

On Mathematical Model of Biological Pest Control of Sorghum Production

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Abstract

In this paper, mathematical model of Sorghum production from seed/planting to harvesting stage was presented with respect to its pests/prey) and the corresponding natural prey enemy (predators) at every stage. The model was characterized, the existence and uniqueness of the model solution was established. And finally numerical applications was carried out using Differential Transform method, and it was found that pests at different stages of sorghum production can be minimized below injury level using biological control which in turn leads to maximization of the sorghum production.

1. Introduction

An important cereal crop known as Sorghum (*Sorghum bicolor L. Moench*) is widely grown in arid and semi-arid ecology worldwide for food and feed purposes. It can be referred to as a security crop because it has a wide adaptability and tolerant to environmental stress. The production of sorghum is drastically affected by different kind of diseases and pests such as Grasshopper, Weevil, Sugarcane beetle, Leafhopper, Mites, Wire worm, Aphids and Army worm, etc. In another development literature revealed that there are some beneficial group of arthropods, such as lady beetle (ladybug or lady bird) which are natural enemies of many sorghum insectpests especially aphids, mites and others. Single lady beetle may eat as many as 5,000 aphids in its lifetime, [10]. Lady bugs has different species and most species are considered beneficial pest because

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they are predators of many harmful pest. This contribute to the high level of regulations of prey population, though ladybird larvae and adults may supplement their normal prey in times of scarcity with other types of food like flower nectar, water and honeydew on sorghum panicle, [6].

Sorghum as a member of the grass family is classified into Grain Sorghum, Sweet Sorghum, Broom Sorghum, and Grass Sorghum. It was ranked second among cereals and fifth among all crops in terms of population in Africa. Its production represents 25% of total cereal production in Nigeria in the year 2012. It is recognized by United State Department of Agriculture as a primary food crop produced in virtually all parts of Northern Nigeria, however demand from industries is the main drive behind increase in sorghum production. Though Boko Haram insurgencies continue to limit land for sorghum production in the northeastern part of Nigeria but farmers have continued to be encouraged by increasing prices and rising sorghum demand, both for food and for industrial use over the last two years, [7, 8, 14, 15].

Sorghum has multiple uses as grain forage, fodder, and more recently as bioenergy crop. In Asia and Africa, sorghum grain is consumed by humans or as animal feed, its stalks are used as animal fodder, or as housing material. It has a high nutritive value, with 70-80% carbohydrate, 11-13% protein, 2.5% fat, 1-3% fiber and 1-2% ash. Protein in sorghum is gluten free and thus, it is a specialty food for people who suffer from celiac (intolerance to food with gluten), including diabetic patients, [17].

Many researchers have worked on sorghum analysis, Sisterson and Stenger [12] carried out a research with the aim of developing a prediction equation describing the cleaning process in a stationary sorghum thresher using a mathematical model based on physical-aerodynamic properties of sorghum and machine characteristics. [3] studied the effect of high temperature on crop growth development, in vitro pollen germination and seed set. [11] studied new sorghum pest in North America (sugarcane aphid). [13] studied the mathematical characterization of growth of a local land-race sorghum from Saudi Arabia (Jazan Region).

In another development, many authors have used the concept of mathematics to model and analyse dynamics of real life situations. Among are: Egbetade and Ibrahim [5], researched on a mathematical model of tuberculosis (TB) incorporating treatment, migration and vaccination, with the focus on vaccination for the treatment of active TB. [9] developed a model to analysed the strategies to control pest through differential equations. [1] investigates the application of differential transformation method (DTM)

and variational iteration method (VIM) in finding the approximate solution of epidemiology (SIR) model. [4] propose a mathematical model on the stability analysis of a non-linear *HIV/AIDS* epidemic with vaccination and antiretroviral therapy to mention a few. However, little or less has been studied on the mathematical dynamics of Sorghum production, hence the present effort is to look at the mathematical model of biological control of Sorghum pest from planting to harvesting stages.

2. Model Formulation

A deterministic dynamical model was formulated into two categories of population, namely: seeds/plant and insect population. The seed/plant population is further classified into seed $S_c(t)$, germination $G(t)$, crop $C(t)$ and harvest (H), while the insect population is categorized into two: the beneficial insect population $P_v(t)$ which could there after become harmful to sorghum seeds and pest population $I_v(t)$ which they are right from time harmful to sorghum. The model shows the movement from one growth stage to another at different rate. Λ_c is the quantity of replenishing seed for plantation, $\frac{\beta_1 I_v}{N + b I_v}$ is the force of infection of seed by pest. γ and μ are the removal rate from germination class where γ is the proportion that produces crop and μ denote natural bad seeds. C is the quantity of matured crop that farmers will be able to harvest.

Basic assumption of the model

In constructing the model the following assumptions were used;

- (i) The growth and development stages of sorghum were formulated in the model from planting of seed to the harvesting stage.
- (ii) The stages of sorghum production examined are in homogeneous form, vectors can attack every stages of sorghum.
- (iii) The Model is considered as a prey-predator population.
- (iv) In the assurance of no pest, there exists no germination.
- (v) The seeds are systemic.
- (vi) Some pests have negative economic importance on sorghum while some have both positive and negative economic importance.

The parameters of the model considered are defined in Table 2.1.

Table 2.1: Description of parameters and variables. The values are hypothetical.

Parameter/ variables	Description	Value
Λ_c	Amount of seed replenishing	50
β_1	Probability of successful harm on sorghum seed.	[0,1]
N	Total number of seed	80
γ	Percentage of germinating seed producing crop.	0.6
ϵ	Fraction of crop which successfully harvest.	0.7
μ	Amount of natural bad seed/natural loss on seeds.	0.1
Λ_p	Relative recruitment rate for insect with both negative and positive impact.	15
Λ_v	Relative recruitment rate for insect with negative impact.	50
bl_v	Proportion of pest interacting/having contact with seeds.	5.1421
μ_1	Natural death rate for insects.	0.1
α	Progressive of destructive insects.	0.02
S_c	Sorghum Seeds	50
G	Germination Stage	35
C	Cropping	30
H	Harvest	20
P_v	Beneficial Pest	7.5
I_v	Pest	10.285

2.1. The simple seed-germination-cropping-harvest (SGCH) model flow diagram

In view of the assumptions we have the systematic diagram of the model.

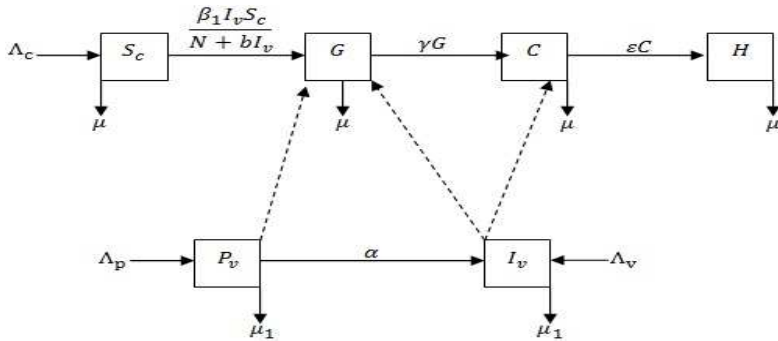


Figure 2.1: Systematic diagram of the sorghum production.

2.2. Model Equation

In view of the aforementioned assumptions, Figure 2.1 is obtained by the following first order nonlinear system of equations;

$$\left. \begin{aligned}
 \frac{dS_c}{dt} &= \Lambda_c - \mu S_c - \frac{\beta_1 I_v S_c}{N + b I_v} \\
 \frac{dG}{dt} &= \frac{\beta_1 I_v S_c}{N + b I_v} - (\gamma + \mu)G \\
 \frac{dC}{dt} &= \gamma G - (\epsilon + \mu)C \\
 \frac{dH}{dt} &= \epsilon C - \mu H \\
 \frac{dP_v}{dt} &= \Lambda_p - \mu_1 P_v - \alpha P_v \\
 \frac{dI_v}{dt} &= \Lambda_v + \alpha P_v - \mu_1 I_v
 \end{aligned} \right\} \tag{1}$$

3. Characterization of the Model

The intention is to investigate the behavior of the model formulated.

3.1. Region of feasibility and positivity of solution

Theorem 3.1. *Let Γ be the feasible region of sorghum model system (1) defined as $\Gamma_1 = \{(S_c, G, C, H) \in R_+^4 : S_c + G + C + H \leq \frac{\Lambda}{\mu}\}$, $\Gamma_2 = \{(P_v, I_v) \in R_+^2 : N_v \leq \frac{\Lambda_v}{\mu_1}\}$ and $\forall S_c > 0, G > 0, C > 0, H > 0, \Gamma = \{\Gamma_1, \Gamma_2\} \in R_+^6$. Then Γ is the positive invariant set.*

Proof. Assume that $N = S_c + C + H$ and $G = 0, I_v = 0$. $G = 0$ (because when the seed started to germinate it turns to a plant and it is no more a seed as we are referring to a seed, until it turn to a crop, that is when you can get a seed again).

Let
$$N = S_c + C + H$$

$$\begin{aligned} \frac{dN}{dt} &= \Lambda_c - \mu S_c - \mu C - \mu H \\ &= \Lambda_c - \mu N \end{aligned}$$

At equilibrium

$$\Lambda_c - \mu N = 0 \quad (2)$$

$$N(t) = \frac{\Lambda_c}{\mu} + C e^{-\mu t} \quad (3)$$

$$\begin{aligned} N(t)|_0^\infty &= \frac{\Lambda_c}{\mu} + C e^{-\mu t} \Big|_0^\infty \\ &= \frac{\Lambda_c}{\mu} + C \end{aligned}$$

$$C = \lim_{t \rightarrow \infty} \left(N(\infty) - N(0) - \frac{\Lambda_c}{\mu} \right) = N_0 - \frac{\Lambda_c}{\mu} \quad (4)$$

Put (4) in (3), we have

$$= \frac{\Lambda_c}{\mu} (1 - e^{-\mu t}) + N_0 e^{-\mu t} > 0, \quad \forall t \quad (4a)$$

Similarly, for pest population;

Let
$$N_v = P_v + I_v$$

Assume

$$\Lambda_p + \Lambda_v = \Lambda, N_v = \Lambda_p + \Lambda_v - \mu_1 N_v,$$

$$\frac{dN_v}{dt} = \Lambda - \mu_1 N_v$$

$$N_v(t) = \frac{\Lambda}{\mu_1} + C e^{-\mu_1 t} \quad (5)$$

At $t = 0$

$$N_v(0) = \frac{\Lambda}{\mu_1} + C$$

$$N_v(t) = \frac{\Lambda}{\mu_1} + \left(N_v(0) - \frac{\Lambda}{\mu_1} \right) e^{-\mu_1 t}$$

$$= \frac{\Lambda}{\mu_1} (1 - e^{-\mu_1 t}) + N_v(0)e^{-\mu_1 t} > 0, \quad \forall t \tag{5a}$$

By (4a) and (5a), hence the result.

3.2. Existence and uniqueness of solution to the model

In this subsection the existence and uniqueness of solution to model (1) shall be investigated using Lipschitz condition and continuous differentiability.

Lemma 3.1 [16]. *Let $D \subset \mathbb{R}^n$ and $f: D \rightarrow \mathbb{R}$ be a nonlinear vector field. f is continuous and Lipschitz in $B = \{x \in D: \|x - x_0\| \leq r\}$ for some real r with $r > 0$. Then, there exists some $\delta > 0$ such that $x' = f(t, x)$, $x(t_0) = x_0$, has unique solution in $[0, \delta]$.*

Theorem 3.2. *Suppose $F_i(t, x)$, $x(t_0) = x_0$, $i = 1, \dots, 6$ exists and unique in solution. Then the system equation (1) satisfies Lipschitz condition.*

Proof. It is sufficient to show that $\frac{\partial f_i}{\partial t_j}$, $i, j = 1, 2, 3, 4, 5, 6$ are continuous and bounded in D .

Let

$$\left. \begin{aligned} f_1 &= \Lambda_c - \mu S_c - \frac{\beta_1 I_v S_c}{N + b I_v} \\ f_2 &= \frac{\beta_1 I_v S_c}{N + b I_v} - (\gamma + \mu)G \\ f_3 &= \gamma G - (\epsilon + \mu)C \\ f_4 &= \epsilon C - \mu H \\ f_5 &= \Lambda_p - \mu_1 P_v - \alpha P_v \\ f_6 &= \Lambda_v + \alpha P_v - \mu_1 I_v \end{aligned} \right\} \tag{6}$$

Taking the partial derivatives of $f_1, f_2, f_3, f_4, f_5, f_6$ in equation (6) to obtain the following:

For f_1 ,

$$\left. \begin{aligned} \left| \frac{\partial f_1}{\partial S_c} \right| &= \left| -\mu - \frac{\beta_1 I_v}{N + b I_v} \right| < \infty \\ \left| \frac{\partial f_1}{\partial G} \right| &= \left| \frac{\partial f_1}{\partial C} \right| = \left| \frac{\partial f_1}{\partial H} \right| = \left| \frac{\partial f_1}{\partial P_v} \right| = 0 < \infty \\ \left| \frac{\partial f_1}{\partial I_v} \right| &= \left| -\frac{N \beta_1 S_c}{(N + b I_v)^2} \right| < \infty; \quad \forall N + b I_v \neq 0 \end{aligned} \right\} \tag{7}$$

For f_2 in (6), we have:

$$\left. \begin{aligned} \left| \frac{\partial f_2}{\partial S_c} \right| &= \left| \frac{\beta_1 I_v}{N + b I_v} \right| < \infty \\ \left| \frac{\partial f_2}{\partial G} \right| &= |-(\gamma + \mu)| < \infty \\ \left| \frac{\partial f_2}{\partial C} \right| &= \left| \frac{\partial f_2}{\partial H} \right| = \left| \frac{\partial f_2}{\partial P_v} \right| = 0 < \infty \\ \left| \frac{\partial f_2}{\partial I_v} \right| &= \left| -\frac{\beta_1 S_c}{(N + b I_v)^2} \right| < \infty; \quad \forall N + b I_v \neq 0 \end{aligned} \right\} \quad (8)$$

Also for f_3 ,

$$\left. \begin{aligned} \left| \frac{\partial f_3}{\partial S_c} \right| &= 0 < \infty \\ \left| \frac{\partial f_3}{\partial G} \right| &= |\gamma| < \infty \\ \left| \frac{\partial f_3}{\partial C} \right| &= |-(\mu + \epsilon)| < \infty \\ \left| \frac{\partial f_3}{\partial H} \right| &= \left| \frac{\partial f_3}{\partial P_v} \right| = \left| \frac{\partial f_3}{\partial I_v} \right| = 0 < \infty \end{aligned} \right\} \quad (9)$$

For f_4 , we have:

$$\left. \begin{aligned} \left| \frac{\partial f_4}{\partial S_c} \right| &= 0 < \infty \\ \left| \frac{\partial f_4}{\partial G} \right| &= 0 < \infty \\ \left| \frac{\partial f_4}{\partial C} \right| &= |\epsilon| < \infty \\ \left| \frac{\partial f_4}{\partial H} \right| &= |-\mu| < \infty \\ \left| \frac{\partial f_4}{\partial P_v} \right| &= \left| \frac{\partial f_4}{\partial I_v} \right| = 0 < \infty \end{aligned} \right\} \quad (10)$$

For f_5 , we have:

$$\left. \begin{aligned} \left| \frac{\partial f_5}{\partial S_c} \right| &= \left| \frac{\partial f_5}{\partial G} \right| = \left| \frac{\partial f_5}{\partial C} \right| = \left| \frac{\partial f_5}{\partial H} \right| = 0 < \infty \\ \left| \frac{\partial f_5}{\partial P_v} \right| &= |-(\mu_1 + \alpha)| < \infty \\ \left| \frac{\partial f_5}{\partial I_v} \right| &= 0 < \infty \end{aligned} \right\} \quad (11)$$

And for f_6 :

$$\left. \begin{aligned} \left| \frac{\partial f_6}{\partial S_c} \right| &= \left| \frac{\partial f_6}{\partial G} \right| = \left| \frac{\partial f_6}{\partial C} \right| = \left| \frac{\partial f_6}{\partial H} \right| = \left| \frac{\partial f_6}{\partial P_v} \right| = |\alpha| < \infty \\ \left| \frac{\partial f_6}{\partial I_v} \right| &= |-\mu_1| < \infty \end{aligned} \right\} \quad (12)$$

Since all the partial derivatives exist, continuous and are bounded, by Lemma 3.1, the model exists and the solution is unique.

4. Numerical Application

Numerical solution of the propose model was investigated for better understanding. The system was solved using Differential Transform Method [1, 2]. The fundamental mathematical operations performed by differential transform method was adopted see [2] for more information.

Using the fundamental operators in [2], the following transform and recurrence relation of equation (1) were obtained

$$\left. \begin{aligned}
 S_c(k+1) &= \frac{1}{k+1} [\Lambda_c - \beta_1 \sum_{m=0}^k I_v(m) S_c(k-m) - \mu S_c(k)] \\
 G(k+1) &= \frac{1}{k+1} [\beta_1 \sum_{m=0}^k I_v(m) S_c(k-m) - \gamma G(k) - \mu G(k)] \\
 C(k+1) &= \frac{1}{k+1} [\gamma G(k) - (\mu + \epsilon) C(k)] \\
 H(k+1) &= \frac{1}{k+1} [\epsilon C(k) - \mu H(k)] \\
 P_v(k+1) &= \frac{1}{k+1} [\Lambda_v - \mu_1 P_v(k) - \alpha P_v(k)] \\
 I_v(k+1) &= \frac{1}{k+1} [\Lambda_v + \alpha P_v(k) - \mu_1 I_v(k)]
 \end{aligned} \right\} \quad (13)$$

Equation (13) was solved with Maple 18 using data on Table 2.1 and the result obtained were shown graphically in Figures 2, 3 and 4.

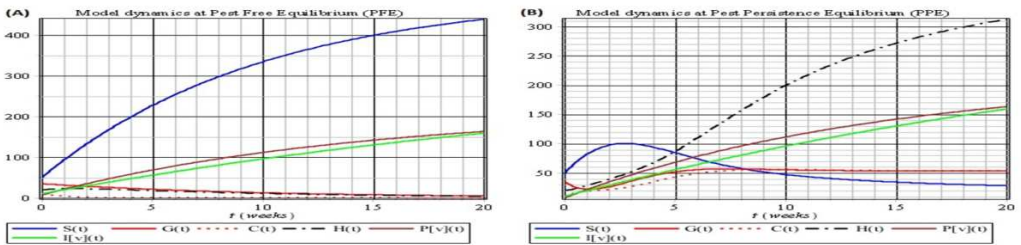


Figure 2: (A) The dynamics for which $I_v = 0$, and (B) $I_v \neq 0$.

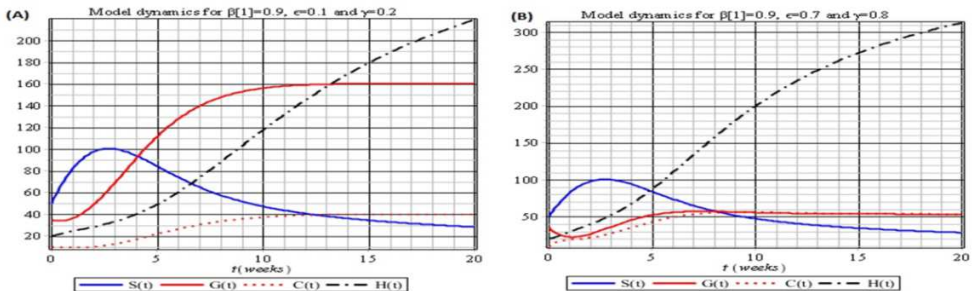


Figure 3: The dynamics for which $I_v \neq 0$.

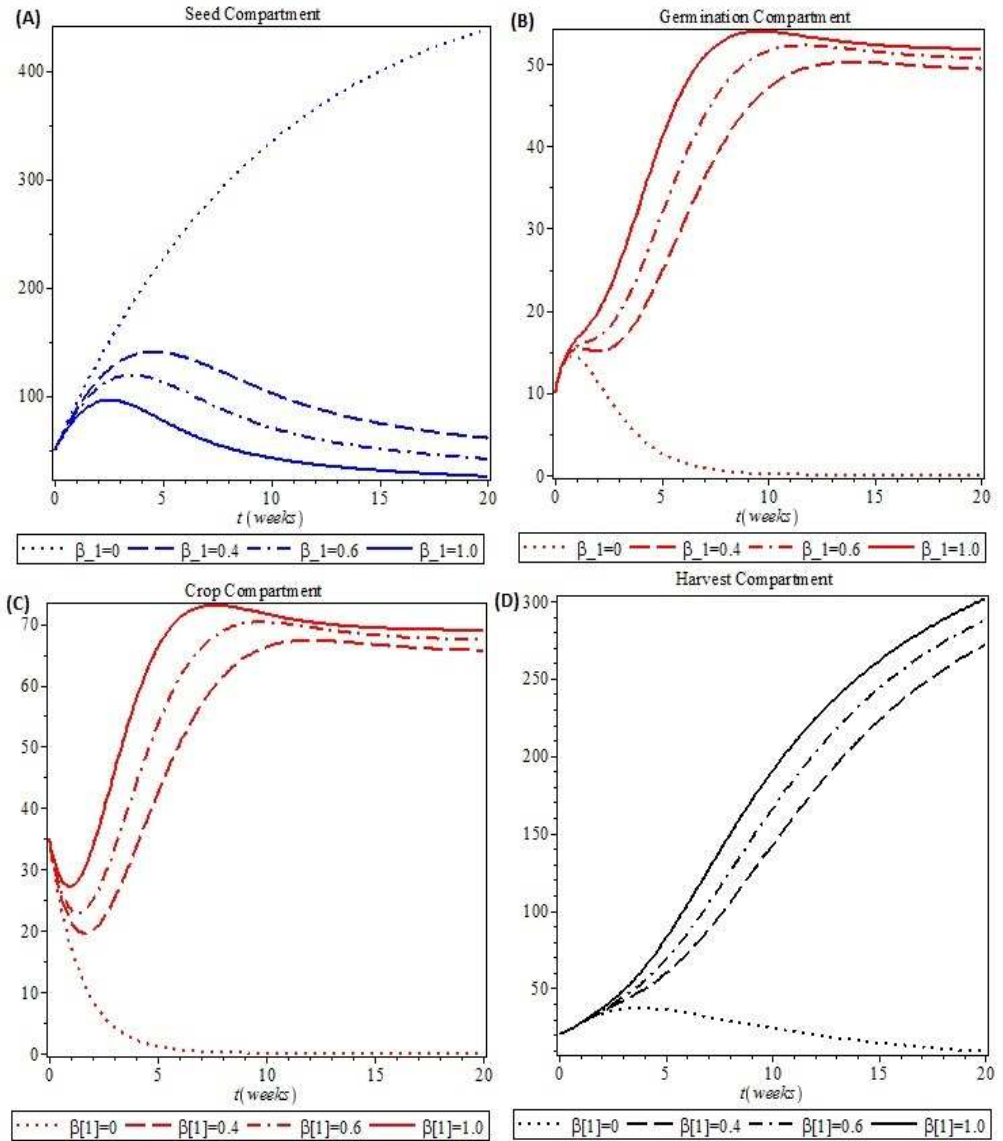


Figure 4: Variation of chances of attack on the sorghum (i.e. β varying from 0, ..., 1).

5. Discussion of Results

Figure 2(A) At Pest free Equilibrium, it was establish that, $S_c = \frac{\Lambda_c}{\mu}$, $G = 0$, $C = 0$, $H = 0$, which implies dynamics when there are no plantation and no pest attack the seed, only pest population and seed exist but there is no interaction between the pest and

the seed since the seed are systemic and they are yet to be planted. While Figure 2(B) depicted the case at Pest Persistent Equilibrium, interaction of pest and sorghum seed after planting, during the germination process, pest was able to attack at about 3 weeks of planting which drastically reduce the germination process which also reduced the cropping process, though farmers were able to harvest but most of the harvest are bad.

From the assumption that P_v helps the sorghum seeds to germinate and I_v causes the damage on the plant. Figure 3(A) clearly explains the scenario which much germination was observed but less crops were produced and this affects the quantity of harvest. Figure 3(B) explains that right from the beginning of germination, the pest had a successful attack on the seed but the survival of the attack produce crop in proportion almost equal to the number of germination seeds. And this case show better harvest than when much germination and less crop. Figure 4 shows variation of chances of attack on the sorghum (i.e., β varying from 0, ..., 1).

6. Conclusion

The sorghum production model was formulated, characterized and analyzed with differential transform method. The result obtained shows that pest attacking the production of sorghum can be controlled at every stage with the introduction of beneficial insect so as to maximize the sorghum production. More so the climatic condition of the soil and states of seeds before planting are essential factor that determines the production of sorghum.

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