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Influence Function and Bootstrap Methods of Estimating the Standard Errors of the Estimators of Mixture Exponential Distribution Parameter

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Abstract

This work estimated the standard error of the maximum likelihood estimator (MLE) and the robust estimators of the exponential mixture parameter (θ) using the influence function and the bootstrap approaches. Mixture exponential random samples of sizes 10, 15, 20, 25, 50, and 100 were generated using 3 mixture exponential models at 2%, 5% and 10% contamination levels. The selected estimators namely: mean, median, alpha-trimmed mean, Huber M-estimate and their standard errors (T_n) were estimated using the two approaches at the indicated sample sizes and contamination levels. The results were compared using the coefficient of variation, confidence interval and the asymptotic relative efficiency of T_n in order to find out which approach yields the more reliable, precise and efficient estimate of T_n. The results of the analysis show that the two approaches do not equally perform at all conditions. From the results, the bootstrap method was found to be more reliable and efficient method of estimating the standard error of the arithmetic mean at all sample sizes and contamination levels. In estimating the standard error of the median, the influence function method was found to be more effective especially when the sample size is small and yet contamination is high. The

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influence function based approach yielded more reliable, precise and efficient estimates of the standard errors of the alpha-trimmed mean and the Huber M-estimate for all sample sizes and levels of contamination although the reliability of the bootstrap method improved better as sample size increased to 50 and above. All simulations and analysis were carried out in R programming language.

1. Introduction

The exponential model has many practical applications especially in life testing. Blischke and Prabhakar Murthy [1]; and Murthy et al. [2] in their different works opined that the exponential distribution is the most commonly employed model in reliability and life testing analysis. Epstein [3] stated that the exponential distribution plays a role in life testing analogous to that of the normal distribution in other areas of statistics. A study on the estimation of exponential distribution parameter is therefore very important.

The maximum likelihood estimator (MLE) of the exponential distribution parameter (θ) is the arithmetic mean and the measure of its long run accuracy is the squared error which is also based on the mean. If the perfect exponential fit assumptions of the classical statistics are fully met the MLE is expected to be optimal in estimating the population parameter in terms of attaining the lowest possible asymptotic variance among a reasonable class of estimators. In the presence of outliers or other forms of contaminations however, the classical estimate can be very suboptimal (Ripley [4]). Writing on the effect of mild deviations from a parametric model on classical estimates, Hampel et al. [5] noted that the effect of contaminations on the squared error is even worse than that on the arithmetic mean.

An estimator of a parameter may have been preferred on the basis of its efficiency, computational, and robustness properties but its realization for a particular set of data is of a little value except there is an accompanying statistic that indicates its accuracy. The standard error is the tool generally employed in assessing the long run accuracy of a given statistical estimator (Staudte and Sheather [6]).

A method that provides accurate probability coverage for one measure of scale or location can perform poorly with another, a guide on which methods performs best for which estimator is therefore very important hence our reason for embarking on this work.

Staudte and Sheather [6], noted that the alpha-trimmed mean is highly efficient and robust for entire neighbourhoods of the exponential model.

This work estimated the standard error of the MLE and robust estimators of the exponential distribution parameter θ using the influence function and the bootstrap approaches and compared the estimates on the basis of their coefficient of variation (CV), asymptotic relative efficiency (ARE) and confidence interval (CI) at various sample sizes and levels of contamination.

2. Theory and Methods

2.1. Mixture exponential models for contamination

One may view mixture models as a weighted average of some probability models. They arise in real life situations and are used to represent chance contamination.

If we presume that x_i is from model F with a probability $1-\varepsilon$ and from a contaminating distribution G with a small probability ε we can write:

$$x \sim (1-\varepsilon)F + \varepsilon G. \quad (1)$$

We may simply take the contaminating distribution G to be the point mass distribution at y so that $G = \delta y$ and $\delta y(x) = 1$ if $x > y$ and 0 otherwise, and as such, the equation (1) can be written as $x \sim (1-\varepsilon)F + \varepsilon\delta y$ as a simple model for proportion ε of outliers or contaminants at y and $F_{x,\varepsilon} = (1-\varepsilon)F + \varepsilon\delta y(x)$ is the mixture model.

Sometimes, contamination could be as a result of a proportion of observation ε coming from the same distribution family say exponential but with a larger scale. When this happens in the case of exponential distribution, it may be referred to as exponential contamination or exponential mixture.

If X has exponential distribution, we may write: $F = F_0(x) = 1 - e^{-\frac{x}{\theta}}$ as the cumulative distribution function F so that $f(x,\theta) = \frac{1}{\theta}e^{-\frac{x}{\theta}}; x > 0$. The cumulative distribution function for G the contaminating model may simply be chosen as $G(x) = 1 - e^{-\frac{x}{c\theta}}; c > 0$; so that $g(x) = \frac{1}{c\theta}e^{-\frac{x}{c\theta}}$ and $F_{x,\varepsilon} = (1-\varepsilon)F\left(\frac{x}{\theta}\right) + \varepsilon F\left(\frac{x}{c\theta}\right) = (1-\varepsilon)\frac{1}{\theta}e^{-\frac{x}{\theta}} + \varepsilon\frac{1}{c\theta}e^{-\frac{x}{c\theta}}$.

Staudte and Sheather [6] noted that if c is not very large, this kind of contamination is difficult to detect. They further asserted that goodness of fit test has power less than 0.15 at 5% alpha to distinguish this form of mixture distribution from exponential distribution even though such mixture models still undermines the optimality of the MLE estimator of θ and its standard error.

There are two approaches mostly employed in estimating the standard error of the estimators of the parameter of mixture exponential distribution are the influence function and the bootstrap based methods.

2.2. Variance of the estimators of the parameters of the mixture exponential distribution via the influence function (IF)

The Influence Function was introduced by Hampel ([7], [8]). It is the relationship between the influence a particular data point has on the value of an estimator and the distance of the data point from the estimate. Huber [9] traced the contributions of robust statistics to classical statistics and noted that among the basic robustness concepts influence function has become a standard tool. Hampel et al. [5] carried out a comprehensive treatment of influence function.

There are two approaches to calculating the influence function.

(1) By estimating a sample parameter $T(x_{n-1})$ from a sample of $n-1$ points and estimating the same parameter $T(x_n)$ from a sample of *n* points obtained by one additional point *x* (a new observation) or (2) by estimating $T(x_n)$ from a sample of *n* points and estimating the same parameter $T(x_n)$ still from a sample of *n* points but by replacing one of the points say x_n with another point say x_r.

The difference between the two estimates using any of the procedures is the influence of the new point. The approximation of this difference as a function of the normalized distance of the new point *x* from $T(x_{n-1})$ yields the influence function (Wainer [10]).

Let $T(x_{n-1}) = \frac{\sum_{i=1}^{n-1} x_i}{n-1}; i = 1,2,\dots,n$. Given an additional value of *x*, we calculate $T(x_n) = \frac{\sum_{i=1}^{n} x_i}{n}$, then $T(x_{n-1}) - T(x_n)$ is the influence of the *n*th point and $\frac{T(x_{n-1}) - T(x_n)}{1/n} = n(T(x_{n-1}) - T(x_n))$ yields the influence function (Wainer [10]).

For the mixture model $F_{x,\varepsilon} = (1-\varepsilon)F + \varepsilon\delta x$, the relative influence on $T(F)$ of proportion ε of "bad" observations at *x* is formulated by: $\frac{T(F_{x,\varepsilon}) - T(F)}{\varepsilon}$ and the influence function of *T* at *F* is defined for each *x* by:

$IF(x;T,F) = \lim_{\varepsilon\downarrow 0}\left(\frac{T(F_{x,\varepsilon}) - T(F)}{\varepsilon}\right)$ provided the limits exist. But $F_{x,\varepsilon} = (1-\varepsilon)F + \varepsilon\delta x$ so that

$$IF(x;T,F) = \lim_{\varepsilon\downarrow 0}\frac{\left[T\left((1-\varepsilon)F + \varepsilon\delta x\right) - T(F)\right]}{\varepsilon}. \quad (2)$$

The asymptotic variance of the estimators are obtained via the formula of IF by $V(T,F) = \frac{1}{n}\sum_{i=1}^{n} IF^2(x_i; T, F)$ and hence the asymptotic relative efficiency of a pair of estimators $\{T_n; n \geq 1\}$ and $\{S_n; n \geq 1\}$ can be obtained as $\mathrm{ARE}_{\mathrm{T,S}} = \frac{V(S,F)}{V(T,F)}$ (Hampel et al. [5], Staudte and Sheather [6], Huber and Ronchetti [11]).

2.2.1. The arithmetic mean

The maximum likelihood estimator of the exponential distribution parameter θ is the arithmetic mean $\bar{X}$ and it is standard error is $s_{\bar{x}}$ (Hampel et al. [5], Staudte and Sheather [6]). Let F_n be the empirical distribution of n observations from F whether or not F is continuous, then $T(F_n) = \int x dF_n(x) = \sum xp\, F_n(x) = \sum_{i=1}^{n} \frac{X_n}{n} = \bar{X}_n$.

$IF(x; T, F) = \frac{T(F_{x,\varepsilon}) - T(F)}{\varepsilon}$; if F has mean $\mu = T(F)$; $F_{x,\varepsilon}$ has mean $(1-\varepsilon)\mu + \varepsilon x$, then $T(F_{x,\varepsilon}) - T(F) = (1-\varepsilon)\mu + \varepsilon x - \mu = \varepsilon(x - \mu) = \varepsilon\big(x - T(F)\big)$ so that $IF(x; T, F) = \frac{\varepsilon(x - T(F))}{\varepsilon} = x - T(F)$.

Therefore for the arithmetic mean $IF(x; T, F) = x - T(F)$ and $V(T,F) = \frac{1}{n}\sum_{i=1}^{n} IF^2(x_i; T, F) = \frac{1}{n}\sum_{i=1}^{n}(x - T(F))^2$ (Staudte and Sheather [6], Wilcox [12], Huber, and Ronchetti [11]).

2.2.2. The trimmed mean ($\bar{X}_{n,\alpha}$)

Let $x_1 \ldots x_n$ be a set of observations such that $x_1 \leq x_2 \ldots \leq x_n$ is the observation in ascending order and let $x_i =$ the ith order statistics of the observation, an $\alpha - trimmed\ mean$ is given by $T_\alpha(F_n) = \frac{1}{n - n\alpha}\sum_{i=1}^{n-[g]} x_i \approx \frac{1}{r}\sum_{i=1}^{r} X_i = \bar{X}_{n,\alpha}$ where: $\alpha \in \left[0, \frac{1}{2}\right]$ is the required amount of trimming, $r = n - [g]$, $g = n\alpha$ and $[.]$ stands for integer part and the influence function estimate of its sample variance is: $V(T_\alpha, F) = \frac{1}{n}\sum_{i=1}^{n} IF^2(x_i; T_\alpha, F) = E_{F_n}(IF(x; T_\alpha, F_n))^2 = \frac{{s_w}^2}{(1-\alpha)^2 n}$ and the standard error is $\sqrt{V(T_\alpha, F)} = \sqrt{\frac{{s_w}^2}{(1-\alpha)^2 n}} = \frac{s_w}{(1-\alpha)\sqrt{n}}$. The sample winsorized variance ${s_w}^2 = \frac{1}{n-\alpha}\sum_{i=1}^{n}(w_i - \bar{w})^2$ and for exponential distribution,

$$w_i = \begin{cases} x_i & \text{if } 0 \leq x_i < x_{(n-g)} \\ x_{(n-g)} & \text{if } x_i \geq x_{(n-g)} \end{cases}.$$

Therefore to estimate the standard error of the trimmed mean using the influence function approach:

1. Compute the winsorized observations w_i.

2. Compute the winsorized sample variance then divide by $(1-\alpha)^2 n$ (Staudte and Sheather [6], Maronna et al. [13], Wilcox [12]).

2.2.3. The Huber's M-Estimate

Let $x_1 \le x_2 \ldots \le x_n$ be a set of observations arranged in ascending order then the Huber's M-estimator $\bar{x}_{mst}$ is given by:

$$T_n = (\bar{\boldsymbol{x}}_{\boldsymbol{mst}}) = \frac{k(\hat{\sigma}) + \sum_{t=i_1+1}^{n-i_2} x_t}{n - i_1 - i_2} = \frac{1.28(MADN(x))(i_2 - i_1) + \sum_{t=i_1+1}^{n-i_2} x_t}{n - i_1 - i_2};$$

where $MADN(x) = \frac{MAD(x)}{z_{.75}} = \frac{MAD(x)}{0.6745}$; $MAD(x) = med\{|x_1 - M|, \ldots |x_n - M|\}$; i_1 is the number of observations x_i for which $\frac{x_i - M}{MADN(x)} < -1.28$ and i_2 is the number of observations x_i for which $\frac{x_i - M}{MADN(x)} > 1.28$. $V(T_{mst}, F) = \frac{1}{n}\sum_{i=1}^{n} IF^2(x_i; T_{mst}, F)$ (Hampel et al. [5], Wilcox [12])

2.2.4. The qth quarntile estimator

Given a set of observations $x_1 \le x_2 \ldots \le x_n$, arranged in ascending order, for any $q, 0 \le q \le 1$, the qth quantile x_q may be defined by $p(x \le x_q) = q$. If we define $m = q[n+1]$ where $q[.]$ is the greatest integer less than or equal to $q(n+1)$, x_q may simply be estimated as $\hat{x}_q = X_{(m)}$ or in our usual notation as $T_{n(q)} = X_{(m)}$ that is the mth observation. If X is continuous and $f(x_q) > 0$, the influence function (IF) of the qth quantile is given as:

$$IF_q(x) = \begin{cases} \dfrac{q-1}{f(x_q)} & \text{if } x < x_q \\ 0 & \text{if } x = x_q. \\ \dfrac{q}{f(x_q)} & \text{if } x > x_q \end{cases}$$

So that $T_{n(q)} = \hat{x}_q = x_q + \frac{1}{n}\sum IF_q(X_i)$ with a remainder term that tends to zero as n tends to infinity and IF based estimated variance is given as $V(\hat{x}_q) = \frac{q(1-q)}{n[f(x_q)]^2}$.

So for $q = 0.5$ ie the median; $V\left(T_{n(.5)}\right) = \frac{1}{4n[f(x_{0.5})]^2}$ and the standard error $SE_{.5} = \frac{1}{2\sqrt{n}f(x_{0.5})}$ (Staudte and Sheather [6], Wilcox [12]).

For the exponential distribution the programme took cognizance of the lower limit of X which is 0.

2.3. Bootstrap estimation of the standard error of T_n

The bootstrap method for estimating a standard error is the second approach we employed for estimating the standard error of the estimates. Efron defined a re-sampling procedure that he coined as bootstrap (Efron [14]).

The idea of bootstrap is to use only what one knows from the data and not introduce extraneous assumptions about the population distribution.

The theoretical mean of the bootstrap distribution is the sample mean (Chernick and LaBudde [15]).

2.3.1. Algorithm for deriving the bootstrap estimate of the standard error of T_n

Let T_n be any estimator based on a random sample of observations: $X_1, X_2, \ldots X_{n.}$; where the observations are Independently and identically distributed as F. We wish to estimate $\sqrt{V(T,F)} = SE_{T_n}(F) = \sqrt{V_F(T_n)}$ using the bootstrap method. The bootstrap estimate of $SE_{T_n}(F)$ is defined by substituting the empirical distribution F_n *by* F (going by our symbol) (Efron [16]) where $SE_{T_n}(F)$ is a known function of F. Therefore:

- First fix F_n the empirical distribution which puts mass $\frac{1}{n}$ on the n data points $x_1, x_2 \ldots x_n$
- Draw a random sample of size n with replacement from the empirical distribution F_n to have $X_1{}^*, X_2{}^*, \ldots, X_n{}^*$ the bootstrap sample, and calculate $T_n{}^* = T_n(X_1{}^*, X_2{}^*, \ldots X_n{}^*)$ which is the bootstrap estimate.
- Repeat the last step independently for B times so that we have: $T_n{}^{*1}, T_n{}^{*2}, \ldots . T_n{}^{*B}$ and $SE_{T_n}(F_n) = \sqrt{\frac{\sum_{b=1}^{B}(T_n{}^{*b} - \bar{T}_{nB}{}^*)^2}{B-1}}$ where $\bar{T}_{nB}{}^* = \frac{\sum_{b=1}^{B} T_n{}^{*b}}{B}$. According to Efron ([16]) most often $B = 100$ will suffice.

2.4. Simulation study

Based on the models, indicated sample sizes and the levels of contamination, we generated data through simulation, estimated T_n the standard error of the estimators of theta, estimated the standard error of the standard errors SET_n and carried out our analysis.

The contaminated data were generated considering the case of contamination due to increased scale for exponential mixture models. Given the general mixture model $F_{x,\varepsilon} = (1-\varepsilon)\frac{1}{\theta}e^{-\frac{x}{\theta}} + \varepsilon\frac{1}{c\theta}e^{-\frac{x}{c\theta}}$ employed in this research work, we choose $c = \text{constant} = 3, 6, \text{and } 9$. Specifically:

(i) $(1-\varepsilon)\frac{1}{\theta}e^{-\frac{x}{\theta}} + \varepsilon\frac{1}{3\theta}e^{-\frac{x}{3\theta}}$ is tagged model one,

(ii) $(1-\varepsilon)\frac{1}{\theta}e^{-\frac{x}{\theta}} + \varepsilon\frac{1}{6\theta}e^{-\frac{x}{6\theta}}$ is model 2 and

(iii) $(1-\varepsilon)\frac{1}{\theta}e^{-\frac{x}{\theta}} + \varepsilon\frac{1}{9\theta}e^{-\frac{x}{9\theta}}$ is model 3.

We worked at 2%, 5%, 10% contamination levels and for sample sizes of, 10, 15, 20, 25, 50, and 100 to make for small, medium and large sample sizes. The multiplicative constants were chosen to make the estimators unbiased for θ. Without loss of generality, we assumed $\theta = 1$ since our estimators are scale equivariant. The "MASS", "mixtools", and the "WRS2" packages for R programming language were used for the simulation and analysis.

3. Method of Comparison

Generally, in evaluating the standard error estimates, what is important to us is for the standard error to be small relative to the estimate itself (Staudte and Sheather [6]). We will therefore compare the methods based on their ability to render the least standard error of the estimate.

3.1. Coefficient of variation (CV)

The relative standard error or the coefficient of variation of T_n is given by:

$$CV(T_n) = \frac{\sqrt{Var(T_n)}}{E(T_n)},$$

where $E(T_n)$ is estimated by T_n itself.

The coefficient of variation of the standard error of the estimator itself is a guide to its effectiveness. Staudte and Sheather [6] recommended the use of CV in evaluating the standard error of estimates. The estimator has a variability which we want to be small relative to what it is estimating. Therefore the approach that returns larger CV is considered less reliable and the one that delivers mostly unreliable estimates will be deemed to be performing poorer than the other.

3.2 Confidence interval (CI)

The $\alpha\%$ confidence interval using the bootstrap method is given as: $\left({T_n}^*_{(\ell+1)}, {T_n}^*_{(u)}\right)$ where ${T_n}^*$ is an estimate of θ based on the bootstrap method, $\ell = {}^{\alpha B}/_{2}$; $u = B - \ell$; α is the confidence level. The procedure that delivers a shorter interval is preferred to the other (Wilcox [12]).

3.3. The asymptotic relative efficiency (ARE)

Again the efficiency of $\hat{\theta}_1$ to $\hat{\theta}_2$ is $Eff\left(\hat{\theta}_1, \hat{\theta}_2\right) = \frac{var(\hat{\theta}_2)}{var(\hat{\theta}_1)}$; if $var\left(\hat{\theta}_2\right) > var\left(\hat{\theta}_1\right)$, $Eff > 1$ meaning that $\hat{\theta}_1$ is preferred however, if $Eff\left(\hat{\theta}_1, \hat{\theta}_2\right) < 1$, then $\hat{\theta}_2$ is preferred (Staudte and Sheather [6]).

4. Result and Discussion

The columns of the tables indicate the approach used for the particular estimation (influence function or bootstrap approach), the sample size used, the model employed, as well as the value of ε so that: $IF_{n_r m_i j}$ means influence function based estimate when $n = r$ for model i at $\varepsilon = j\%$ and $B_{n_r\, m_i\, j}$ = bootstrap based estimate at $n = r$, for model i and at $\varepsilon = j\%$.

An abridged version of the results of the analysis for the coefficient of variation and the confidence interval are included in the body of the work while the unabridged results are provide as appendix. This is to avoid producing an unnecessarily long report.

4.1. Coefficient of variation (CV) of the standard error estimate T_n

When CV is used as a tool for analysis, interest is typically to find out which procedure returns the smaller CV.

Table 1. Result of the Coefficient of Variation of T_n

When n=10 and ε= 2%	IFn10m12	Bn10m12	IFn10m22	Bn10m22	IFn10m32	Bn10m32
CV Mean T_n	0.4307355	0.05696358	1.008673	0.107728	0.9956117	0.1860713
CV Med T_n	0.05722874	0.121998	0.0905039	0.09064958	0.06205067	0.09914904
CV 10% tmnT_n	0.03957395	0.0632623	0.04406564	0.0624341	0.04854952	0.1781203

CV 20% tmnT_n	0.03680817	0.0636865	0.04574305	0.05412152	0.04991686	0.1939246
CV HmestT_n	0.09763311	0.1086214	0.1366441	0.1200062	0.1318419	0.1351329
When n=10 and ε= 5%	**IFn10m15**	**Bn10m15**	**IFn10m25**	**Bn10m25**	**IFn10m35**	**Bn10m35**
CV Mean Tn	0.3835547	0.06819947	0.7765613	0.0406069	1.165643	0.1502246
CV Med Tn	0.04123198	0.1037717	0.08191376	0.06791203	0.1288774	0.01871698
CV 10% tmnTn	0.0380557	0.06369623	0.04329522	0.041706	0.0430507	0.1000914
CV 20% tmnTn	0.03641615	0.06390621	0.03917372	0.0516966	0.04075297	0.03744649
CV HmestTn	0.1350465	0.1116545	0.1111417	0.203535	0.130325	1.090695
When n=10 and ε= 10%	**IFn10m110**	**Bn10m110**	**IFn10m210**	**Bn10m210**	**IFn10m310**	**Bn10m310**
CV Mean Tn	0.4816624	0.06094511	0.7969795	0.110284	1.010452	0.1155721
CV Med Tn	0.05622038	0.1265833	0.06216959	0.09464207	0.1492849	0.0997504
CV 10% tmnTn	0.04267457	0.08178365	0.04565435	0.1537206	0.05749545	0.1232185
CV 20% tmnTn	0.04173948	0.09724286	0.04741101	0.08494325	0.04833963	0.1113086
CV HmestTn	0.1224908	0.1316121	0.1103881	0.09389521	0.1216276	0.1513644
. . .	. . .	. . .	. . .	. . .	. . .	. . .
When n=100 and ε= 2%	**IFn100m12**	**Bn100m12**	**IFn100m22**	**Bn100m22**	**IFn100m32**	**Bn100m32**
CV Mean Tn	0.1851841	0.02423298	0.3812978	0.04538339	0.5056724	0.0719155
CV Med Tn	0.02557197	0.02198737	0.026072	0.02660919	0.02224238	0.02415154
CV 10% tmnTn	0.01346576	0.01772905	0.01228351	0.01839356	0.01823878	0.018725
CV 20% tmnTn	0.01207272	0.0169637	0.01214709	0.01635217	0.01563112	0.01996117
CV HmestTn	0.1047202	0.075984	0.100963	0.08272461	0.1326444	0.1091139
When n=100 and ε= 5%	**IFn100m15**	**Bn100m15**	**IFn100m25**	**Bn100m25**	**IFn100m35**	**Bn100m35**
CV Mean Tn	0.2288451	0.02423298	0.3734015	0.04959519	0.4261752	0.05920616
CV Med Tn	0.02581634	0.02198737	0.02190671	0.0185176	0.0221687	0.02864529
CV 10% tmnTn	0.01400197	0.01772905	0.01258529	0.01505818	0.01662085	0.02075339
CV 20% tmnTn	0.01223093	0.0169637	0.01345667	0.01828625	0.01564633	0.01934215
CV HmestTn	0.1050312	0.075984	0.09988048	0.0788894	0.1023692	0.09093356
When n=100, ε= 10%	**IFn100m110**	**Bn100m110**	**IFn100m210**	**Bn100m210**	**IFn100m310**	**Bn100m310**
CV Mean Tn	0.2684579	0.02644563	0.379446	0.04186255	0.2506887	0.05894393
CV Med Tn	0.02862773	0.04120912	0.02312763	0.02202045	0.01910866	0.02822895
CV 10% tmnTn	0.01552349	0.01808345	0.01674969	0.02455759	0.01715469	0.03570538
CV 20% tmnTn	0.01305047	0.01795251	0.01684379	0.02159811	0.01686232	0.02625656
CV HmestTn	0.1022002	0.06615164	0.113449	0.07977504	0.1179325	0.06194797

Here CV Mean T_n= the CV of the standard error estimate T_n for the mean, CV Med T_n = the CV of the standard error estimate T_{nn} for the median, CV10% tmnT_n = the CV of the standard error estimate T_n for 10% trimmed mean, CV20%tmnT_n = the CV of the standard error estimate T_n for the 20% trimmed mean, and CVHmestT_n = the CV of the standard error estimate T_n for the Huber's M-estimate

From Table 1 above, the highest coefficient of variation values returned by the influence function approach for model 1, model 2 and model 3 are for the standard error of the arithmetic mean and mostly at the lower levels of contamination and sample sizes.

The highest CV values returned by the bootstrap approach are for the standard error of the Huber's M-estimate at lower sample sizes. The bootstrap approach was however more reliable than the influence function for the standard error of the arithmetic mean at all sample sizes and levels of contamination.

The influence function method was more reliable for the standard error of the median, the 10% and the 20% trimmed mean as well as the Huber's M-estimate even though the bootstrap approach gained more reliability with Huber's M-estimate with increased sample size. This implies that for a mixture exponential distribution the bootstrap approach is more effective than the influence function approach essentially for the standard error of the arithmetic mean under the conditions covered in this study.

4.2. The 95% confidence interval for the standard error estimate T_n

Table 2. Result at 95% confidence interval for T_n.

When n=10 & ε= 2%	**IFn10m12**	**B n10m12**	**IFn10m22**	**Bn10m22**	**IFn10m32**	**Bn10m32**
CI Mean Tn	0.2766077 1.6211948	0.8314 1.0511	-0.7482768 3.0188106	0.783 1.166	-1.033458 3.536181	0.871 1.796
CI Med Tn	0.4754683 0.5742846	0.1303 0.2202	0.4337110 0.5854254	0.1505 0.2359	0.4815322 0.5909993	0.1773 0.2591
CI 10% tmnTn	0.1984403 0.2320555	0.6226 0.7940	0.2078346 0.2451616	0.5794 0.7540	0.2402176 0.2828713	0.670 1.384
CI 20% tmnTn	0.1582650 0.1782971	0.4288 0.5625	0.1562903 0.1812813	0.4198 0.5335	0.2205365 0.2630283	0.3742 0.9056
CI HmestTn	0.2155397 0.2980122	0.4288 0.5625	0.1978728 0.3126084	4.673 7.740	0.1913133 0.2972672	4.967 9.117
When n=10 & ε= 5%	**IFn10m15**	**Bn10m15**	**IFn10m25**	**Bn10m25**	**IFn10m35**	**Bn10m35**
CI Mean Tn	0.3608059 1.5942060	0.899 1.183	-0.3662668 3.0076489	0.7361 0.8654	-1.814372 5.003946	0.893 1.535
CI Med Tn	0.2845203 0.3259207	0.899 1.183	0.4871484 0.6388628	0.1718 0.2273	0.4543144 0.6987449	0.1388 0.2192
CI 10% tmnTn	0.1418869 0.1629802	0.6563 0.8288	0.2079713 0.2486066	0.5459 0.6496	0.1968265 0.2346458	0.7030 1.0939
CI 20% tmnTn	0.1234329 0.1399589	0.5090 0.6686	0.1669658 0.1898464	0.4263 0.5240	0.1615816 0.1846629	0.4796 0.6338
CI HmestTn	0.1631623 0.2563490	5.349 8.488	0.2261599 0.3273476	2.151 4.908	0.2075221 0.3207697	6.544 10.596
When n=10 & ε= 10%	**IFn10m110**	**Bn10m110**	**IFn10m210**	**Bn10m210**	**IFn10m310**	**Bn10m310**
CI Mean Tn	0.2390448 2.0623850	1.113 1.411	-3.238097 7.202947	1.258 2.000	-1.906115 6.374884	1.880 2.979
CI Med Tn	0.5434780 0.6542357	0.1711 0.2803	0.5995978 0.8840185	0.1633 0.2389	0.5668662 0.9358643	0.1777 0.2756
CI 10% tmnTn	0.2430693 0.2854576	0.7684 1.0749	0.2451516 0.2953039	0.624 1.425	0.2117765 0.2691653	0.954 1.582

CI 20% tmnTn	0.1813074 0.2074969	0.5379 0.8113	0.1562903 0.1812813	0.5272 0.7495	0.1687207 0.1973796	0.6287 1.0281
CI HmestTn	0.2378621 0.3578948	4.440 8.424	0.2638496 0.380922	5.575 8.235	0.2339003 0.3508940	7.44 14.41
. . .	. . .	. . .	. . .	. . .	. . .	. . .
When n=100 and ε= 5%	**IFn100m15**	**Bn100m15**	**IFn20m25**	**Bn100m25**	**IFn100m35**	**Bn100m35**
CI Mean Tn	0.6233571 1.5306104	1.040 1.143	0.7286027 3.0484079	1.52 1.84	0.7685911 4.3724153	1.940 2.425
CI Med Tn	0.09535659 0.10381420	0.0897 0.0973	0.1019220 0.1095417	0.0900 0.0982	0.1022051 0.1099408	0.0870 0.0984
CI 10% tmnTn	0.06022304 0.06350337	0.6017 0.6488	0.06331783 0.06639718	0.6417 0.6816	0.06253255 0.06667619	0.6665 0.7291
CI 20% tmnTn	0.04993297 0.05197984	0.4431 0.4748	0.04921317 0.05153343	0.4557 0.4886	0.05239614 0.05489366	0.4758 0.5159
CI HmestTn	0.07814902 0.11079037	1.995 2.734	0.08294792 0.11556065	2.807 3.807	0.08476724 0.11909386	3.626 5.350
When n=100-& ε= 10%	**IFn100m110**	**Bn100m110**	**IFn100m210**	**Bn100m210**	**IFn100m310**	**Bn100m310**
CI Mean Tn	0.8041398 2.0758830	1.312 1.471	0.8992901 3.8858617	2.080 2.492	1.556163 5.819326	3.126 4.059
CI Med Tn	0.1027302 0.1128831	0.0920 0.1054	0.1089969 0.1176177	0.0944 0.1040	0.1136669 0.1233243	0.0895 0.1024
CI 10% tmnTn	0.05910087 0.06311327	0.6708 0.7272	0.06721233 0.07160005	0.7462 0.8250	0.06908224 0.07390841	0.7797 0.9053
CI 20% tmnTn	0.04791090 0.05028046	0.4866 0.5242	0.05647432 0.05930473	0.5167 0.5668	0.05784181 0.06142697	0.5204 0.5790
CI HmestTn	0.08882558 0.12472424	2.257 2.938	0.0914879 0.1334659	3.884 5.512	0.09309415 0.13790347	7.077 9.150

CI Mean Tn = the confidence interval for Tn on the Mean, CI Med Tn = Confidence Interval for Tn on the median, CI 10% tmnTn= confidence interval for Tn on the 10% trimmed, CI 20% tmnTn= confidence interval for Tn on 20% trimmed, CI HmestTn=confidence interval for Tn on the Huber M-estimator

Our interest is to find out which approach returns a shorter CI for T_n. From Table 2, the bootstrap approach returned shorter confidence intervals for the mean for all three levels of contamination and all sample sizes. At the lower contamination levels and sample sizes, the bootstrap method returned shorter interval for the standard error of the median than the influence function approach, as the contamination level increased however this advantage in precision was lost. The influence function approach was more precise for the standard error of the 10%, and 20% trimmed mean as well as the Huber's M-estimate for a mixture exponential distribution.

4.3. The asymptotic relative efficiency (ARE) of the standard error of T_n

The efficiency of the bootstrap based estimates relative to the influence function approach is presented on the table below.

Table 3. Result of the asymptotic relative efficiency (ARE) of the standard error of T_n

	At ε= 2%			**At ε= 5%**			**At ε= 10%**		
When n=10 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	58.65502	111.8286	22.95648	27.38818	1010.447	2.137431	50.87081	73.96608	66.66479
eff2	1.762772	6.629202	2.626477	0.30727	10.58428	0.166457	1.317895	18.26337	20.94128
eff3	0.051531	0.07369	0.0044	0.01838	0.24928	0.00017	0.018886	0.133036	0.012299
eff4	0.035367	0.086576	0.00324	0.018315	0.079033	0.00016	0.013846	0.031354	0.008517
eff5	0.001377	0.002139	0.000982	0.001381	0.001749	1.68E-05	0.001665	0.002773	0.000394
When n=15 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	75.67057	179.9077	85.88122	35.97352	104.5063	515.3845	94.1612	120.1907	97.19749
eff2	0.829153	30.00991	1.753307	0.550306	0.841251	2.10143	0.902593	0.936079	0.916384
eff3	0.072208	0.26764	0.046113	0.0245	0.046074	0 06962	0.033734	0.020495	0.003846
eff4	0.047766	0.378383	0.031969	0.025305	0.031311	0.052087	0.023216	0.025839	0.004163
eff5	0.00278	0.019881	0.004112	0.002381	0.001946	0.004981	0.002369	0.00098	0.000192
When n=20 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	95.43625	175.0107	1.926993	57.71506	0.955302	145.4732	95.85688	89.53016	279.5066
eff2	0.728706	0.940562	0.002022	0.949277	1.654461	0.395601	0.950944	0.533721	171.5596
eff3	0.07505	0.015082	9.37E-05	0.011418	0.027316	0.017459	0.033854	0.008201	0.067411
eff4	0.046226	0.02781	0.000245	0.02594	0.042305	0.037204	0.039797	0.008011	0.067332
eff5	0.00169	0.001276	4.66E-05	0.002689	0.002088	0.001762	0.003685	0.000749	0.004801
When n=25 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	72.85275	188.9274	313.2703	72.85275	102.3673	50.98293	42.66317	912.996	91.73421
eff2	0.252735	0.703354	0.148683	0.202461	0.426959	0.258227	0.708104	1.461085	0.322092
eff3	0.024739	0.013715	0.017135	0.01768	0.019185	0.024388	0.017192	0.032514	0.003071
eff4	0.020503	0.026637	0.014005	0.016779	0.010356	0.050419	0.019897	0.034353	0.027031
eff5	0.003646	0.002848	0.002453	0.001851	0.001295	0.001888	0.003547	0.005466	0.02677
When n=50 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	42.66317	912.996	91.73421	79.2282	130.966	146.664	63.60464	74.03	209.5211
eff2	0.708104	1.461085	0.322092	1.492036	1.113834	1.690552	1.364737	0.229131	0.147486
eff3	0.017192	0.032514	0.003071	0.017944	0.01213	0.013304	0.014843	0.004192	0.004795
eff4	0.019897	0.034353	0.027031	0.015947	0.013628	0.018184	0.025398	0.002671	0.002626
eff5	0.003547	0.005466	0.02677	0.008959	0.003186	0.004286	0.003514	0.000603	0.001238
When n=100 and ε= 2%, 5%, & 10%	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**	**M1**	**M2**	**M3**
eff1	57.30409	76.81346	60.31253	49.45757	70.15957	72.49108	108.6669	88.37365	35.68032
eff2	1.523525	1.167904	1.025961	1.060915	1.761512	0.772278	0.57809	1.394134	1.092401

eff3	0.006833	0.005688	0.011465	0.010472	0.00884	0.007594	0.009326	0.004821	0.002331
eff4	0.005769	0.006559	0.007258	0.007646	0.00665	0.006249	0.006321	0.012981	0.005654
eff5	0.002748	0.002523	0.001407	0.002538	0.001448	0.000644	0.003817	0.001098	0.000732

Here eff1= the efficiency of the bootstrap based estimates relative to the influence function with respect to the standard error of the mean, eff2 = the efficiency of the bootstrap based estimates relative to the influence function with respect to the standard error of the median, eff3 = the efficiency of the bootstrap based estimates relative to the influence function with respect to the standard error of the 10% trimmed mean, eff4 = the efficiency of the bootstrap based estimates relative to the influence function with respect to the standard error of the 20% trimmed mean, eff5 = the efficiency of the bootstrap based estimates relative to the influence function with respect to the standard error of the Huber M-estimate. Also let M1, M2, M3 stand for model1, model2 and model3 as defined earlier

Table 4. The number of times bootstrap approach was more efficient than IF at different ε

E	mean	median	10% trimmed mean	20% trimmed mean	Huber M-estimate
2%	54/54	9/54	0	0	0
5%	54/54	7/54	0	0	0
10%	54/54	8/54	0	0	0

From Table 3, we noticed that the bootstrap method is more efficient than the influence function approach for the estimation of the standard error of the mean, the influence function approach is more efficient for the estimation of the standard error of the median, the 10%, & 20% trimmed mean and the Huber's m-estimator for a mixture exponential distribution.

5. Summary of Finding and Conclusion

5.1. Summary of finding

The following were the findings of the study:

The bootstrap approach yielded a more reliable, more precise and more efficient result for the estimation of the standard error of the arithmetic mean than the influence function approach did. This could possibly be explained by the claims of Chernick and LaBudde ([15]) that the arithmetic mean is the theoretical mean of the bootstrap distribution.

The coefficient of variation for the standard error of the arithmetic mean was high even more than 100% in some cases at lower sample sizes and higher contamination levels when the influence function approach was used which indicates a poor performance. This however improved with increase in the sample size especially for the IF method.

The influence function approach was more reliable for the standard error of the median especially at the higher levels of contamination.

For the 10% and 20% trimmed mean, the influence function approach also yielded more reliable, more precise and more efficient estimates than the bootstrap approach however, the results of the 20% trimmed mean are preferred.

At lower sample sizes and particularly with higher contamination levels, the influence function approach performed better than the bootstrap approach for the standard error of the Huber's m-estimate particularly, in terms of effectiveness and reliability. This however greatly improved for the bootstrap method as the sample size increased to 50 and above.

5.2. Conclusion

In conclusion, the two procedures for estimating the standard error considered in this work do not equally perform under all conditions. For mixture exponential distribution and under the stipulated conditions, the influence function approach is generally preferred in estimating the standard error having been found more efficient, more reliable and precise in estimation in most cases especially at higher levels of contamination except for the arithmetic mean.

We therefore recommend that the bootstrap method should be the first choice preferred to the influence function approach in estimating the standard error of the arithmetic mean for mixture exponential distribution. For the standard error of the median, the alpha trimmed mean and the Huber M-estimator, the influence function approach should be the first choice. The standard error of 20% trimming performed better than 10% trimming.

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Appendix A

Result of the Coefficient of Variation of T_n

When n=10 and ε= 2%	IFn10m12	Bn10m12	IFn10m22	Bn10m22	IFn10m32	Bn10m32
CV Mean T_n	0.4307355	0.05696358	1.008673	0.107728	0.9956117	0.1860713
CV Med T_n	0.05722874	0.121998	0.0905039	0.09064958	0.06205067	0.09914904
CV 10% tmnT_n	0.03957395	0.0632623	0.04406564	0.0624341	0.04854952	0.1781203
CV 20% tmnT_n	0.03680817	0.0636865	0.04574305	0.05412152	0.04991686	0.1939246
CV HmestT_n	0.09763311	0.1086214	0.1366441	0.1200062	0.1318419	0.1351329
When n=10 and ε= 5%	**IFn10m15**	**Bn10m15**	**IFn10m25**	**Bn10m25**	**IFn10m35**	**Bn10m35**
CV Mean Tn	0.3835547	0.06819947	0.7765613	0.0406069	1.165643	0.1502246
CV Med Tn	0.04123198	0.1037717	0.08191376	0.06791203	0.1288774	0.01871698
CV 10% tmnTn	0.0380557	0.06369623	0.04329522	0.041706	0.0430507	0.1000914
CV 20% tmnTn	0.03641615	0.06390621	0.03917372	0.0516966	0.04075297	0.03744649
CV HmestTn	0.1350465	0.1116545	0.1111417	0.203535	0.130325	1.090695
When n=10 and ε= 10%	**IFn10m110**	**Bn10m110**	**IFn10m210**	**Bn10m210**	**IFn10m310**	**Bn10m310**
CV Mean Tn	0.4816624	0.06094511	0.7969795	0.110284	1.010452	0.1155721
CV Med Tn	0.05622038	0.1265833	0.06216959	0.09464207	0.1492849	0.0997504
CV 10% tmnTn	0.04267457	0.08178365	0.04565435	0.1537206	0.05749545	0.1232185
CV 20% tmnTn	0.04173948	0.09724286	0.04741101	0.08494325	0.04833963	0.1113086
CV HmestTn	0.1224908	0.1316121	0.1103881	0.09389521	0.1216276	0.1513644
When n=15 and ε= 2%	**IFn15m12**	**Bn15m12**	**IFn15m22**	**Bn15m22**	**IFn15m32**	**Bn15m32**
CV Mean Tn	0.977506	0.9267116	1.196078	1.3005	1.337995	1.095413
CV Med Tn	0.3052205	0.1875868	0.3007642	0.0924575	0.3253976	0.1677188
CV 10% tmnTn	0.1524336	0.5872462	0.1601059	0.62407	0.1685818	0.567053
CV 20% tmnTn	0.1316959	0.4953583	0.1345257	0.4618157	0.1377779	0.4849248
CV HmestTn	0.2097556	4.17356	0.2104178	2.261791	0.2243302	4.391644
When n=15 and ε= 5%	**IFn15m15**	**B n15m15**	**IF n15m25**	**B n15m25**	**IF n15m35**	**B n15m35**
CV Mean Tn	1.079279	1.068706	1.367796	1.292602	2.142445	1.498106
CV Med Tn	0.3134978	0.2082372	0.3156534	0.2055432	0.3412486	0.189422
CV 10% tmnTn	0.1702153	0.6755228	0.176446	0.6918751	0.307145	0.567053
CV 20% tmnTn	0.1408037	0.5435577	0.142751	0.5597664	0.23049	0.4849248
CV HmestTn	0.2264072	4.193927	0.2259005	5.211548	0.2243302	4.391644
When n=15 and ε= 10%	**IFn15m110**	**Bn15m110**	**IFn15m210**	**Bn15m210**	**IFn15m310**	**Bn15m310**
CV Mean Tn	1.060691	0.9674195	1.265927	1.232958	1.620429	1.121002
CV Med Tn	0.2285878	0.1819971	0.225282	0.1987781	0.2377125	0.169723
CV 10% tmnTn	0.1332488	0.6686231	0.1372087	0.6892312	0.1376816	0.6747918
CV 20% tmnTn	0.1163239	0.5002373	0.1142462	0.5132513	0.1182036	0.4743597
CV HmestTn	0.1934155	3.719893	0.18325	4.184207	0.2000695	3.508061

When n=20 and ε= 2%	**IFn20m12**	**Bn20m12**	**IFn20m22**	**Bn20m22**	**IFn20m32**	**Bn20m32**
CV Mean Tn	1.060691	0.9674195	1.265927	1.232958	1.620429	1.121002
CV Med Tn	0.2285878	0.1819971	0.225282	0.1987781	0.2377125	0.169723
CV 10% tmnTn	0.1332488	0.6686231	0.1372087	0.6892312	0.1376816	0.6747918
CV 20% tmnTn	0.1163239	0.5002373	0.1142462	0.5132513	0.1182036	0.4743597
CV HmestTn	0.1934155	3.719893	0.18325	4.184207	0.2000695	3.508061
When n=20 and ε= 5%	**IFn20m15**	**Bn20m15**	**IFn20m25**	**Bn20m25**	**IFn20m35**	**Bn20m35**
CV Mean Tn	1.164085	1.183336	1.495955	1.530552	2.214	1.505431
CV Med Tn	0.2614947	0.2055002	0.2328776	0.1810504	0.2590309	0.1902197
CV 10% tmnTn	0.1444517	0.7647136	0.1376903	0.8330986	0.1487749	0.812842
CV 20% tmnTn	0.11661	0.5286491	0.11661	0.5669414	0.1257121	0.5291056
CV HmestTn	0.1984061	3.582698	0.1984061	4.341629	0.2092236	4.577884
When n=20 and ε= 10%	**IFn20m110**	**Bn20m110**	**IFn20m210**	**Bn20m210**	**IFn20m310**	**Bn20m310**
CV Mean Tn	1.36348	1.234779	2.371529	2.072068	2.234384	1.671181
CV Med Tn	0.2805284	0.2207353	0.2774771	0.2180895	0.7513653	0.2012939
CV 10% tmnTn	0.02531264	0.04068042	0.02891495	0.04492138	0.03320726	0.04719746
CV 20% tmnTn	0.02393301	0.03849992	0.02560195	0.0508134	0.03482178	0.03398128
CV HmestTn	0.1128538	0.1284312	0.1232576	0.1230948	0.1272732	0.1399542
When n=25 and ε= 2%	**IFn25m12**	**Bn25m12**	**IFn25m22**	**Bn25m22**	**IFn25m32**	**Bn25m32**
CV Mean Tn	0.3239635	0.04003858	0.7058304	0.05864381	0.7988325	0.05662681
CV Med Tn	0.03823712	0.09434729	0.0295303	0.04191963	0.03068519	0.1032371
CV 10% tmnTn	0.02531264	0.03625988	0.02883281	0.0508088	0.02626749	0.04354824
CV 20% tmnTn	0.02393301	0.03585624	0.02950344	0.03906192	0.02463783	0.04437831
CV HmestTn	0.1128538	0.1048696	0.1243911	0.1334689	0.1152748	0.115891
When n=25and ε= 5%	**IFn25m15**	**Bn25m15**	**IFn25m25**	**Bn25m25**	**IFn25m35**	**Bn25m35**
CV Mean Tn	0.3239635	0.0350895	0.701652	0.08306027	0.7864148	0.09999554
CV Med Tn	0.03823712	0.09980895	0.04077204	0.0672462	0.05620511	0.06836956
CV 10% tmnTn	0.02531264	0.04068042	0.02891495	0.04492138	0.03320726	0.04719746
CV 20% tmnTn	0.02393301	0.03849992	0.02560195	0.0508134	0.03482178	0.03398128
CV HmestTn	0.1128538	0.1284312	0.1232576	0.1230948	0.1272732	0.1399542
When n=25 and ε= 10%	**IFn25m110**	**Bn25m110**	**IFn25m210**	**Bn25m210**	**IFn25m310**	**Bn25m310**
CV Mean Tn	0.4445413	0.04155351	0.7748815	0.04155351	0.7172928	0.08155868
CV Med Tn	0.0300703	0.05279967	0.05092504	0.05279967	0.03378353	0.07286246
CV 10% tmnTn	0.02615154	0.04541698	0.03685324	0.04541698	0.03411938	0.1039639
CV 20% tmnTn	0.02564059	0.03958788	0.03433863	0.03958788	0.03040753	0.03620111
CV HmestTn	0.109033	0.100869	0.1343914	0.100869	0.1156228	0.05661229
When n=50 and ε= 2%	**IFn50m12**	**Bn50m12**	**IFn50m22**	**Bn50m22**	**IFn50m32**	**Bn50m32**
CV Mean Tn	0.1866186	0.02284893	0.545211	0.04709388	0.8420415	0.07455753
CV Med Tn	0.03078831	0.03327397	0.02971557	0.03704365	0.03547128	0.03552073
CV 10% tmnTn	0.01935963	0.02439447	0.01907923	0.0288382	0.02395395	0.03342763

CV 20% tmnTn	0.01940844	0.02424277	0.01909364	0.02627209	0.02157181	0.02523321
CV HmestTn	0.1168254	0.08878587	0.1114721	0.0887918	0.1189673	0.08038695
When n=50 and ε= 5%	**IFn50m15**	**Bn50m15**	**IFn50m25**	**Bn50m25**	**IFn50m35**	**Bn50m35**
CV Mean Tn	0.2662618	0.03358665	0.5112604	0.08306027	0.6055528	0.08190364
CV Med Tn	0.03114767	0.03453115	0.03947838	0.0672462	0.03065495	0.07621776
CV 10% tmnTn	0.02122502	0.02823881	0.01811836	0.04492138	0.02199127	0.05931312
CV 20% tmnTn	0.02183945	0.02161244	0.01891118	0.0508134	0.02092531	0.06237495
CV HmestTn	0.1153136	0.09966965	0.1113039	0.1230948	0.1170926	0.1030478
When n=50 and ε= 10%	**IFn50m110**	**Bn50m110**	**IFn50m210**	**Bn50m210**	**IFn50m310**	**Bn50m310**
CV Mean Tn	1.291617	1.387103	2.367365	2.378448	3.435958	3.390528
CV Med Tn	0.1778646	0.1360042	0.1778275	0.1356485	0.1909091	0.1490424
CV 10% tmnTn	0.09638346	0.742071	0.05821992	0.8649104	0.08774134	0.9590077
CV 20% tmnTn	0.08109131	0.5313989	0.05459124	0.5494784	0.08698404	0.5889044
CV HmestTn	0.1444413	2.69772	0.1592606	5.137981	0.163959	8.115525
When n=100 and ε= 2%	**IFn100m12**	**Bn100m12**	**IFn100m22**	**Bn100m22**	**IFn100m32**	**Bn100m32**
CV Mean Tn	0.1851841	0.02423298	0.3812978	0.04538339	0.5056724	0.0719155
CV Med Tn	0.02557197	0.02198737	0.026072	0.02660919	0.02224238	0.02415154
CV 10% tmnTn	0.01346576	0.01772905	0.01228351	0.01839356	0.01823878	0.018725
CV 20% tmnTn	0.01207272	0.0169637	0.01214709	0.01635217	0.01563112	0.01996117
CV HmestTn	0.1047202	0.075984	0.100963	0.08272461	0.1326444	0.1091139
When n=100 and ε= 5%	**IFn20m15**	**Bn20m15**	**IFn20m25**	**Bn20m25**	**IFn20m35**	**Bn20m35**
CV Mean Tn	0.2288451	0.02423298	0.3734015	0.04959519	0.4261752	0.05920616
CV Med Tn	0.02581634	0.02198737	0.02190671	0.0185176	0.0221687	0.02864529
CV 10% tmnTn	0.01400197	0.01772905	0.01258529	0.01505818	0.01662085	0.02075339
CV 20% tmnTn	0.01223093	0.0169637	0.01345667	0.01828625	0.01564633	0.01934215
CV HmestTn	0.1050312	0.075984	0.09988048	0.0788894	0.1023692	0.09093356
When n=100, ε= 10%	**IFn100m110**	**Bn100m110**	**IFm210**	**Bm210**	**IFm310**	**Bm310**
CV Mean Tn	0.2684579	0.02644563	0.379446	0.04186255	0.2506887	0.05894393
CV Med Tn	0.02862773	0.04120912	0.02312763	0.02202045	0.01910866	0.02822895
CV 10% tmnTn	0.01552349	0.01808345	0.01674969	0.02455759	0.01715469	0.03570538
CV 20% tmnTn	0.01305047	0.01795251	0.01684379	0.02159811	0.01686232	0.02625656
CV HmestTn	0.1022002	0.06615164	0.113449	0.07977504	0.1179325	0.06194797

Where CV Mean T_n = the CV of the standard error estimate T_n for the mean, CV Med T_n = the CV of the standard error estimate T_{nn} for the median, CV10% tmnT_n = the CV of the standard error estimate T_n for 10% trimmed mean, CV20%tmnT_n = the CV of the standard error estimate T_n for the 20% trimmed mean, and CVHmestT_n = the CV of the standard error estimate T_n for the Huber's M-estimate

The Result of 95% Confidence Interval for the Standard Error Estimate T_n

When n=10 & ε=2%	IFn10m12	B n10m12	IFn10m22	Bn10m22	IFn10m32	Bn10m32
CI Mean Tn	0.2766077 1.6211948	0.8314, 1.0511	-0.7482768 3.0188106	0.783, 1.166	-1.033458 3.536181	0.871 1.796
CI Med Tn	0.4754683 0.5742846	0.1303, 0.2202	0.4337110 0.5854254	0.1505, 0.2359	0.4815322 0.5909993	0.1773, 0.2591
CI 10% tmnTn	0.1984403 0.2320555	0.6226, 0.7940	0.2078346 0.2451616	0.5794 0.7540	0.2402176 0.2828713	0.670 1.384
CI 20% tmnTn	0.1582650 0.1782971	0.4288, 0.5625	0.1562903 0.1812813	0.4198 0.5335	0.2205365 0.2630283	0.3742, 0.9056
CI HmestTn	0.2155397 0.2980122	0.4288, 0.5625	0.1978728 0.3126084	4.673 7.740	0.1913133 0.2972672	4.967 9.117
CI Mean Tn	0.3608059 1.5942060	0.899 1.183	-0.3662668 3.0076489	0.7361 0.8654	-1.814372 5.003946	0.893 1.535
CI Med Tn	0.2845203 0.3259207	0.899 1.183	0.4871484 0.6388628	0.1718 0.2273	0.4543144 0.6987449	0.1388 0.2192
CI 10% tmnTn	0.1418869 0.1629802	0.6563 0.8288	0.2079713 0.2486066	0.5459 0.6496	0.1968265 0.2346458	0.7030 1.0939
CI 20% tmnTn	0.1234329 0.1399589	0.5090 0.6686	0.1669658 0.1898464	0.4263 0.5240	0.1615816 0.1846629	0.4796 0.6338
CI HmestTn	0.1631623 0.2563490	5.349 8.488	0.2261599 0.3273476	2.151 4.908	0.2075221 0.3207697	6.544 10.596
When n=10 & ε=10%	**IFn10m110**	**Bn10m110**	**IFn10m210**	**Bn10m210**	**IFn10m310**	**Bn10m310**
CI Mean Tn	0.2390448 2.0623850	1.113 1.411	-3.238097 7.202947	1.258 2.000	-1.906115 6.374884	1.880 2.979
CI Med Tn	0.5434780 0.6542357	0.1711 0.2803	0.5995978 0.8840185	0.1633 0.2389	0.5668662 0.9358643	0.1777 0.2756
CI 10% tmnTn	0.2430693 0.2854576	0.7684 1.0749	0.2451516 0.2953039	0.624 1.425	0.2117765 0.2691653	0.954 1.582
CI 20% tmnTn	0.1813074 0.2074969	0.5379 0.8113	0.1562903 0.1812813	0.5272 0.7495	0.1687207 0.1973796	0.6287 1.0281
CI HmestTn	0.2378621 0.3578948	4.440 8.424	0.2638496 0.380922	5.575 8.235	0.2339003 0.3508940	7.44 14.41
CI Mean Tn	0.3608059 1.5942060	0.899 1.183	-0.3662668 3.0076489	0.7361 0.8654	-1.814372 5.003946	0.893 1.535
When n=15 & ε=2%	**IFn15m12**	**Bn15m12**	**IFn15m22**	**Bn15m22**	**IFn15m32**	**Bn15m32**
CI Mean Tn	0.3608059 1.5942060	0.8422 1.0056	-0.1060672 2.4982238	1.168, 1.398	-0.02960048 2.70559015	0.923 1.268
CI Med Tn	0.2845203 0.3259207	0.1607 0.2130	0.2785959 0.3229326	0.0885 0.0983	0.2975560 0.3532392	0.1356 0.1876
CI 10% tmnTn	0.1418869 0.1629802	0.5396 0.6372	0.1503380 0.1698738	0.5973 0.6451	0.1567019 0.1804617	0.4951 0.6175
CI 20% tmnTn	0.1234329 0.1399589	0.4500, 0.5434	0.1268849 0.1421664	0.4462 0.4765	0.1288823 0.1466735	0.4268 0.5462
CI HmestTn	0.1631623 0.2563490	3.212 5.625	0.1670238 0.2538117	1.938 2.626	0.1765948 0.2720656	3.358 5.231

When n=15 & ε=5%	IFn15m15	Bn15m15	IFn15m25	Bn15m25	IFn15m35	Bn15m35
CI Mean Tn	0.3676821 1.7908757	0.919 1.197	-0.4375336 3.1731260	1.054 1.476	-1.207781 5.492672	0.923 1.268
CI Med Tn	0.2923364 0.3346593	0.1653 0.2341	0.2948008 0.3365061	0.1636 0.2273	0.3107680 0.3717291	0.1356 0.1876
CI 10% tmnTn	0.1607799 0.1796506	0.5903 0.7367	0.1654728 0.1874192	0.6061 0.7465	0.1546379 0.1838348	0.4951 0.6175
CI 20% tmnTn	0.1338850 0.1477225	0.4900 0.5968	0.1344723 0.1510297	0.4944 0.6099	0.1299385 0.1526479	0.4268 0.5462
CI HmestTn	0.1862512 0.2665632	3.158 5.275	0.1793545 0.2724466	3.970 6.384	0.1822979 0.2873716	3.358 5.231
When n=15 & ε=10%	**IFn15m110**	**Bn15m110**	**IFn15m210**	**Bn15m210**	**IFn15m310**	**Bn15m310**
CI Mean Tn	0.1173302 2.4965768	1.007 1.303	-0.8457995 4.7710390	1.387 2.011	-1.336987 7.650459	1.978 3.124
CI Med Tn	0.3104606 0.3545854	0.1898 0.2434	0.3434046 0.3899084	0.1428 0.2077	0.3568073 0.4167929	0.1711 0.2316
CI 10% tmnTn	0.1699933 0.1953590	0.5990 0.7786	0.1875470 0.2105669	0.6401 0.8423	0.3868449 0.5052325	0.833 1.342
CI 20% tmnTn	0.1395646 0.1575026	0.4800 0.6203	0.1564214 0.1732927	0.5164 0.6453	0.2169007 0.3279791	0.5345 0.9285
CI HmestTn	0.1631967 0.2563146	4.496 6.587	0.2127586 0.3056034	5.459 9.303	0.1764229 0.2722375	7.04 14.80
When n=20 & ε=2%	**IFn20m12**	**Bn20m12**	**IFn20m22**	**Bn20m22**	**IFn20m32**	**Bn20m32**
CI Mean Tn	0.4700585 1.6513242	0.8837 1.0379	-0.6422091 3.1740637	1.058 1.404	-0.939180 4.180038	0.936, 1.276
CI Med Tn	0.2106040 0.2465715	0.1529 0.1975	0.2056712 0.2448929	0.1736 0.2350	0.2251585 0.2502666	0.1474 0.1886
CI 10% tmnTn	0.1226453 0.1438522	0.6168 0.7169	0.1309992 0.1434182	0.6178 0.7463	0.1290620 0.1463012	0.5845 0.7462
CI 20% tmnTn	0.1095626 0.1230852	0.4483 0.5293	0.1089400 0.1195524	0.4709 0.5526	0.1126304 0.1237767	0.4387 0.5175
CI HmestTn	0.1519345 0.2348965	2.651 5.184	0.1503184 0.2161816	2.826 5.169	0.160681 0.239458	2.633 4.126
When n=20 & ε=5%	**IFn20m15**	**Bn20m15**	**IFn20m25**	**Bn20m25**	**IFn20m35**	**Bn20m35**
CI Mean Tn	0.2011777 2.1269923	1.019 1.350	-0.197459 3.189368	1.252 1.746	-0.8341707 5.2621707	1.127 1.797
CI Med Tn	0.2430208 0.2799686	0.1860 0.2407	0.2172181 0.2485372	0.1550 0.1942	0.2412306 0.2768312	0.1606 0.2227
CI 10% tmnTn	0.1371944 0.1517090	0.6734 0.8497	0.1264300 0.1489506	0.7217 0.9396	0.1373912 0.160158	0.7276 0.9226
CI 20% tmnTn	0.1194342 0.1316477	0.4864 0.5739	0.1095808 0.1236392	0.4640 0.6416	0.1189174 0.1325068	0.4828 0.5688
CI HmestTn	0.1635663 0.2452434	2.589 4.431	0.1607249 0.2360872	2.543 5.278	0.1730241 0.2454231	3.319 5.566

When n=20 & ε=10%	**IFn20m110**	**Bn20m110**	**IFn20m210**	**Bn20m210**	**IFn20m310**	**Bn20m310**
CI Mean Tn	0.3007132 2.4262476	1.119 1.386	-0.7337578 5.4768157	1.630 2.475	-1.906115 6.374884	1.381 1.929
CI Med Tn	2.4262476 0.3019793	0.1978 0.2425	0.2609117 0.2940425	0.1861 0.2448	0.5668662 0.9358643	0.1794 0.2189
CI 10% tmnTn	0.1460895 0.1707578	0.7272 0.8939	0.1446940 0.1706794	0.835 1.168	0.2117765 0.2691653	0.8308 1.0982
CI 20% tmnTn	0.1314148 0.1472259	0.7272 0.8939	0.1066295 0.1218628	0.5703 0.7706	0.1687207 0.1973796	0.4965 0.6545
CI HmestTn	0.1723200 0.2775047	0.5185 0.6170	0.1796197 0.2758760	4.177 8.305	0.2339003	4.257 6.645
When n=25 & ε=2%	**IFn20m12**	**Bn20m12**	**IFn20m22**	**Bn20m22**	**IFn20m32**	**Bn20m32**
CI Mean Tn	0.4890923 1.6049499	0.9310 1.0877	-0.2106751 2.8279571	0.963 1.266	-0.442431 3.260799	0.972 1.246
CI Med Tn	0.1987287 0.2254043	0.1415 0.2063	0.1853379 0.2042620	0.1468 0.1792	0.1924017 0.2128562	0.1209 0.1810
CI 10% tmnTn	0.1180613 0.1297018	0.5756 0.6641	0.1111825 0.1239449	0.5746 0.7162	0.1188499 0.1306036	0.5647 0.6755
CI 20% tmnTn	0.1023193 0.1106642	0.4526 0.5250	0.09772582 0.10766067	0.4400 0.5157	0.09998503 0.10838865	0.4363 0.5219
CI HmestTn	0.1453086 0.2115518	2.513 3.943	0.1376865 0.2085227	1.934 3.737	0.1418061 0.2081641	2.461 4.168
When n=25 & ε=5%	**IFn25m15**	**Bn25m15**	**IFn25m25**	**Bn25m25**	**IFn25m35**	**Bn25m35**
CI Mean Tn	0.4890923 1.6049499	0.9310 1.0877	-0.2385952 3.3349249	1.054 1.476	-0.6875088 5.3718115	1.204 1.849
CI Med Tn	0.1987287 0.2254043	0.1514 0.2225	0.2066585 0.2363698	0.1636, 0.2273	0.2018331 0.2429532	0.1616 0.2065
CI 10% tmnTn	0.1180613 0.1297018	0.5961 0.7116	0.1104827 0.1246446	0.6061, 0.7465	0.1182939 0.1342376	0.5824, 0.7156
CI 20% tmnTn	0.1023193 0.1106642	0.4651 0.5431	0.09793224 0.10745425	0.4944 0.6099	0.1046814 0.1172028	0.4640 0.5324
CI HmestTn	0.1453086 0.2115518	2.653 4.440	0.1493066 0.2252431	3.970 6.384	0.1500870 0.2295657	2.616 4.809
When n=25 & ε=10%	**IFn25m110**	**Bn25m110**	**IFn25m210**	**Bn25m210**	**IFn25m310**	**Bn25m310**
CI Mean Tn	0.3764002 2.4242522	1.151 1.353	-0.6349792 5.2602973	1.151 1.353	-0.5333497 6.4646766	2.253 3.159
CI Med Tn	0.2003985 0.2237346	0.1574 0.1989	0.2095668 0.2478849	0.1574 0.1989	0.2100119 0.2304663	0.1536 0.2107
CI 10% tmnTn	0.1298334 0.1414738	0.6195 0.7455	0.1275750 0.1461411	0.6195 0.7455	0.1235024 0.1409299	0.6867 1.0778
CI 20% tmnTn	0.1137315 0.1237166	0.4997 0.5832	0.1106894 0.1238097	0.4997 0.5832	0.09834936 0.11002432	0.5249 0.6586
CI HmestTn	0.1669713 0.2399499	2.783 4.385	0.1596007 0.2501857	2.783 4.385	0.1657117 0.2435453	2.296 2.854

When n=50 & ε=2%	**IFn50m12**	**Bn50m12**	**IFn50m22**	**Bn50m22**	**IFn50m32**	**Bn50m32**
CI Mean Tn	0.7251775 1.3675670	0.9107 0.9992	0.1319687 2.4253716	1.177 1.401	-0.6200821 3.8409951	1.261 1.696
CI Med Tn	0.1406456 0.1556507	0.1037 0.1161	0.1504817 0.1659481	0.1098 0.1285	0.1481759 0.1665379	0.1101 0.1298
CI 10% tmnTn	0.08096043 0.08737572	0.5636 0.6252	0.08688539 0.09334865	0.5826 0.6575	0.1294313 0.1380395	0.5899 0.6862
CI 20% tmnTn	0.06813431 0.07261176	0.4232 0.4657	0.07088998 0.07549378	0.4350 0.4784	0.1262487 0.1353394	0.4467 0.4912
CI HmestTn	0.1036087 0.1528995	1.508 2.079	0.1050807 0.1522667	2.327 3.327	0.1063871 0.1581533	2.426 3.366
When n=50 & ε=5%	**IFn50m15**	**Bn50m15**	**IFn50m25**	**Bn50m25**	**IFn50m35**	**Bn50m35**
CI Mean Tn	0.4337382 1.9516650	1.071 1.236	0.2873809 3.3263020	1.054 1.476	0.008480444 4.280688744	0.923 1.268
CI Med Tn	0.1959973 0.2226730	0.1129 0.1303	0.1567094 0.1784749	0.1636 0.2273	0.1520699 0.1682198	0.1356 0.1876
CI 10% tmnTn	0.1152711 0.1279774	0.6136 0.6850	0.08790194 0.09452166	0.6061 0.7465	0.08427305 0.09193453	0.4951 0.6175
CI 20% tmnTn	0.1006778 0.1099809	0.4474 0.4855	0.07558114 0.08041675	0.4944 0.6099	0.07198057 0.07707923	0.4268 0.5462
CI HmestTn	0.1471182 0.2142982	1.887 3.117	0.1155991 0.1674125	3.970 6.384	0.1097833 0.1621595	3.358 5.231
When n=50 & ε=10%	**IFn50m110**	**Bn50m110**	**IFn50m210**	**Bn50m210**	**IFn50m310**	**Bn50m310**
CI Mean Tn	0.6780164 1.9052167	1.276 1.476	0.3595472 4.3751835	2.098 2.641	0.3409801 6.5309352	2.886 3.748
CI Med Tn	0.1706519 0.1850774	0.1191 0.1466	0.1670200 0.1886351	0.1252 0.1438	0.1818391 0.1999792	0.1397 0.1595
CI 10% tmnTn	0.09231594 0.10045098	0.6982 0.7849	0.05821992 0.05821992	0.7745 0.9337	0.09689876 0.10658946	0.8685 1.0482
CI 20% tmnTn	0.07862220 0.08356043	0.4972 0.5640	0.05459124 0.05459124	0.5192 0.5854	0.08367776 0.09029031	.5587 0.6236
CI HmestTn	0.1185139 0.1703687	2.316 3.194	0.1335169 0.185004	4.108 6.312	0.1303807 0.1975373	6.606 9.600
When n=100 and ε= 2%	**IFn100m12**	**Bn100m12**	**IFn100m22**	**Bn100m22**	**IFn100m32**	**Bn100m32**
CI Mean Tn	0.7489337 1.4050338	1.040 1.143	0.5057804 2.2074813	1.168 1.398	0.3075835 3.3488384	1.429 1.894
CI Med Tn	0.09460185 0.10290960	0.0897 0.0973	0.09760391 0.10635042	0.0885 0.0983	0.09807634 0.10552519	0.0854 0.0955
CI 10% tmnTn	0.05959635 0.06261780	0.6017 0.6488	0.06045759 0.06330553	0.5973 0.6451	0.06008269 0.06429406	0.6167 0.6620
CI 20% tmnTn	0.04812407 0.05006730	0.4431 0.4748	0.04936733 0.05137927	0.4462 0.4765	0.04893748 0.05151757	0.4437 0.4774
CI HmestTn	0.07395161 0.10472928	1.995 2.734	0.07762542 0.10854216	1.938 2.626	0.07260207 0.11312388	2.224 3.666

When n=100 and ε= 5%	**IFn100m15**	**Bn100m15**	**IFn20m25**	**Bn100m25**	**IFn100m35**	**Bn100m35**
CI Mean Tn	0.6233571 1.5306104	1.040 1.143	0.7286027 3.0484079	1.52 1.84	0.7685911 4.3724153	1.940 2.425
CI Med Tn	0.09535659 0.10381420	0.0897 0.0973	0.1019220 0.1095417	0.0900 0.0982	0.1022051 0.1099408	0.0870 0.0984
CI 10% tmnTn	0.06022304 0.06350337	0.6017 0.6488	0.06331783 0.06639718	0.6417 0.6816	0.06253255 0.06667619	0.6665 0.7291
CI 20% tmnTn	0.04993297 0.05197984	0.4431 0.4748	0.04921317 0.05153343	0.4557 0.4886	0.05239614 0.05489366	0.4758 0.5159
CI HmestTn	0.07814902 0.11079037	1.995 2.734	0.08294792 0.11556065	2.807 3.807	0.08476724 0.11909386	3.626 5.350
When n=100 & ε= 10%	**IFn100m110**	**Bn100m110**	**IFn100m210**	**Bn100m210**	**IFn100m310**	**Bn100m310**
CI Mean Tn	0.8041398 2.0758830	1.312 1.471	0.8992901 3.8858617	2.080 2.492	1.556163 5.819326	3.126 4.059
CI Med Tn	0.1027302 0.1128831	0.0920 0.1054	0.1089969 0.1176177	0.0944 0.1040	0.1136669 0.1233243	0.0895 0.1024
CI 10% tmnTn	0.05910087 0.06311327	0.6708 0.7272	0.06721233 0.07160005	0.7462 0.8250	0.06908224 0.07390841	0.7797 0.9053
CI 20% tmnTn	0.04791090 0.05028046	0.4866 0.5242	0.05647432 0.05930473	0.5167 0.5668	0.05784181 0.06142697	0.5204 0.5790
CI HmestTn	0.08882558 0.12472424	2.257 2.938	0.0914879 0.1334659	3.884 5.512	0.09309415 0.13790347	7.077 9.150

CI Mean T_n = the confidence interval for T_n on the Mean, CI Med T_n = Confidence Interval for T_n on the median, CI 10% tmnT_n = confidence interval for T_n on the 10% trimmed, CI 20% tmnT_n = confidence interval for T_n on 20% trimmed, CI HmestT_n = confidence interval for T_n on the Huber M-estimator