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Bipolar Complex Intuitionistic Fuzzy Sets

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Abstract

The primary motivation behind this paper is to present a brief overview of the bipolar complex intuitionistic fuzzy sets (in short BCIFS) which is an extension of bipolar intuitionistic fuzzy set theory.

1. Introduction

Fuzzy sets are a sort of useful mathematical structure representing a vague collection of objects. There are various types of fuzzy sets in the fuzzy set theory, such as intuitional fuzzy sets, valued fuzzy sets, vague sets, etc. Atanassov [10] introduced intuitionistic fuzzy sets, which measure both membership degree and non-membership degree.

Zhang [8] introduced bipolar fuzzy sets in 1998. Positive information in a bipolar fuzzy set is what is guaranteed to be possible, while negative information is what is impossible or forbidden or certainly false and Zhang extended the fuzzy sets as bipolar fuzzy sets by assigning the membership value in the range $[-1,1]$. Bipolar valued fuzzy set by Lee [2] introduced a further generalization of fuzzy sets in which the degree of membership between $[0,1]$ and $[-1,1]$ increased. In bipolar fuzzy sets, membership degree 0 means that elements are irrelevant to corresponding property, membership degree belongs to $(0,1]$ indicate that somewhat elements are satisfying the corresponding property and membership degree belongs to $[-1,0)$ indicate that somewhat elements are satisfying implicit counter property.

Ramot et al. [4] introduced a new innovative concept in 2002 and called it a complex

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fuzzy set (CFS). This approach is absolutely different from other researchers, where Ramot et al. extended the range of membership function to unit disc in the complex plane, unlike the others who limited to $[0,1]$. Hence, Ramot et al. [4] added an additional term called the phase term to solve the enigma in translating some complex-valued functions on physical terms to human language and vice versa. In 2012, Alkouri and Salleh [9] introduced a new concept of complex intuitionistic fuzzy set which is generalized from the innovative concept of a complex fuzzy set by adding the non-membership term to the definition of complex fuzzy set. The novelty of complex intuitionistic fuzzy set lies in its ability for membership and non-membership functions to achieve more range of values. The ranges of values are extended to the unit disc in complex plane for both membership and non-membership functions instead of $[0,1]$ as in the conventional intuitionistic fuzzy functions.

In 2020, Al-Husban et al. [16] introduced a new concept of bipolar complex fuzzy set and some properties of the set theoretic operations of bipolar complex fuzzy sets. Ezhilmaran and Sankar [11] introduced the notion of bipolar intuitionistic fuzzy sets as well as some properties of the set theoretic operations of bipolar intuitionistic fuzzy sets.

In this work, we introduce some workable concepts about our bipolar complex intuitionistic fuzzy set concept, which are intersection, union and complement.

2. Preliminaries

In this section, we recall the definitions and related results that this work requires.

Definition 2.1 [8]**.** Let X be a non-empty set. A *bipolar fuzzy set* (BFS) A in X is an objective having the form $A=\{(x,r_A^+(x),r_A^-(x)):x\in X\}$, where $r^+:X\to[0,1]$ and $r^-:X\to[-1,0]$ are mappings.

Definition 2.2 [4]**.** A *complex fuzzy set* (CFS) A, defined on a universe of discourse U, is characterized by a membership function $\mu_A(x)$ that assigns to any element $x\in U$ a complex-valued grade of membership in A. By definition, the values of $\mu_A(x)$, may receive all lying within the unit circle in the complex plane, and are thus of the form $\mu_A(x)=r_A(x)e^{i\omega_A(x)}$, where $i=\sqrt{-1}$, each of $r_A(x)$ and $\omega_A(x)$ are both real-valued, and $r_A(x)\in[0,1]$. The CFS A may be represented as the set of ordered pairs $A=\{(x,\mu_A(x)):x\in U\}=\{(x,r_A(x)e^{i\omega_A(x)}):x\in U\}$.

Definition 2.3 [9]**.** A complex intuitionistic fuzzy set S, defined on a universe of discourse U, is characterized by membership and non-membership functions $\mu_S(x)$ and

$\gamma_S(x)$, respectively, that assign to any element $x \in U$ a complex-valued grade of both membership and non-membership in S. By definition, the values of $\mu_S(x)$, $\gamma_S(x)$, and their sum may receive all lying within the unit circle in the complex plane, and are on the form $\mu_S(x) = r_S(x) \cdot e^{i\omega_{\mu_S}(x)}$ for membership function in S and $\gamma_S(x) = k_S(x) \cdot e^{i\omega_{\gamma_S}(x)}$ for non-membership function in S, where $i = \sqrt{-1}$, each of $r_S(x)$ and $k_S(x)$ are real-valued and both belong to the interval $[0,1]$ such that $0 \leq r_S(x) + k_S(x) \leq 1$, also $\omega_{\mu_s}(x)$ and $\omega_{\gamma_s}(x)$ are real-valued. We represent the CIFS S as

$$S = \{\langle x, \mu_S(x), \gamma_S(x)\rangle : x \in U\},$$

where $\mu_S(x): U \to \{a | a \in C, |a| \leq 1\}$, $\gamma_S(x): U \to \{a' | a' \in C, |a'| \leq 1\}$ and $|\mu_S(x) + \gamma_S(x)| \leq 1$.

Definition 2.4 [16]**.** Let X be a non-empty set. A *bipolar complex fuzzy set* (BCFS) A in X is an objective having the form $A = \left\{\left(x, {r_A}^+ e^{i{\theta_A}^+}, {r_A}^- e^{i{\theta_A}^-}\right) : x \in X\right\}$, where ${r_A}^+: X \to [0,1]$ and ${r_A}^-: X \to [-1, 0]$ are mappings, $({r_A}^+ e^{i{\theta_A}^+})$, the positive complex membership degree and the $({r_A}^- e^{i{\theta_A}^-})$, negative complex membership degree. Also the phase term of bipolar complex positive membership function and bipolar complex negative membership function belongs to $(0,2\pi]$ and ${r_A}^+ \in [0,1], {r_A}^- \in [-1,0]$.

Definition 2.5 [11]**.** Let X be a non-empty set. A *bipolar intuitionistic fuzzy set* A in X is an objective having the form $A = \{\langle x, \mu_A^P(x), \gamma_A^P(x), \mu_A^N(x), \gamma_A^N(x)\rangle : x \in X\}$, where $\mu_A^P(x): X \to [0,1]$, $\gamma_A^P(x): X \to [0,1]$, $\mu_A^N(x): X \to [-1,0]$, $\gamma_A^N(x): X \to [-1,0]$ are mappings such that $0 \leq \mu_A^P(x) + \gamma_A^P(x) \leq 1$ and $-1 \leq \mu_A^N(x) + \gamma_A^N(x) \leq 0$.

Definition 2.6 [11]**.** For any two bipolar intuitionistic fuzzy sets

$$A = \{\langle x, \mu_A^P(x), \gamma_A^P(x), \mu_A^N(x), \gamma_A^N(x)\rangle : x \in X\}$$

and

$$B = \{\langle x, \mu_B^P(x), \gamma_B^P(x), \mu_B^N(x), \gamma_B^N(x)\rangle : x \in X\}$$

we have the following:

i. $A \cap B = \{\langle x, \mu_A^P(x) \wedge \mu_B^P(x), \gamma_A^P(x) \vee \gamma_B^P(x), \mu_A^N(x) \vee \mu_B^N(x), \gamma_A^N(x) \wedge \gamma_B^N(x)\rangle : x \in X\}$

ii. $A \cup B = \{\langle x, \mu_A^P(x) \vee \mu_B^P(x), \gamma_A^P(x) \wedge \gamma_B^P(x), \mu_A^N(x) \wedge \mu_B^N(x), \gamma_A^N(x) \vee \gamma_B^N(x)\rangle : x \in X\}$

iii. $A^c = \{\langle x, \gamma_A^P(x), \mu_A^P(x), \gamma_A^N(x), \mu_A^N(x), \rangle : x \in X\}$.

3. Bipolar Complex Intuitionistic Fuzzy Sets

In this section, we introduce bipolar complex intuitionistic fuzzy set, complement of a bipolar intuitionistic fuzzy set, and union of a bipolar intuitionistic fuzzy set intersection of a bipolar intuitionistic fuzzy set.

Definition 3.1. Let X be a non-empty set. A *bipolar complex intuitionistic fuzzy set* (BCIFS) A in X is an objective having the form

$$A = \left\{\left(x, r_A^+ e^{i\theta_A^+} r_A^- e^{i\theta_A^-}, s_A^+ e^{i\theta_A^+}, s_A^- e^{i\theta_A^-}\right) : x \in X\right\},$$

where $r_A^+ : X \to [0,1], r_A^- : X \to [-1, 0], s_A^+ : X \to [0,1]$ and $s_A^- : X \to [-1, 0]$ are mappings such that $0 \le r_A^+ + s_A^+ \le 1$ and $-1 \le r_A^- + s_A^- \le 0$.

We use the positive complex membership degree $\left(r_A^+ e^{i\theta_A^+}\right)$, which denote how for an element x satisfies the property corresponding to a bipolar complex intuitionistic fuzzy set A and the negative complex membership degree $\left(r_A^- e^{i\theta_A^-}\right)$, which denote how for an element x satisfies the implicit counter property corresponding to a bipolar complex intuitionistic fuzzy set. We use the positive complex non-membership degree $(s_A^+ e^{i\theta_A^+})$, which is one minus the positive complex membership degree and the negative complex non-membership degree $(r_A^- e^{i\theta_A^-})$, which is one minus the negative complex membership degree.

Also the phase term of bipolar positive complex membership function, bipolar negative complex membership function, bipolar positive complex non-membership degree, and negative complex non-membership degree belongs to $(0, 2\pi]$.

Example 3.1. Let

$$A = \{(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}, 0.3e^{\pi i}, -0.2e^{-0.2\pi i}),$$
$$(b, 0.1e^{1.3\pi i}, -0.2e^{2\pi i} 0.1e^{0.3\pi i}, -0.2e^{0.3i}),$$
$$(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}, 0.2e^{\pi i}, -0.1e^{-0.5\pi i})\}$$

be a bipolar complex intuitionistic fuzzy set of $X = \{a, b, c\}$.

Definition 3.2. The complement of a bipolar complex intuitionistic fuzzy set $A = \{(x, r_A^+ e^{i\theta_A^+} r_A^- e^{i\theta_A^-}, s_A^+ e^{i\theta_A^+}, s_A^- e^{i\theta_A^-}) : x \in X\}$ is denoted by A^c and defined by

$$A^c = \{(x, r_A^- e^{i\theta_A^-}, r_A^+ e^{i\theta_A^+}, s_A^- e^{i\theta_A^-}, s_A^+ e^{i\theta_A^+}) : x \in X\}.$$

Example 3.2. Let A is a bipolar complex intuitionistic fuzzy sets. Determine the complement of A, A^c if

$$A = \{\big(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}, 0.3e^{\pi i}, -0.2e^{-0.2\pi i}\big),$$
$$\big(b, 0.1e^{1.3\pi i}, -0.2e^{2\pi i}0.1e^{0.3\pi i}, -0.2e^{0.3\pi i}\big),$$
$$\big(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}, 0.2e^{\pi i}, -0.1e^{-0.5\pi i}\big)\}.$$

Solution. By applying Definition 3.2 on the bipolar complex intuitionistic fuzzy sets A, we have

$$A^c = \{\big(a, -0.4e^{-1.2\pi i}, 0.2e^{2\pi i}, -0.2e^{-0.2\pi i}, 0.3e^{\pi i},\big),$$
$$\big(b, -0.2e^{2\pi i}, 0.1e^{1.3\pi i}, -0.2e^{0.3\pi i}, 0.1e^{0.3\pi i},\big),$$
$$\big(c, -0.4e^{-1.5\pi i}, 0.3e^{\pi i}, -0.1e^{-0.5\pi i}, 0.2e^{\pi i}\big)\}.$$

Definition 3.3. The union of two bipolar complex intuitionistic fuzzy sets as follows:

Let A and B be two bipolar complex intuitionistic fuzzy sets in X, where

$$A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}, s_A^+ e^{i\theta_A^+}, s_A^- e^{i\theta_A^-}): x \in X\}$$

and

$$B = \{(x, r_B^+ e^{i\theta_B{}^+}, r_B^- e^{i\theta_B{}^-}, s_B^+ e^{i\theta_B{}^+}, s_B^- e^{i\theta_B{}^-}): x \in X\}.$$

Then, the union of A and B is denoted as $A \cup B$ and is given as:

$$(A \cup B) = \{max(\,r_A^+, r_B{}^+)e^{i\,max(\theta_A^+,\theta_A{}^+)}, min(\,r_A{}^-, r_B{}^-)e^{i\,max(\theta_A^-,\theta_A^-)}$$
$$min(\,r_A^+, r_B{}^+)e^{i\,min(\theta_A^+,\theta_A{}^+)}\,max(\,(r_A^+, r_B{}^+)e^{i\,max(\theta_A^+,\theta_A{}^+)}))\}.$$

Example 3.3. Let $X = \{a, b, c\}$ be a universe of discourse. Let A and B be two bipolar complex intuitionistic fuzzy sets in X, where

$$A = \{\big(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}, 0.3e^{\pi i}, -0.2e^{-0.2\pi i}\big),$$
$$\big(b, 0.1e^{1.3\pi i}, -0.2e^{2\pi i}0.1e^{0.3\pi i}, -0.2e^{0.3i}\big),$$
$$\big(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}, 0.2e^{\pi i}, -0.1e^{-0.5\pi i}\big)\}$$

and

$$B = \{\big(a, 0.1e^{\pi i}, -0.2e^{-1.3\pi i}, 0.2e^{2\pi i}, -0.3e^{-0.6\pi i}\big),$$

$$\left(b, 0.2e^{1.4\pi i}, -0.3e^{2\pi i} 0.3e^{0.6\pi i}, -0.1e^{1.3i}\right),$$

$$\left(c, 0.2e^{2\pi i}, -0.3e^{-1.6\pi i}, 0.3e^{1.6\pi i}, -0.6e^{-1.5\pi i}\right)\}$$

Then

$$A \cup B = \{\left(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}, 0.2e^{\pi i}, -0.2e^{-0.2\pi i}\right),$$

$$\left(b, 0.2e^{1.4\pi i}, -0.3e^{2\pi i}, 0.1e^{0.3\pi i}, -0.1e^{1.3i}\right),$$

$$\left(c, 0.3e^{2\pi i}, -0.4e^{-1.6\pi i}, 0.2e^{1.6\pi i}, -0.1e^{-0.5\pi i}\right)\}.$$

Definition 3.4. The intersection of two bipolar complex intuitionistic fuzzy sets as follows where

$$A = \{(x, r_A^+ e^{i\theta_A^+} r_A^- e^{i\theta_A^-}, s_A^+ e^{i\theta_A^+}, s_A^- e^{i\theta_A^-}) : x \in X\}$$

and

$$B = \left\{\left(x, r_B^+ e^{i\theta_B{}^+} r_B^- e^{i\theta_B{}^-}, s_B^+ e^{i\theta_B{}^+}, s_B^- e^{i\theta_B{}^-}\right) : x \in X\right\}.$$

Then, the intersection of A and B denoted by $A \cap B$ is given as:

$$A \cap B = \{min(r_A^+, r_B^+) e^{i\, min(\theta_A^+, \theta_A{}^+)}, max(r_A^-, r_B^-) e^{i\, max(\theta_A{}^-, \theta_A{}^-)}$$

$$max(r_A^+, r_B^+) e^{i\, max(\theta_A^+, \theta_A{}^+)}\, min((r_A^+, r_B^+) e^{i\, min(\theta_A^+, \theta_A{}^+)}))\}.$$

Example 3.4. Let $X = \{a, b, c\}$ be a universe of discourse. Let A and B be two bipolar complex intuitionistic fuzzy sets in X, where

$$A = \{\left(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}, 0.3e^{\pi i}, -0.2e^{-0.2\pi i}\right),$$

$$\left(b, 0.1e^{1.3\pi i}, -0.2e^{2\pi i} 0.1e^{0.3\pi i}, -0.2e^{0.3i}\right),$$

$$\left(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}, 0.2e^{\pi i}, -0.1e^{-0.5\pi i}\right)\}$$

and

$$B = \{\left(a, 0.1e^{2\pi i}, -0.2e^{-1.2\pi i}, 0.2e^{\pi i}, -0.3e^{-0.2\pi i}\right),$$

$$\left(b, 0.2e^{1.3\pi i}, -0.3e^{2\pi i} 0.3e^{0.3\pi i}, -0.1e^{0.3i}\right),$$

$$\left(c, 0.2e^{\pi i}, -0.3e^{-1.5\pi i}, 0.3e^{\pi i}, -0.6e^{-0.5\pi i}\right)\}$$

Then

$$A \cap B = \{\left(a, 0.2e^{2\pi i}, -0.2e^{-1.2\pi i}, 0.3e^{2\pi i}, -0.3e^{-0.6\pi i}\right),$$
$$\left(b, 0.1e^{1.3\pi i}, -0.2e^{2\pi i} 0.1e^{0.3\pi i}, -0.2e^{0.3i}\right),$$
$$\left(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}, 0.2e^{\pi i}, -0.1e^{-0.5\pi i}\right)\}.$$

Conclusion

In this work, we introduced bipolar complex intuitionistic fuzzy sets (BCIFS) and defined new operations over the bipolar complex fuzzy set (BCFS) and discussed some of their properties.

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