcal{H})$, then we have that $\mathcal{L}$ is P^*-skew-bi-normal operator and $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

Let η be scalar. Then

$$\begin{aligned}[(\eta\mathcal{L})^*\eta\mathcal{L}\eta\mathcal{L}(\eta\mathcal{L})^*]((\eta\mathcal{L})^*)^P &= [\bar{\eta}\mathcal{L}^*\eta\mathcal{L}\eta\mathcal{L}\bar{\eta}\mathcal{L}^*](\bar{\eta}\mathcal{L}^*)^P \\ &= [\bar{\eta}\mathcal{L}^*\eta\mathcal{L}\eta\mathcal{L}\bar{\eta}\mathcal{L}^*](\bar{\eta})^P(\mathcal{L}^*)^P \\ &= \bar{\eta}\bar{\eta}\eta\eta(\bar{\eta})^P[\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*](\mathcal{L}^*)^P \\ &= \bar{\eta}\bar{\eta}\eta\eta(\bar{\eta})^P(\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L}) \\ &= (\bar{\eta}\mathcal{L}^*)^P[\eta\mathcal{L}\bar{\eta}\mathcal{L}^*\bar{\eta}\mathcal{L}^*\eta\mathcal{L}] \\ &= ((\eta\mathcal{L})^*)^P[\eta\mathcal{L}(\eta\mathcal{L})^*(\eta\mathcal{L})^*\eta\mathcal{L}].\end{aligned}$$

Hence, $\eta\mathcal{L} \in \Omega(\mathcal{H})$.

Let $\mathcal{L}_k$ be a sequence in $\Omega(\mathcal{H})$ and converge to $\mathcal{L}$. Then we can prove that

$$\|[(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P] - [(\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})]\|$$

$$= \|[(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P] - [(\mathcal{L}_k^*\mathcal{L}_k\mathcal{L}_k\mathcal{L}_k^*)(\mathcal{L}_k^*)^P] + [(\mathcal{L}_k^*)^P(\mathcal{L}_k\mathcal{L}_k^*\mathcal{L}_k^*\mathcal{L}_k)] - [(\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})]\|$$

$$\leq \|[(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P] - [(\mathcal{L}_k^*\mathcal{L}_k\mathcal{L}_k\mathcal{L}_k^*)(\mathcal{L}_k^*)^P]\| + \|[(\mathcal{L}_k^*)^P(\mathcal{L}_k\mathcal{L}_k^*\mathcal{L}_k^*\mathcal{L}_k)] - [(\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})]\|$$

$\to 0$ as $k \to \infty$.

Hence, $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

Therefore $\mathcal{L} \in \Omega(\mathcal{H})$.

Then, $\Omega(\mathcal{H})$ is closed subset.

Theorem 1.4. *If $\mathcal{L}$ and ξ are normal, P^*-skew-bi-normal operators on $\mathcal{H}$, and let $\mathcal{L}$ commute with ξ then $(\mathcal{L}\xi)$ is P^*-skew-bi-normal operator on $\mathcal{H}$.*

Proof. Since $\mathcal{L}$ and ξ are P^*-skew-bi-normal operators, we have $(\xi^*\xi\xi\xi^*)(\xi^*)^P = (\xi^*)^P(\xi\xi^*\xi^*\xi)$ and $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

$$\begin{aligned}
[(\mathcal{L}\xi)^*(\mathcal{L}\xi)(\mathcal{L}\xi)(\mathcal{L}\xi)^*][((\mathcal{L}\xi)^*)^P] &= [(\xi^*\mathcal{L}^*)(\mathcal{L}\xi)(\mathcal{L}\xi)(\xi^*\mathcal{L}^*)][(\mathcal{L}^*)^P(\xi^*)^P] \\
&= [\xi^*\mathcal{L}^*\mathcal{L}\xi\ \mathcal{L}\xi\ \xi^*\mathcal{L}^*][(\mathcal{L}^*)^P(\xi^*)^P] \\
&= [\mathcal{L}\xi^*\mathcal{L}^*\xi\ \mathcal{L}\ \xi^*\mathcal{L}^*\xi]\ [(\mathcal{L}^*)^P(\xi^*)^P] \\
&= [\mathcal{L}\xi^*\xi\mathcal{L}^*\ \xi^*\mathcal{L}\mathcal{L}^*\xi][(\mathcal{L}^*)^P\ (\xi^*)^P] \\
&= [\mathcal{L}\xi\xi^*\mathcal{L}^*\ \xi^*\mathcal{L}^*\mathcal{L}\xi][(\mathcal{L}^*)^P\ (\xi^*)^P] \\
&= \mathcal{L}\xi\xi^*\mathcal{L}^*\ \xi^*\mathcal{L}^*\mathcal{L}(\mathcal{L}^*)^P\ \ \xi(\xi^*)^P \\
&= \mathcal{L}\xi\xi^*\mathcal{L}^*\ \xi^*\mathcal{L}^*\mathcal{L}(\mathcal{L}^*)^P(\xi^*)^P\xi \\
&= \mathcal{L}\xi\xi^*\mathcal{L}^*\ \xi^*\mathcal{L}^*(\mathcal{L}^*)^P\mathcal{L}(\xi^*)^P\xi \\
&= \mathcal{L}\xi\xi^*\mathcal{L}^*\ \xi^*\mathcal{L}^*(\mathcal{L}^*)^P(\xi^*)^P\mathcal{L}\xi \\
&\ \vdots \\
&= [(\mathcal{L}^*)^P(\xi^*)^P].[(\mathcal{L}\xi)(\xi^*\mathcal{L}^*)(\xi^*\mathcal{L}^*)(\mathcal{L}\xi)] \\
&= [((\mathcal{L}\xi)^*)^P].[(\mathcal{L}\xi)(\xi^*\mathcal{L}^*)(\xi^*\mathcal{L}^*)(\mathcal{L}\xi)].
\end{aligned}$$

Hence $(\mathcal{L}\xi)$ P^*-skew-bi-normal operator on $\mathcal{H}$.

Theorem 1.5. *If $\mathcal{L}^{-1}$ exists and $\mathcal{L}$ is a P^*-skew-bi-normal operator, then $\mathcal{L}^{-1}$ is P^*-skew-bi-normal operator.*

Proof. Since $\mathcal{L}$ is P^*-skew-bi-normal operator, we have $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

Let

$$[(\mathcal{L}^{-1})^*\mathcal{L}^{-1}\mathcal{L}^{-1}(\mathcal{L}^{-1})^*]((\mathcal{L}^{-1})^*)^P = [(\mathcal{L}^*)^{-1}\mathcal{L}^{-1}\mathcal{L}^{-1}(\mathcal{L}^*)^{-1}]((\mathcal{L}^*)^P)^{-1}$$

$$= [(\mathcal{L}\mathcal{L}^*)^{-1}(\mathcal{L}^*\mathcal{L})^{-1})]((\mathcal{L}^*)^P)^{-1}$$

$$= [(\mathcal{L}^*\mathcal{L})(\mathcal{L}\mathcal{L}^*)]^{-1}((\mathcal{L}^*)^P)^{-1}$$

$$= [(\mathcal{L}^*)^P[(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)]]^{-1}, \text{since } \mathcal{L} \text{ is binormal}$$

$$= [[(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})](\mathcal{L}^*)^P]^{-1},$$

$$\text{since } \mathcal{L} \text{ is a } P^* - \text{skew binormal},$$

$$= [(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P]^{-1}, \text{ since } \mathcal{L} \text{ is binormal}$$

$$= [(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})(\mathcal{L}^*)^P]^{-1}$$

$$= ((\mathcal{L}^*)^P)^{-1}[(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})]^{-1}$$

$$= ((\mathcal{L}^*)^P)^{-1}[(\mathcal{L}^*\mathcal{L})^{-1}(\mathcal{L}\mathcal{L}^*)^{-1}]$$

$$= ((\mathcal{L}^*)^P)^{-1}[\mathcal{L}^{-1}(\mathcal{L}^*)^{-1}(\mathcal{L}^*)^{-1}\mathcal{L}^{-1}]$$

$$= ((\mathcal{L}^{-1})^*)^P[\mathcal{L}^{-1}(\mathcal{L}^{-1})^*(\mathcal{L}^{-1})^*\mathcal{L}^{-1}].$$

Hence, $\mathcal{L}^{-1}$ is P^*-skew-bi-normal operator.

Definition 1.6 [4]**.** If $\mathcal{L}, \xi$ are bounded operators on $\mathcal{H}$, then $\mathcal{L}, \xi$ are *unitary equivalent* if there is an isomorphism $U: \mathcal{H} \to \mathcal{H}$ such that $\xi = U\mathcal{L}U^*$.

Theorem 1.7. *Let $\mathcal{L}$ is P^*-skew-bi-normal operator. Then*

1. *The operator $\tau\mathcal{L}$ is P^*-skew-bi-normal operator for every real scalar τ.*

2. *If $\zeta \in \mathcal{B}(\mathcal{H})$ is unitary equivalent to $\mathcal{L}$, then ζ is P^*-skew-bi-normal operator.*

Proof.

1. Let $\mathcal{L}$ be P^*-skew-bi-normal operator. Then we have $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

$$\left[(\tau\mathcal{L})^*\tau\mathcal{L}\tau\mathcal{L}(\tau\mathcal{L})^*\right]((\tau\mathcal{L})^*)^P = \left[\bar{\tau}\mathcal{L}^*\tau\mathcal{L}\tau\mathcal{L}\bar{\tau}\mathcal{F}^*\right](\bar{\tau}\mathcal{L}^*)^P$$
$$= \left[\bar{\tau}\mathcal{L}^*\tau\mathcal{L}\tau\mathcal{L}\bar{\tau}\mathcal{L}^*\right](\bar{\tau})^P(\mathcal{L}^*)^P$$
$$= \bar{\tau}\bar{\tau}\tau\tau\,(\bar{\tau})^P[\mathcal{L}^*\mathcal{L}\,\mathcal{L}\,\mathcal{L}^*](\mathcal{L}^*)^P$$
$$= \bar{\tau}\bar{\tau}\tau\tau(\bar{\tau})^P(\mathcal{L}^*)^P(\mathcal{L}\,\mathcal{L}^*\mathcal{L}^*\,\mathcal{L})$$
$$= (\bar{\tau}\mathcal{L}^*)^P\left[\tau\mathcal{L}\bar{\tau}\mathcal{L}^*\bar{\tau}\mathcal{L}^*\tau\mathcal{L}\right]$$
$$= ((\tau\mathcal{L})^*)^P\left[\tau\mathcal{L}(\tau\mathcal{L})^*(\tau\mathcal{L})^*\tau\mathcal{L}\right].$$

So, $\tau\mathcal{L}$ is P^*-skew-bi-normal operator.

2. Since ζ is unitary equivalent to $\mathcal{L}$, we have $\zeta = U\mathcal{L}U^*$, $(U\mathcal{L}U^*)^n = U\mathcal{L}^nU^*$ and since $\mathcal{L}$ is P^*-skew-bi-normal operator, we have $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

$$(\zeta^*\zeta\zeta\zeta^*)(\zeta^*)^P = [((U\mathcal{L}U^*)^*(U\mathcal{L}U^*)(U\mathcal{L}U^*)(U\mathcal{L}U^*)^*)]((U\mathcal{L}U^*)^*)^P$$
$$= [(U\mathcal{L}^*U^*)(U\mathcal{L}U^*)(U\mathcal{L}U^*)(U\mathcal{L}^*U^*)](U\mathcal{L}^PU^*)$$
$$= U[\,(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P\,]\,.\,U^*,$$

since $\mathcal{L}$ is P^*-skew- bi-normal operator, we have

$$= U[(\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})]U^*$$
$$= (U\mathcal{L}^PU^*)[(U\mathcal{L}^*U^*)(U\mathcal{L}U^*)(U\mathcal{L}U^*)(U\mathcal{L}^*U^*)]$$
$$= (\zeta^*)^P(\zeta\zeta^*\zeta^*\zeta).$$

Hence ζ is P^*-skew-bi-normal operator.

Theorem 1.8. *If $\mathcal{L}$ is a self adjoint and P^*-skew-bi-normal operator, then $\mathcal{L}^*$ is P^*-skew-bi-normal operator.*

Proof. Since $\mathcal{L}$ is P^*-skew-binormal operator, we have $(\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P = (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L})$.

$$[(\mathcal{L}^*)^*\mathcal{L}^*\mathcal{L}^*(\mathcal{L}^*)^*]((\mathcal{L}^*)^*)^P = (\mathcal{L}^*\mathcal{L}\mathcal{L}\mathcal{L}^*)(\mathcal{L}^*)^P, \text{ since } \mathcal{L} \text{ is self adjoint}$$
$$= (\mathcal{L}^*)^P(\mathcal{L}\mathcal{L}^*\mathcal{L}^*\mathcal{L}), \text{ since } \mathcal{L} \text{ is } P^*\text{-skew-bi-normal}$$
$$= ((\mathcal{L}^*)^*)^P[\,\mathcal{L}^*(\mathcal{L}^*)^*(\mathcal{L}^*)^*\,\mathcal{L}^*], \text{ since } \mathcal{L} \text{ is a self adjoint}$$

Hence, $\mathcal{L}^*$ is P^*-skew-bi-normal operator.

Theorem 1.9. *Consider $\aleph_1, \aleph_2, \dots, \aleph_r$ are P^*-skew-bi-normal operators on $\mathcal{H}$. Then the direct sum $(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)$ is P^*-skew-bi-normal operators on $\mathcal{H}$.*

Proof. Since every operator of $\aleph_1, \aleph_2, \dots, \aleph_r$ is a P^*-skew-bi-normal operator, we have $\left(\aleph_j^* \aleph_j \aleph_j \aleph^*\right)(\aleph_j^*)^P = (\aleph_j^*)^P\left(\aleph_j \aleph_j^* \aleph_j^* \aleph_j\right)$ for all $j = 1,2, \dots, r$.

$$[(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)^* (\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)^*] \times ((\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)^*)^P$$

$$= [((\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r))^* (\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)((\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r))^*] \times [({\aleph_1}^*)^P \oplus ({\aleph_2}^*)^P \oplus \dots \oplus ({\aleph_r}^*)^P]$$

$$= (({\aleph_1}^* \aleph_1 \aleph_1 {\aleph_1}^*)({\aleph_1}^*)^P) \oplus (({\aleph_2}^* \aleph_2 \aleph_2 {\aleph_2}^*)({\aleph_2}^*)^P) \oplus \dots \oplus (({\aleph_r}^* \aleph_r \aleph_r {\aleph_r}^*)({\aleph_r}^*)^P.$$

Since every operator of $\aleph_1, \aleph_2, \dots, \aleph_r$ is a P^*-skew-bi-normal operator, we have

$$= (({\aleph_1}^*)^P (\aleph_1 {\aleph_1}^* {\aleph_1}^* \aleph_1)) \oplus (({\aleph_2}^*)^P (\aleph_2 {\aleph_2}^* {\aleph_2}^* \aleph_2)) \oplus \dots \oplus (({\aleph_r}^*)^P (\aleph_r {\aleph_r}^* {\aleph_r}^* \aleph_r))$$

$$= (((\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r))^*)^P [(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)^* \times (\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)^* (\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)]$$

Hence, $(\aleph_1 \oplus \aleph_2 \oplus \dots \oplus \aleph_r)$ is a P^*-skew-bi-normal operators on $\mathcal{H}$.

Theorem 1.10. *Suppose $\aleph_1, \aleph_2, \dots, \aleph_m$ are P^*-skew-bi-normal operators on $\mathcal{H}$. Then the tenser product $(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)$ is P^*-skew-bi-normal operators on $\mathcal{H}$.*

Proof. Since every operator of $\aleph_1, \aleph_2, \dots, \aleph_m$ is P^*-skew-bi-normal operator, we have $\left(\aleph_j^* \aleph_j \aleph_j \aleph^*\right)(\aleph_j^*)^P = (\aleph_j^*)^P\left(\aleph_j \aleph_j^* \aleph_j^* \aleph_j\right)$ for all $j = 1,2, \dots, m$.

$$[[(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)^* (\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)^*] \times [(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)^*]^P](\boldsymbol{x_1} \otimes \boldsymbol{x_2} \otimes \dots \otimes \boldsymbol{x_\mathrm{m}})$$

$$= [[({\aleph_1}^* \otimes {\aleph_2}^* \otimes \dots \otimes {\aleph_m}^*)(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)(\aleph_1 \otimes \aleph_2 \otimes \dots \otimes \aleph_m)] \times [({\aleph_1}^*)^P \otimes ({\aleph_2}^*)^P \otimes \dots \otimes ({\aleph_m}^*)^P]](\boldsymbol{x_1} \otimes \boldsymbol{x_2} \otimes \dots \otimes \boldsymbol{x_\mathrm{m}})$$

$$= [({\aleph_1}^* \aleph_1 \aleph_1 {\aleph_1}^*)({\aleph_1}^*)^P]\boldsymbol{x_1} \otimes [({\aleph_2}^* \aleph_2 \aleph_2 {\aleph_2}^*)({\aleph_2}^*)^P]\boldsymbol{x_2} \otimes \quad \dots \otimes [({\aleph_m}^* \aleph_m \aleph_m {\aleph_m}^*)({\aleph_m}^*)^P]\boldsymbol{x_\mathrm{m}}$$

Since every operator of $\aleph_1, \aleph_2, \dots, \aleph_m$ is P^*-skew- bi-normal operator, we have

$$= [(\aleph_1{}^*)^P(\aleph_1\aleph_1{}^*\aleph_1{}^*\aleph_1)]\boldsymbol{x_1} \otimes [(\aleph_2{}^*)^P(\aleph_2\aleph_2{}^*\aleph_2{}^*\aleph_2)]\boldsymbol{x_2} \otimes \quad \ldots$$

$$\otimes [(\aleph_m{}^*)^P(\aleph_m\aleph_m{}^*\aleph_m{}^*\aleph_m)]\boldsymbol{x_m}$$

$$= [[(\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m)^*]^P [((\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m))((\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m))^*$$

$$\times ((\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m))^*((\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m))]]](\boldsymbol{x_1} \otimes \boldsymbol{x_2} \otimes \ldots \otimes \boldsymbol{x_m}).$$

Then $(\aleph_1 \otimes \aleph_2 \otimes \ldots \otimes \aleph_m))$ is P^*-skew-bi-normal operators on $\mathcal{H}$.

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