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Strong Differential Sandwich Results for Bazilevic-Sakaguchi Type Functions Associated with Admissible Functions

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Abstract

In the present article, we define a new family for holomorphic functions (so-called Bazilevic-Sakaguchi type functions) and determinate strong differential subordination and superordination results for these new functions by investigating certain suitable classes of admissible functions. These results are applied to obtain strong differential sandwich results.

1. Introduction and Preliminaries

Indicate by $\mathfrak{U}$ the open unit disk $\mathfrak{U}=\{t\in\mathbb{C}:|t|<1\}$, $\overline{\mathfrak{U}}=\{t\in\mathbb{C}:|t|\leq 1\}$ the closed unit disk and let $\wp(\mathfrak{U})$ stand for the family of holomorphic functions in $\mathfrak{U}$. For $n\in\mathbb{Z}^+$ and $a\in\mathbb{C}$, let $\wp[a,n]$ be the subfamily of $\wp(\mathfrak{U})$ consisting of functions of the form:

$$\mathcal{M}(t)=a+a_n t^n+a_{n+1}t^{n+1}+\cdots,$$

with $\wp=\wp[1,1]$.

Denote by $\mathbb{T}$ the collection of all holomorphic functions in the open unit disk $\mathfrak{U}$ that have the form:

$$\mathcal{M}(t)=t+\sum_{k=2}^{\infty}a_k t^k.$$

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With a view to recalling the principle of subordination between holomorphic functions, let the functions $\mathcal{M}$ and $\mathcal{N}$ be members of $\wp(\mathfrak{U})$. We say that the function $\mathcal{M}$ is said to be subordinate to $\mathcal{N}$, if there exists a Schwarz function ω holomorphic in $\mathfrak{U}$ with $\omega(0) = 0$ and $|\omega(t)| < 1$ $(t \in \mathfrak{U})$ such that $\mathcal{M}(t) = \mathcal{N}(\omega(t))$. This subordination is denoted by $\mathcal{M} \prec \mathcal{N}$ or $\mathcal{M}(t) \prec \mathcal{N}(t)(t \in \mathfrak{U})$. It is well known that, if the function $\mathcal{N}$ is univalent in $\mathfrak{U}$, then we have the following equivalent (see [10])

$$\mathcal{M}(t) \prec \mathcal{N}(t) \quad \Leftrightarrow \quad \mathcal{M}(0) = \mathcal{N}(0) \text{ and } \mathcal{M}(\mathfrak{U}) \subset \mathcal{N}(\mathfrak{U}).$$

Let $\mathcal{G}(t,\zeta)$ be holomorphic in $\mathfrak{U} \times \overline{\mathfrak{U}}$ and let $\mathcal{M}(t)$ be holomorphic and univalent in $\mathfrak{U}$. Then the function $\mathcal{G}(t,\zeta)$ is said to be strongly subordinate to $\mathcal{M}(t)$ or $\mathcal{M}(t)$ is said to be strongly superordinate to $\mathcal{G}(t,\zeta)$, written as $\mathcal{G}(t,\zeta) \prec\prec \mathcal{M}(t)$, if for $\zeta \in \overline{\mathfrak{U}} = \{t \in \mathbb{C} : |t| \le 1\}$, $\mathcal{G}(t,\zeta)$ as a function of t is subordinate to $\mathcal{M}(t)$. We note that

$$\mathcal{G}(t,\zeta) \prec\prec \mathcal{M}(t) \quad \Leftrightarrow \quad \mathcal{G}(0,\zeta) = \mathcal{M}(0) \text{ and } \mathcal{G}(\mathfrak{U} \times \overline{\mathfrak{U}}) \subset \mathcal{M}(\mathfrak{U}).$$

Definition 1.1 [11]**.** Let $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ and let h be univalent function in $\mathfrak{U}$. If $\mathbb{p}$ is holomorphic in $\mathfrak{U}$ and fulfills the following (second-order) strong differential subordination:

$$\phi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta) \prec\prec h(t), \tag{1.1}$$

then $\mathbb{p}$ is namely a solution of the strong differential subordination (1.1). The univalent function $\mathbb{q}$ is called a dominant of the solutions of the strong differential subordination or more simply a dominant if $\mathbb{p}(t) \prec \mathbb{q}(t)$ for all $\mathbb{p}$ satisfying (1.1). A dominant $\tilde{\mathbb{q}}$ that fulfills $\tilde{\mathbb{q}}(t) \prec \mathbb{q}(t)$ for all dominants $\mathbb{q}$ of (1.1) is said to be the best dominant.

Definition 1.2 [12]**.** Let $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ and let h be holomorphic function in $\mathfrak{U}$. If $\mathbb{p}$ and $\phi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta)$ are univalent in $\mathfrak{U}$ for $\zeta \in \overline{\mathfrak{U}}$ and satisfy the following (second-order) strong differential superordination:

$$h(t) \prec\prec \phi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta), \tag{1.2}$$

then $\mathbb{p}$ is namely a solution of the strong differential superordination (1.2). A holomorphic function $\mathbb{q}$ is called a subordinant of the solutions of the strong differential superordination or more simply a subordinant if $\mathbb{q}(t) \prec \mathbb{p}(t)$ for all $\mathbb{p}$ satisfying (1.2). A univalent subordinant $\tilde{\mathbb{q}}$ that fulfills $\mathbb{q}(t) \prec \tilde{\mathbb{q}}(t)$ for all subordinants $\mathbb{q}$ of (1.2) is said to be the best subordinant.

Definition 1.3 [11]**.** Denote by Q the set consisting of all functions $\mathbb{q}$ that are holomorphic and injective on $\overline{\mathfrak{U}} \setminus E(\mathbb{q})$, where

$$E(\mathfrak{q}) = \left\{\xi \in \partial\mathfrak{U}: \lim_{t\to\xi} \mathfrak{q}(t) = \infty\right\},$$

and are such that $\mathfrak{q}'(\xi) \neq 0$ for $\xi \in \partial\mathfrak{U}\backslash E(\mathfrak{q})$.

Further, let the subclass of Q for which $\mathfrak{q}(0) = a$ be denoted by $Q(a)$, $Q(0) \equiv Q_0$ and $Q(1) \equiv Q_1$.

Definition 1.4 [14]**.** Let Π be a collection in $\mathbb{C}$, $\mathfrak{q} \in Q$ and $n \in \mathbb{N}$. The class of admissible functions $\Psi_n[\Pi, \mathfrak{q}]$ consists of those functions $\psi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ that satisfy the following admissibility condition:

$$\psi(\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3; t, \zeta) \notin \Pi,$$

whenever

$$\mathfrak{r}_1 = \mathfrak{q}(\xi),\ \ \mathfrak{r}_2 = k\xi\mathfrak{q}'(\xi) \text{ and } Re\left\{\frac{\mathfrak{r}_3}{\mathfrak{r}_2} + 1\right\} \geq\ k\, Re\left\{\frac{\xi\mathfrak{q}''(\xi)}{\mathfrak{q}'(\xi)} + 1\right\},$$

$t \in \mathfrak{U}, \xi \in \partial\mathfrak{U}\backslash E(\mathfrak{q}), \zeta \in \overline{\mathfrak{U}}$ and $k \geq n$.

We simply write $\Psi_1[\Pi, \mathfrak{q}] = \Psi[\Pi, \mathfrak{q}]$.

Definition 1.5 [13]**.** Let Π be a collection in $\mathbb{C}$ and $\mathfrak{q} \in \wp[a, n]$ with $\mathfrak{q}'(t) \neq 0$. The class of admissible functions $\Psi'_n[\Pi, \mathfrak{q}]$ consists of those functions $\psi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ that satisfy the following admissibility condition:

$$\psi(\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3; \xi, \zeta) \in \Pi,$$

whenever

$$\mathfrak{r}_1 = \mathfrak{q}(t),\ \ \mathfrak{r}_2 = \frac{t\mathfrak{q}'(t)}{m} \text{ and } Re\left\{\frac{\mathfrak{r}_3}{\mathfrak{r}_2} + 1\right\} \leq \frac{1}{m}\, Re\left\{\frac{t\mathfrak{q}''(t)}{\mathfrak{q}'(t)} + 1\right\},$$

$t \in \mathfrak{U}, \xi \in \partial\mathfrak{U},\ \zeta \in \overline{\mathfrak{U}}$ and $m \geq n \geq 1$.

In particular, we write $\Psi'_1[\Pi, \mathfrak{q}] = \Psi'[\Pi, \mathfrak{q}]$.

Definition 1.6 [19]**.** Let Π be a collection in $\mathbb{C}$ and $\mathfrak{q} \in Q_1 \cap \wp$. The class of admissible functions $\Phi_{\mathcal{L}}[\Pi, \mathfrak{q}]$ consists of those functions $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ that satisfy the admissibility condition:

$$\phi(\mathcal{U}, \mathcal{V}, \mathcal{W}; t, \zeta) \notin \Pi,$$

whenever

$$\mathcal{U} = \mathbb{q}(\xi), \ \ \mathcal{V} = \frac{k\xi\mathbb{q}'(\xi)}{\mathbb{q}(\xi)}, \ \ \mathbb{q}(\xi) \neq 0 \ \text{and} \ Re\left\{\frac{\mathcal{W}+\mathcal{V}^2}{\mathcal{V}}\right\} \geq \ k\ Re\left\{\frac{\xi\mathbb{q}''(\xi)}{\mathbb{q}'(\xi)}+1\right\},$$

where $t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}, \xi \in \partial\mathfrak{U}\backslash E(\mathbb{q})$ and $k \geq 1$.

Definition 1.7 [19]**.** Let Π be a collection in $\mathbb{C}$ and $\mathbb{q} \in \wp$. The class of admissible functions $\Phi'_{\mathcal{L}}[\Pi, \mathbb{q}]$ consists of those functions $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ that satisfy the admissibility condition:

$$\phi(\mathcal{U}, \mathcal{V}, \mathcal{W}; \xi, \zeta) \in \Pi,$$

whenever

$$\mathcal{U} = \mathbb{q}(t), \ \ \mathcal{V} = \frac{t\mathbb{q}'(t)}{m\ \mathbb{q}(t)}, \ \ \mathbb{q}(t) \neq 0 \ \text{and} \ Re\left\{\frac{\mathcal{W}+\mathcal{V}^2}{\mathcal{V}}\right\} \leq \ \frac{1}{m}\ Re\left\{\frac{t\mathbb{q}''(t)}{\mathbb{q}'(t)}+1\right\},$$

where $t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}, \xi \in \partial\mathfrak{U}$ and $m \geq 1$.

In our investigations we shall need the following lemmas:

Lemma 1.1 [14]**.** *Let* $\psi \in \Psi_n[\Pi, \mathbb{q}]$ *with* $\mathbb{q}(0) = a$. *If* $\mathbb{p} \in \wp[a, n]$ *fulfills*

$$\psi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta) \in \Pi,$$

then $\mathbb{p}(t) \prec \mathbb{q}(t)$.

Lemma 1.2 [13]**.** *Let* $\psi \in \Psi'_n[\Pi, \mathbb{q}]$ *with* $\mathbb{q}(0) = a$. *If* $\mathbb{p} \in Q(a)$ *and* $\psi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta)$ *is univalent in* $\mathfrak{U}$ *for* $\zeta \in \overline{\mathfrak{U}}$, *then*

$$\Pi \subset \{\psi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta) : t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}\}$$

implies $\mathbb{q}(t) \prec \mathbb{p}(t)$.

In recent years, several authors obtained many interesting results in strong differential subordination and superordination [1,2,3,6,8,9,20,21,22,23]. The purpose of the present investigation is to consider certain suitable classes of admissible functions and investigate some strong differential subordination and superordination properties of Bazilevic-Sakaguchi type functions.

2. Main Results

We begin this section by defining a new family for functions $\mathcal{M} \in \mathbb{T}$ (so-called

Bazilevic-Sakaguchi type functions) as follows:

Definition 2.1. Let $\mathcal{M} \in \mathbb{T}$, the function $\mathcal{M}$ is said to be in the family $S(\alpha, \lambda, \delta, x, y)$ if it fulfills the following geometrical condition:

$$Re\left\{\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\right\} > \alpha,$$

where $0 \leq \delta < 1$, $\lambda \geq 0$, $0 \leq \alpha < 1$, $x, y \in \mathbb{C}$ with $x \neq y, |y| \leq 1$ and $t \in \mathfrak{U}$.

Remark 2.1. It should be remarked that the family $S(\alpha, \lambda, \delta, x, y)$ is a generalization of well-known families considered earlier. These families are:

(1) For $\delta = 0$ and $\lambda = 1$, the family $S(\alpha, \lambda, \delta, x, y)$ reduce to the family $S(\alpha, x, y)$ which was introduced recently by Frasin [7].

(2) For $\delta = 0$ and $\lambda = x = 1$, the family $S(\alpha, \lambda, \delta, x, y)$ reduce to the family $S(\alpha, y)$ which was investigated by Owa et al. [15].

(3) For $\delta = 0$, $\lambda = x = 1$ and $y = -1$, the family $S(\alpha, \lambda, \delta, x, y)$ reduce to the family $S_s(\alpha)$ of Sakaguchi type functions of order $\alpha (0 \leq \alpha < 1)$ which was studied by Sakaguchi [17] (see also [5,16]).

(4) For $\alpha = y = 0$ and $\lambda = x = 1$, the family $S(\alpha, \lambda, \delta, x, y)$ reduce to the family B_δ of Bazilevic functions which was given by Singh [18].

(5) For $\delta = y = 0$ and $x = 1$, the family $S(\alpha, \lambda, \delta, x, y)$ reduce to the family $\mathcal{L}_\lambda(\alpha)$ of λ-pseudo-starlike function of order $\alpha (0 \leq \alpha < 1)$ which was introduced by Babalola [4].

Our first main result is asserted by Theorem 2.1 below.

Theorem 2.1. *Suppose that* $\phi \in \Phi_{\mathcal{L}}[\Pi, \mathbb{q}]$. *If* $\mathcal{M} \in \mathbb{T}$ *fulfills*

$$\left\{\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.\right.$$

$$+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$

$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$

$$\left.\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\right):t\in\mathfrak{U},\zeta\in\overline{\mathfrak{U}}\right\}$$

$$\subset \Pi\,, \tag{2.1}$$

then

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec \mathbb{q}(t).$$

Proof. Assume that the function $\mathbb{p}$ be defined by

$$\mathbb{p}(t)=\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}. \tag{2.2}$$

It is obvious that $\mathbb{p}$ is holomorphic in $\mathfrak{U}$.

After some calculation from (2.2), we deduce that

$$\frac{t\mathbb{p}'(t)}{\mathbb{p}(t)}=\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right). \tag{2.3}$$

More computations show that

$$\frac{t^2\mathbb{p}''(t)}{\mathbb{p}(t)}+\frac{t\mathbb{p}'(t)}{\mathbb{p}(t)}-\left(\frac{t\mathbb{p}'(t)}{\mathbb{p}(t)}\right)^2$$

$$=t\left[\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right]'$$

$$=\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$

$$\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right]. \tag{2.4}$$

Define the transforms from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$\mathcal{U} = \mathfrak{r}_1, \qquad \mathcal{V} = \frac{\mathfrak{r}_2}{\mathfrak{r}_1}, \qquad \mathcal{W} = \frac{\mathfrak{r}_1(\mathfrak{r}_3 + \mathfrak{r}_2) - {\mathfrak{r}_2}^2}{{\mathfrak{r}_1}^2}.$$

Assume that

$$\psi\,(\mathfrak{r}_1, \mathfrak{r}_2, \mathfrak{r}_3; t, \zeta) = \phi(\mathcal{U}, \mathcal{V}, \mathcal{W}; t, \zeta) = \phi\left(\mathfrak{r}_1, \frac{\mathfrak{r}_2}{\mathfrak{r}_1}, \frac{\mathfrak{r}_1(\mathfrak{r}_3 + \mathfrak{r}_2) - {\mathfrak{r}_2}^2}{{\mathfrak{r}_1}^2}; t, \zeta\right). \tag{2.5}$$

In the light of equations (2.2), (2.3) and (2.4), we find from (2.5) that

$$\begin{aligned}
&\psi\,(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta)\\
&= \phi\left(\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)^{1-\delta}}, \frac{\lambda t \mathcal{M}''(t)}{\mathcal{M}'(t)}\right.\\
&\quad + (1-\delta)\left(1 - \frac{t\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\right), \frac{\lambda t^2 \mathcal{M}'''(t)}{\mathcal{M}'(t)}\\
&\quad + \frac{\lambda t \mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1 - \frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\\
&\quad - (1-\delta)\left[\frac{t^2\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)''}{\mathcal{M}(xt) - \mathcal{M}(yt)} + \frac{t\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\right.\\
&\qquad\qquad \left.\left.\left(1 - \frac{t\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\right)\right]; t, \zeta\right).
\end{aligned} \tag{2.6}$$

Therefore (2.1) becomes $\psi\,(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta) \in \Pi$.

We next show that the admissibility condition for $\phi \in \Phi_{\mathcal{L}}[\Pi, \mathbb{q}]$ is equivalent to the admissibility condition for ψ as given in Definition 1.4.

Note that

$$\frac{\mathfrak{r}_3}{\mathfrak{r}_2} + 1 = \frac{\mathcal{W} + \mathcal{V}^2}{\mathcal{V}},$$

and hence $\psi \in \Psi[\Pi, \mathbb{q}]$. By applying Lemma 1.1, we conclude that $\mathbb{p}(t) \prec \mathbb{q}(t)$ or equivalently

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec q(t).$$

We consider the special situation when $\Pi \neq \mathbb{C}$ is a simply connected domain. In this case $\Pi = h(\mathfrak{U})$, for some conformal mapping h of $\mathfrak{U}$ onto Π and the family $\Phi_{\mathcal{L}}[h(\mathfrak{U}), q]$ is written as $\Phi_{\mathcal{L}}[h, q]$. The following result is an immediate consequence of Theorem 2.1.

Theorem 2.2. *Suppose that* $\phi \in \Phi_{\mathcal{L}}[h, q]$. *If* $\mathcal{M} \in \mathbb{T}$ *fulfills*

$$\phi\Bigg(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)} + (1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)} + \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right) - (1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)} + \frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right]; t, \zeta\Bigg) \prec\prec h(t), \tag{2.7}$$

then

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec q(t).$$

By selecting $\phi(\mathcal{U}, \mathcal{V}, \mathcal{W}; t, \zeta) = \mathcal{U} + \frac{\mathcal{V}}{\eta\mathcal{U}+\ell}$, $(\eta, \ell \in \mathbb{C})$ in Theorem 2.2, we state the following corollary:

Corollary 2.1. *Let* $\eta, \ell \in \mathbb{C}$ *and let* h *be convex in* $\mathfrak{U}$ *with* $h(0) = 1$ *and* $Re\{\eta h(t) + \ell\} > 0$. *If* $\mathcal{M} \in \mathbb{T}$ *fulfills*

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}$$

$$+\frac{\left(1-\delta+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}-(1-\delta)\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{\delta}}}{\ell\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}+\eta\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}$$

$\prec\prec h(t)$,

then

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\prec \mathbb{q}(t).$$

The next result is an extension of Theorem 2.1 to the case where the behavior of $\mathbb{q}$ on ∂U is not known.

Corollary 2.2. *Suppose that* $\Pi\in\mathbb{C}$ *and* $\mathbb{q}$ *be univalent in* $\mathfrak{U}$ *with* $\mathbb{q}(0)=1$. *Let* $\phi\in\Phi_{\mathcal{L}}[h,\mathbb{q}_\rho]$ *for some* $\rho\in(0,1)$, *where* $\mathbb{q}_\rho(t)=\mathbb{q}(\rho t)$. *If* $\mathcal{M}\in\mathbb{T}$ *fulfills*

$$\phi\Bigg(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\Bigg)\in\Pi,$$

then

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\prec \mathbb{q}(t).$$

Proof. Theorem 2.1 produces $\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\prec\mathbb{q}_\rho(t)$. The result is now deduced from the fact that $\mathbb{q}_\rho(t)\prec\mathbb{q}(t)$.

Theorem 2.3. *Assume that* h *and* $\mathbb{q}$ *be univalent in* $\mathfrak{U}$ *with* $\mathbb{q}(0) = 1$ *and put* $\mathbb{q}_\rho(t) = \mathbb{q}(\rho t)$ *and* $h_\rho(t) = h(\rho t)$. *Let* $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ *satisfy one of the following conditions:*

(1) $\phi \in \Phi_{\mathcal{L}}[h, \mathbb{q}_\rho]$ *for some* $\rho \in (0,1)$,

(2) *there exists* $\rho_0 \in (0,1)$ *such that* $\phi \in \Phi_{\mathcal{L}}[h_\rho, \mathbb{q}_\rho]$ *for all* $\rho \in (\rho_0, 1)$.

If $\mathcal{M} \in \mathbb{T}$ *fulfills* (2.7), *then*

$$\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}} \prec \mathbb{q}(t).$$

Proof.

Case (1): An application of Theorem 2.1, we conclude that $\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec \mathbb{q}_\rho(t)$, since $\mathbb{q}_\rho(t) \prec \mathbb{q}(t)$, we find that

$$\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}} \prec \mathbb{q}(t).$$

Case (2): Define $\mathbb{p}(t) = \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}$ and $\mathbb{p}_\rho(t) = \mathbb{p}(\rho t)$. Then

$$\phi\big(\mathbb{p}_\rho(t), t\mathbb{p}'_\rho(t), t^2\mathbb{p}''_\rho(t); \rho t, \zeta\big) = \phi(\mathbb{p}(\rho t), t\mathbb{p}'(\rho t), t^2\mathbb{p}''(\rho t); \rho t, \zeta) \in h_\rho(\mathfrak{U}).$$

By making use of Theorem 2.1 and the comment associated with

$$\phi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); \omega(t), \zeta) \in \Pi,$$

where ω is any function mapping $\mathfrak{U}$ into $\mathfrak{U}$, with $\omega(t) = \rho t$, we obtain $\mathbb{p}_\rho(t) \prec \mathbb{q}_\rho(t)$ for $\rho \in (\rho_0, 1)$. By taking $\rho \longrightarrow 1^-$, we deduce $\mathbb{p}(t) \prec \mathbb{q}(t)$. Thus

$$\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}} \prec \mathbb{q}(t).$$

The next result offers the best dominant of the strong differential subordination (2.7):

Theorem 2.4. *Suppose that* h *be univalent in* $\mathfrak{U}$ *and* $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$. *Let the differential equation*

$$\phi\left(\mathbb{q}(t), \frac{t\mathbb{q}'(t)}{\mathbb{q}(t)}, \frac{t^2\mathbb{q}''(t)}{\mathbb{q}(t)} + \frac{t\mathbb{q}'(t)}{\mathbb{q}(t)} - \left(\frac{t\mathbb{q}'(t)}{\mathbb{q}(t)}\right)^2 ; t, \zeta\right) = h(t) \qquad (2.8)$$

has a solution $\mathbb{q}$ *with* $\mathbb{q}(0) = 1$ *and fulfills one of the following conditions:*

(1) $\mathbb{q} \in Q_1$ *and* $\phi \in \Phi_{\mathcal{L}}[h, \mathbb{q}]$,

(2) $\mathbb{q}$ *is univalent in* $\mathfrak{U}$ *and* $\phi \in \Phi_{\mathcal{L}}[h, \mathbb{q}_\rho]$ *for some* $\rho \in (0,1)$,

(3) $\mathbb{q}$ *is univalent in* $\mathfrak{U}$ *and there exists* $\rho_0 \in (0,1)$ *such that* $\phi \in \Phi_{\mathcal{L}}[h_\rho, \mathbb{q}_\rho]$ *for all* $\rho \in (\rho_0, 1)$.

If $\mathcal{M} \in \mathbb{T}$ *fulfills* (2.7), *then*

$$\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt) - \mathcal{M}(yt)\big)^{1-\delta}} \prec \mathbb{q}(t).$$

and $\mathbb{q}$ *is the best dominant.*

Proof. By applying Theorem 2.2 and Theorem 2.3, we conclude that $\mathbb{q}$ is a dominant of (2.7). Since $\mathbb{q}$ fulfills (2.8), it is also a solution of (2.7) and thus $\mathbb{q}$ will be dominated by all dominants. Hence $\mathbb{q}$ is the best dominant of (2.7).

In the particular case $\mathbb{q}(t) = 1 + Mt,\ M > 0$ and in view of Definition 1.6, the family of admissible functions $\Phi_{\mathcal{L}}[\Pi, \mathbb{q}]$ indicated by $\Phi_{\mathcal{L}}[\Pi, M]$ can be expressed in the following form:

Definition 2.2. Suppose that Π be a collection in $\mathbb{C}$ and $M > 0$. The family of admissible functions $\Phi_{\mathcal{L}}[\Pi, M]$ consists of those functions $\phi : \mathbb{C}^3 \times \mathfrak{U} \times \overline{\mathfrak{U}} \longrightarrow \mathbb{C}$ such that

$$\phi\left(1 + Me^{i\theta}, \frac{kM}{M + e^{-i\theta}}, \frac{kM + Le^{-i\theta}}{M + e^{-i\theta}} - \left(\frac{kM}{M + e^{-i\theta}}\right)^2 ; t, \zeta\right) \notin \Pi, \qquad (2.9)$$

whenever $t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}, \theta \in \mathbb{R}, Re\{Le^{-i\theta}\} \geq k(k-1)M$ for all θ and $k \geq 1$.

Corollary 2.3. *Let* $\phi \in \Phi_{\mathcal{L}}[\Pi, M]$. *If* $\mathcal{M} \in \mathbb{T}$ *fulfills*

$$\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.$$
$$+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$
$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$
$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$
$$\left.\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\right)\in\Pi,$$

then

$$\left|\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}-1\right|<M.$$

When $\Pi=\mathbb{q}(\mathfrak{U})=\{\omega:\ |\omega-1|<M\}$, the family $\Phi_{\mathcal{L}}[\Pi,M]$ is simply indicated by $\Phi_{\mathcal{L}}[M]$, then Corollary 2.3 takes the following form:

Corollary 2.4. *If* $\phi\in\Phi_{\mathcal{L}}[M]$ *and* $\mathcal{M}\in\mathbb{T}$ *fulfills*

$$\left|\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.\right.$$
$$+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$
$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$
$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$
$$\left.\left.\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\right)-1\right|$$
$$<M,$$

then

$$\left|\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}}-1\right|<M.$$

Example 2.1. If $M>0$ and $\mathcal{M}\in\mathbb{T}$ fulfills

$$\left|\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}-(1-\delta)\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right|<\frac{M}{M+1},$$

then

$$\left|\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}}-1\right|<M.$$

This implication follows from Corollary 2.3 by taking $\phi(\mathcal{U},\mathcal{V},\mathcal{W};t,\zeta)=\mathcal{V}+\delta$ and $\Pi=h(\mathfrak{U})$ where $h(t)=\frac{M}{M+1}t,\ M>0$. To apply Corollary 2.3, we need to show that $\phi\in\Phi_{\mathcal{L}}[\Pi,M]$, that is the admissibility condition (2.9) is satisfied follows from

$$\left|\phi\left(1+Me^{i\theta},\frac{kM}{M+e^{-i\theta}},\frac{kM+Le^{-i\theta}}{M+e^{-i\theta}}-\left(\frac{kM}{M+e^{-i\theta}}\right)^{2};t,\zeta\right)\right|=\frac{kM}{M+1}\geq\frac{M}{M+1},$$

for $t\in\mathfrak{U},\zeta\in\overline{\mathfrak{U}},\theta\in\mathbb{R}$ and $k\geq 1$.

Example 2.2. If $M>0$ and $\mathcal{M}\in\mathbb{T}$ fulfills

$$\left|\frac{\lambda t^{2}\mathcal{M}'''(t)}{\mathcal{M}'(t)}-\lambda\left(\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)^{2}\right.$$
$$\left.+(1-\delta)\left(\left(\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)^{2}-\frac{t^{2}\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right|<M,$$

then

$$\left|\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}}-1\right|<M.$$

This implication follows from Corollary 2.4 by selecting $\phi(\mathcal{U},\mathcal{V},\mathcal{W};t,\zeta)=\mathcal{W}-\mathcal{V}+\delta+2$.

Theorem 2.5. *Suppose that* $\phi \in \Phi_{\mathcal{L}}^{'}[\Pi, \mathbb{q}]$. *If* $\mathcal{M} \in \mathbb{T}$, $\frac{((x-y)t)^{1-\delta}(\mathcal{M}'(t))^{\lambda}}{(\mathcal{M}(xt)-\mathcal{M}(yt))^{1-\delta}} \in Q_1$ *and*

$$\phi\left(\frac{((x-y)t)^{1-\delta}(\mathcal{M}'(t))^{\lambda}}{(\mathcal{M}(xt)-\mathcal{M}(yt))^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)} + (1-\delta)\left(1-\frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)} + \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right) - (1-\delta)\left[\frac{t^2(\mathcal{M}(xt)-\mathcal{M}(yt))''}{\mathcal{M}(xt)-\mathcal{M}(yt)} + \frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right]; t, \zeta\right)$$

is univalent in $\mathfrak{U}$, *then*

$$\Pi \subset \left\{\phi\left(\frac{((x-y)t)^{1-\delta}(\mathcal{M}'(t))^{\lambda}}{(\mathcal{M}(xt)-\mathcal{M}(yt))^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)} + (1-\delta)\left(1-\frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)} + \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right) - (1-\delta)\left[\frac{t^2(\mathcal{M}(xt)-\mathcal{M}(yt))''}{\mathcal{M}(xt)-\mathcal{M}(yt)} + \frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t(\mathcal{M}(xt)-\mathcal{M}(yt))'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right]; t, \zeta\right) : t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}\right\}, \quad (2.10)$$

implies

$$\mathbb{q}(t) \prec \frac{((x-y)t)^{1-\delta}(\mathcal{M}'(t))^{\lambda}}{(\mathcal{M}(xt)-\mathcal{M}(yt))^{1-\delta}}.$$

Proof. Assume that $\mathbb{p}$ defined by (2.2) and $\psi\,(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta)$ defined by (2.6). Since $\phi \in \Phi_{\mathcal{L}}^{'}[\Pi, \mathbb{q}]$, then we find from (2.6) and (2.10) that

$$\Pi \subset \{\psi(\mathbb{p}(t), t\mathbb{p}'(t), t^2\mathbb{p}''(t); t, \zeta) : t \in \mathfrak{U}, \zeta \in \overline{\mathfrak{U}}\}.$$

In view of (2.5), we see that the admissibility condition for $\phi \in \Phi'_{\mathcal{L}}[\Pi, \mathbb{q}]$ is equivalent to the admissibility condition for ψ as given in Definition 1.5. Hence $\psi \in \Psi'[\Pi, \mathbb{q}]$ and by apply Lemma 1.2, we obtain $\mathbb{q}(t) \prec \mathbb{p}(t)$ or equivalently

$$\mathbb{q}(t) \prec \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)^{1-\delta}}.$$

We consider the special situation when $\Pi \neq \mathbb{C}$ is a simply connected domain. In this case $\Pi = h(\mathfrak{U})$, for some conformal mapping h of $\mathfrak{U}$ onto Π and the family $\Phi'_{\mathcal{L}}[h(\mathfrak{U}), \mathbb{q}]$ is written as $\Phi'_{\mathcal{L}}[h, \mathbb{q}]$. The following result is an immediate consequence of Theorem 2.5.

Theorem 2.6. *Suppose that $\phi \in \Phi'_{\mathcal{L}}[h, \mathbb{q}]$, $\mathbb{q} \in \wp$ and h be holomorphic in $\mathfrak{U}$. If $\mathcal{M} \in \mathbb{T}$, $\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \in Q_1$ and*

$$\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)} + (1-\delta)\left(1 - \frac{t\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)} + \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1 - \frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right) - (1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)''}{\mathcal{M}(xt) - \mathcal{M}(yt)} + \frac{t\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\left(1 - \frac{t\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)'}{\mathcal{M}(xt) - \mathcal{M}(yt)}\right)\right]; t, \zeta\right)$$

is univalent in $\mathfrak{U}$, then

$$h(t) \prec\prec \left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt) - \mathcal{M}(yt)\right)^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.$$

$$+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^{2}\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$

$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^{2}\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$

$$\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\Bigg), \qquad (2.11)$$

implies

$$\mathbb{q}(t)\prec\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}.$$

By selecting $\phi(\mathcal{U},\mathcal{V},\mathcal{W};t,\zeta)=\mathcal{U}+\frac{\mathcal{V}}{\eta\mathcal{U}+\ell}$, $(\eta,\ell\in\mathbb{C})$ in Theorem 2.6, we state the following corollary:

Corollary 2.5. *Suppose that* $\eta,\ell\in\mathbb{C}$ *and let* h *be convex in* $\mathfrak{U}$ *with* $h(0)=1$. *Let the differential equation* $\mathbb{q}(t)+\frac{t\mathbb{q}'(t)}{\eta\mathbb{q}(t)+\ell}=h(t)$ *has univalent solution* $\mathbb{q}$ *that fulfills* $\mathbb{q}(0)=1$ *and* $\mathbb{q}(t)\prec h(t)$. *If* $\mathcal{M}\in\mathbb{T}$, $\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\in\wp\cap Q_1$ *and*

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}$$

$$+\frac{\left(1-\delta+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}-(1-\delta)\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{\delta}}}{\ell\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}+\eta\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}$$

is univalent in $\mathfrak{U}$, *then*

$$h(t)\prec\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}$$

$$+\frac{\left(1-\delta+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}-(1-\delta)\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{\delta}}}{\ell\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}+\eta\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}},$$

implies

$$\mathbb{q}(t)\prec\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}.$$

The next result gives the best subordinant of the strong differential superordination (2.11):

Theorem 2.7. *Suppose that* h *be holomorphic in* $\mathfrak{U}$ *and* $\phi:\mathbb{C}^3\times\mathfrak{U}\times\overline{\mathfrak{U}}\longrightarrow\mathbb{C}$. *Let the differential equation*

$$\phi\left(\mathbb{q}(t),\frac{t\mathbb{q}'(t)}{\mathbb{q}(t)},\frac{t^2\mathbb{q}''(t)}{\mathbb{q}(t)}+\frac{t\mathbb{q}'(t)}{\mathbb{q}(t)}-\left(\frac{t\mathbb{q}'(t)}{\mathbb{q}(t)}\right)^2;t,\zeta\right)=h(t)$$

has a solution $\mathbb{q}\in Q_1$. *If* $\phi\in\Phi_{\mathcal{L}}'[h,\mathbb{q}]$, $\mathcal{M}\in\mathbb{T}$, $\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}\in Q_1$ *and*

$$\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.$$

$$+(1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$

$$\left.\left.+\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\right),$$

is univalent in $\mathfrak{U}$, *then*

$$h(t)\prec\prec\phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.$$

$$+(1-\delta)\left(1-\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$

$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^2\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)''}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right.$$

$$\left.+\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(+\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\Bigg),$$

implies

$$\mathbb{q}(t)\prec\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}}.$$

and $\mathbb{q}$ *is the best subordinant.*

Proof. The proof is similar to that of Theorem 2.4 and is omitted.

By combining Theorem 2.2 and Theorem 2.6, we obtain the following sandwich theorem:

Theorem 2.8. *Suppose that* h_1 *and* $\mathbb{q}_1$ *be holomorphic functions in* $\mathfrak{U}$, h_2 *be univalent in* $\mathfrak{U}$, $\mathbb{q}_2\in Q_1$ *with* $\mathbb{q}_1(0)=\mathbb{q}_2(0)=1$ *and* $\phi\in\Phi_{\mathcal{L}}[h_2,\mathbb{q}_2]\cap\Phi'_{\mathcal{L}}[h_1,\mathbb{q}_1]$. *If* $\mathcal{M}\in\mathbb{T}$, $\frac{((x-y)t)^{1-\delta}(\mathcal{M}'(t))^{\lambda}}{(\mathcal{M}(xt)-\mathcal{M}(yt))^{1-\delta}}\in\wp\cap Q_1$ *and*

$$\phi\left(\frac{\big((x-y)t\big)^{1-\delta}\big(\mathcal{M}'(t)\big)^{\lambda}}{\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)^{1-\delta}},\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right.$$

$$+(1-\delta)\left(1-\frac{t\big(\mathcal{M}(xt)-\mathcal{M}(yt)\big)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right),\frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)}$$

$$+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)$$

$$-(1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)} + \frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\Bigg)$$

is univalent in $\mathfrak{U}$, *then*

$$h_1(t) \prec \phi\left(\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}}, \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)} + (1-\delta)\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right), \frac{\lambda t^2\mathcal{M}'''(t)}{\mathcal{M}'(t)} + \frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\left(1-\frac{t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right) - (1-\delta)\left[\frac{t^2\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)''}{\mathcal{M}(xt)-\mathcal{M}(yt)} + \frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\left(1-\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\mathcal{M}(xt)-\mathcal{M}(yt)}\right)\right];t,\zeta\right)$$

$$\prec h_2(t),$$

implies

$$\mathfrak{q}_1(t) \prec \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec \mathfrak{q}_2(t).$$

By combining Corollary 2.1 and Corollary 2.5, we obtain the following sandwich corollary:

Corollary 2.6. *Suppose that* $\eta, \ell \in \mathbb{C}$ *and let* h_1, h_2 *be convex in* $\mathfrak{U}$ *with* $h_1(0) = h_2(0) = 1$. *Suppose that the differential equations* $\mathfrak{q}_1(t) + \frac{t\mathfrak{q}_1'(t)}{\eta\mathfrak{q}_1(t)+\ell} = h_1(t)$, $\mathfrak{q}_2(t) + \frac{t\mathfrak{q}_2'(t)}{\eta\mathfrak{q}_2(t)+\ell} = h_2(t)$ *have a univalent solutions* $\mathfrak{q}_1$ *and* $\mathfrak{q}_2$, *respectively, that fulfills* $\mathfrak{q}_1(0) = \mathfrak{q}_2(0) = 1$ *and* $\mathfrak{q}_1(t) \prec h_1(t), \mathfrak{q}_2(t) \prec h_2(t)$. *If* $\mathcal{M} \in \mathbb{T}, \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \in \wp \cap Q_1$

and

$$\frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} + \frac{\left(1-\delta+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}-(1-\delta)\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{\delta}}}{\ell\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}+\eta\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}$$

is univalent in $\mathfrak{U}$, *then*

$$h_1(t) \prec \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} + \frac{\left(1-\delta+\frac{\lambda t\mathcal{M}''(t)}{\mathcal{M}'(t)}\right)\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}-(1-\delta)\frac{t\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)'}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{\delta}}}{\ell\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}+\eta\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}} \prec h_2(t),$$

implies

$$q_1(t) \prec \frac{\left((x-y)t\right)^{1-\delta}\left(\mathcal{M}'(t)\right)^{\lambda}}{\left(\mathcal{M}(xt)-\mathcal{M}(yt)\right)^{1-\delta}} \prec q_2(t).$$

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