Mathematical Analysis of the Effect of Control on the Dynamics of Diabetes Mellitus and Its Complications

Patrick Olabanji Aye

Department of Mathematical Sciences, Adekunle Ajasin University, Akungba-Akoko,
Ondo State, Nigeria; e-mail: patrick.aye@aaua.edu.ng; ayepatricko@gmail.com

Abstract

Diabetes is a disorder in which the body becomes unable to control the amount of sugar in the blood. The pancreas (beta-cells) is not functioning normally, resulting in a partial or total lack of insulin which is the key to the mechanism converting sugar to energy. In this study, mathematical model for the dynamics of diabetes mellitus and its complications incorporating control is developed and analyzed. Positive lifestyle, which includes abstinence from alcohol, smoking and glutoning, and effective management of diabetes condition are incorporated as controls. The analytical solution of the model equations is obtained using Homotopy Perturbation Method. Numerical simulation of the model solution was done using Maple 18 Mathematical software. The parameters are varied and their effects on the model dynamics are presented graphically. The results showed that the two control measures can effectively be used to reduce the evolution of incidence of diabetes and occurrence of complications of diabetes thereby reducing the rate of morbidity and mortality due to diabetes complications.

1. Introduction

Diabetes is a metabolic disorder characterized by the inability of the body system to regulate the amount of glucose (special sugar in the body) in the body. This occurs when the insulin secretes by the organ (pancreas) in the beta cell is not sufficient or the body does not utilise the insulin produced effectively. The resulting effect is that the body system cannot function effectively and glucose level in the blood goes beyond normal [37], [36], [34], and [24]. The menace of diabetes and it effect on human, economy and social impact have become a serious problem to the world. International Diabetes

Federation [27] and World Health Organization [36], reported that 2.9 - 8.4 million deaths worldwide are attributable to diabetes. An undiagnosed or uncontrolled diabetes leads to complications and complications lead to deaths. Rising cases of cardiovascular disease, renal failure, blindness, cognitive and psychiatric illness and infections are attributable to diabetes. Diabetes causes loss of sensory and motor function, uneven blood circulation in the feet and hand, and poor wound healing.

Diabetes coexists with infections. This can be seen in diabetes and tuberculosis and group B Streptococci [11], [28]. The relationship between diabetes and tuberculosis slow down recovery rate and increase risk of tuberculosis relapse, resistance and death [28]. Diabetes also coexists with Human Immunodeficiency Virus (HIV) thereby causing early death [4]. The number of deaths attributable to diabetes yearly worldwide is alarming. Recently, there has been an increase in life expectancy around the world. Death rate caused by other leading non-communicable diseases such as Cancer, Cardiovascular disease and Stroke have been decreasing but that of diabetes is rising [5]. According to WHO, 2.8 percent of all deaths worldwide in 2010 are attributable to diabetes excluding deaths caused by cardiovascular disease and chronic kidney disease. 21 percent of coronary heart disease and 13 percent of stroke mortality worldwide are caused by diabetes [9]. Also, International Diabetes Federation revealed that 5 million adult deaths yearly, or 8.4 percent of all deaths worldwide are attributable to diabetes [26], [32].

This paper expanded the scope of the dynamics of diabetes mellitus by adding three classes and incorporates control measures. The classes added are healthy class, susceptible class and diabetics with complications undergoing treatment class while the control measures are positive lifestyle and effective management of diabetes conditions. In section two, we described the proposed model and its parameters and obtained the analytical solution of the model equations. In section three, we carried out numerical simulation of the results obtained in section two and presents the graphical profile of the system responses. In section four, we discussed our results.

1.1. Homotopy perturbation method (HPM)

The method of Homotopy perturbation for solving problems of nonlinear equation was first introduced by He [16], [17] and subsequently, it was improved by [18], [19], [20], [21] and [22]. The method was successfully applied to solve problems made up of nonlinear and linear functional equations. In recent years, scientists, technologists and engineers have applied homotopy perturbation method to deform problem of nonlinear and linear differential equations that seem to be difficult into simpler one that is easier to

solve. The method is a combination of homotopy method in topology and traditional perturbation method [23]. The method of homotopy perturbation have provides a very convenient approach for obtaining analytical or approximate solution of series of problems in various field.

Many researchers have successfully applied the method to physical problems such as non-linear wave equations, bifurcation, asymptotology, SIR infection disease model [1], equation of Schrodinger [6], diabetes model [15], heat radiation equation and reaction-diffusion equation [13], [14], oscillators with discontinuities [20], [21], Sine-Gordon and Klein-Gordon equations [29], [8].

1.2. Fundamental of homotopy perturbation method

The basic illustration of the method according to [17], using non-linear differential equation is stated as follows:

$$D(u) - k(x) = 0, \qquad x \in \Lambda \tag{1.1}$$

with boundary condition of:

$$E\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad x \in \Pi. \tag{1.2}$$

D denotes general differential operator, the boundary operator is E, the known analytical function is $k(x)$ and Π denotes domain boundary of Λ. The operator D can be divided into two parts of G and H, the linear part is G, while H is a non-linear one. Equation (1.1) can be rewritten as follows:

$$G(u) + H(u) - k(x) = 0, \qquad x \in \Lambda. \tag{1.3}$$

The stated HPM equation is as follows:

$$Q(w, r) = (1 - r)[G(w) - G(u_0)] + r[G(w) + H(w) - k(x)] = 0. \tag{1.4}$$

The embedding parameter is $r \in [0, 1]$ in equation (1.4) and the first assumption that meets the boundary condition is μ_0. The assumed solution of equation (1.4) can be expressed in power series as follows:

$$w = w_0 + rw_1 + r^2 w_2 + \cdots. \tag{1.5}$$

The correct solution of (1.1) is obtained by setting $r = 1$

$$u = \lim_{p \to 1} w = w_0 + w_1 + w_2 + \cdots. \tag{1.6}$$

In most cases, the series (1.5) converges. The convergent rate depends on the nonlinear operator $D(w)$ [17].

2. Materials and Methods

2.1. Model formulation

The study considered the model proposed by [15] with some modifications and incorporate two control measures. The control measures are positive lifestyle and effective management of diabetes conditions. Based on their health status, the model population are classified into five classes. They are healthy class $H(t)$, susceptible class $S(t)$, diabetic without complications class $D(t)$, diabetic with complications class $C(t)$, and diabetic with complications that undergo treatment class $T(t)$. The assumption of the model was that the healthy individual will give birth to a healthy children that will be born into healthy compartment while parent who is diabetic or have history of diabetes will give birth to children with genetic factors that will be born into susceptible compartment. The proportion of children born into healthy compartment is denoted by θ while proportion of children that are born into susceptible compartment is denoted by $1 - \theta$. The control parameters are ϕ_1, ϕ_2. ϕ_1 is a measure of positive lifestyle in the susceptible class, such that $0 \le \phi_1 \le 1$. $\phi_1 = 0$ indicate negative lifestyle and $\phi_1 = 1$ indicate positive lifestyle. ϕ_2 is a measure of effective management of diabetes condition in the compartment of diabetics without complications, such that $0 \le \phi_2 \le 1$. $\phi_2 = 0$ indicate ineffective management of diabetes condition and $\phi_2 = 1$ indicate effective management of diabetes condition.

Table 2.1. Definition of variables of the model.

S/N	Variables	Description
1	$H(t)$	Healthy class
2	$S(t)$	Susceptible class
3	$D(t)$	Diabetics without complications class
4	$C(t)$	Diabetics with complications class
5	$T(t)$	Diabetics with complications undergoing treatment class
6	$N(t)$	Total population

Table 2.2. Definition of parameters of the model.

| S/N	Parameters	Description
1	α	Probability rate of incidence of diabetes
2	β	Birth rate
3	μ	Natural mortality rate
4	τ	Rate at which healthy individual become susceptible
5	σ	Rate at which susceptible individual become healthy
6	λ	Rate at which $D(t)$ develop a complications
7	γ	Rate at which $C(t)$ are treated
8	ω	Rate at which $C(t)$ after treatment return to $D(t)$
9	δ	Mortality rate due to complications
10	θ	Proportion of children born into the healthy class
11	ϕ_1	Measure of positive lifestyle in $S(t)$ class
12	ϕ_2	Measure of effective management of diabetes condition in $D(t)$ class
13	$1-\theta$	Proportion of children born into the susceptible class
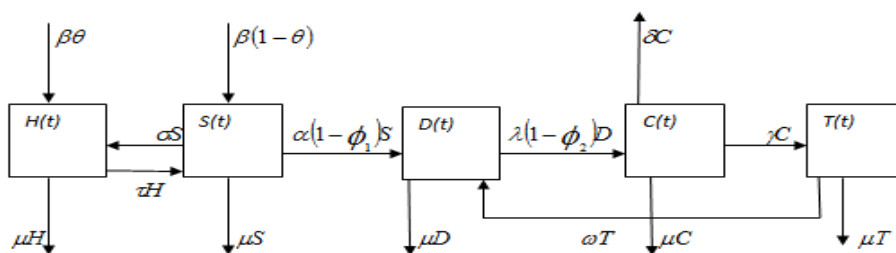
Figure 2.1. Schematic diagram of the model.

2.2. The model equations

The model equations are stated as follows in (2.1) to (2.5)

$$\frac{dH(t)}{dt} = \sigma S(t) - \mu H(t) - \tau H(t) + \beta\theta, \tag{2.1}$$

$$\frac{dS(t)}{dt} = \beta(1 - \theta) - \mu S(t) + \tau H(t) - \alpha(1 - \phi_1)S(t) - \sigma S(t), \tag{2.2}$$

$$\frac{dD(t)}{dt} = \alpha(1 - \phi_1)S(t) + \omega T(t) - \lambda(1 - \phi_2)D(t) - \mu D(t), \tag{2.3}$$

$$\frac{dC(t)}{dt} = \lambda(1 - \phi_2)D(t) - \gamma C(t) - \delta C(t) - \mu C(t), \tag{2.4}$$

$$\frac{dT(t)}{dt} = \gamma C(t) - \omega T(t) - \mu T(t). \tag{2.5}$$

The initial values conditions are $H(o) = H_o,\ \ S(o) = S_o,\ \ D(o) = D_o,\ \ C(o) = C_o$ and $T(o) = T_o.$

2.3. Analytical solution of the model equations by homotopy perturbation method (HPM)

Consider the systems:

$$\frac{dH}{dt} = \beta\theta - (\mu + \tau)H + \sigma S, \tag{2.6}$$

$$\frac{dS}{dt} = \beta(1 - \theta) + \tau H - \mu S - \sigma S - \alpha(1 - \phi_1)S, \tag{2.7}$$

$$\frac{dD}{dt} = \alpha(1 - \phi_1)S - \mu D - \lambda(1 - \phi_2)D + \omega T, \tag{2.8}$$

$$\frac{dC}{dt} = \lambda(1 - \phi_2)D - (\mu + \delta + \gamma)C \tag{2.9}$$

$$\frac{dT}{dt} = \gamma C - (\mu + \omega)T. \tag{2.10}$$

Equations (2.1) to (2.5) can be written as

$$\frac{dH}{dt} + Q_1H - \sigma S - \beta\theta = 0, \tag{2.11}$$

$$\frac{dS}{dt} - \tau H + Q_2S - \beta(1 - \theta) = 0, \tag{2.12}$$

$$\frac{dD}{dt} - \alpha(1 - \phi_1)S + Q_3 D - \omega T = 0, \tag{2.13}$$

$$\frac{dC}{dt} - \lambda(1 - \phi_2)D + Q_4 C = 0, \tag{2.14}$$

$$\frac{dT}{dt} - \gamma C + Q_5 T = 0, \tag{2.15}$$

where

$$Q_1 = \mu + \tau, \tag{2.16}$$

$$Q_2 = \mu + \sigma + \alpha - \alpha\phi_1, \tag{2.17}$$

$$Q_3 = \mu + \lambda - \lambda\phi_2, \tag{2.18}$$

$$Q_4 = \mu + \delta + \gamma, \tag{2.19}$$

$$Q_5 = \mu + \omega, \tag{2.20}$$

with initial conditions $H(0) = H_0,\ S(0) = S_0,\ D(0) = D_0,\ C(0) = C_0$ and $T(0) = T_0$.

Let

$$H(t) = b_0 + pb_1 + p^2 b_2 + \cdots \tag{2.21}$$

$$S(t) = e_0 + pe_1 + p^2 e_2 + \cdots \tag{2.22}$$

$$D(t) = f_0 + pf_1 + p^2 f_2 + \cdots \tag{2.23}$$

$$C(t) = g_0 + pg_1 + p^2 g_2 + \cdots \tag{2.24}$$

$$T(t) = j_0 + pj_1 + p^2 j_2 + \cdots. \tag{2.25}$$

Applying HPM to (2.11) gives

$$(1 - p)\frac{dH}{dt} + p\left[\frac{dH}{dt} + Q_1 H - \sigma S - \beta\theta\right] = 0. \tag{2.26a}$$

Substituting (2.21) to (2.25) into (2.26a) and simplify gives

$$(1-p)[b_0' + pb_1' + p^2 m_2' + \cdots] + p\begin{bmatrix} b_0' + pb_1' + p^2 b_2' + \cdots \\ + Q_1(b_0 + pb_1 + p^2 b_2 + \cdots) \\ - \sigma(e_0 + pe_1 + p^2 e_2 + \cdots) - \beta\theta \end{bmatrix} = 0,$$

$$b_0' + pb_1' + p^2 b_2' + \cdots + pQ_1(b_0 + pb_1 + p^2 b_2 + \cdots)$$

$$- p\sigma(e_0 + pe_1 + p^2 e_2 + \cdots) - p\beta\theta = 0. \tag{2.26b}$$

Collecting the coefficients of power of p in (2.26b) gives

$$\left. \begin{array}{l} p^0 : b_0' = 0 \\ p^1 : b_1' + Q_1 b_0 - \sigma e_0 - \beta\theta = 0 \\ p^2 : b_2' + Q_1 b_1 - \sigma e_1 = 0 \end{array} \right\}. \tag{2.27}$$

Applying HPM to (2.12) gives

$$(1-p)\frac{dS}{dt} + p\left[\frac{dS}{dt} - \tau H + Q_2 S + \beta\theta - \beta\right] = 0. \tag{2.28}$$

Substituting (2.21) to (2.25) into (2.28) and simplify gives

$$(1-p)[e_0' + pe_1' + p^2 e_2' + \cdots] + p\begin{bmatrix} e_0' + pe_1' + p^2 e_2' + \cdots \\ - \tau(b_0 + pb_1 + p^2 b_2 + \cdots) \\ + Q_2(e_0 + pe_1 + p^2 e_2 + \cdots) \\ + \beta\theta - \beta \end{bmatrix} = 0,$$

$$e_0' + pe_1' + p^2 e_2' + \cdots - p\tau(b_0 + pb_1 + p^2 b_2 + \cdots)$$

$$+ pQ_2(e_0 + pe_1 + p^2 e_2 + \cdots) + p\beta\theta - p\beta = 0. \tag{2.29}$$

Collecting the coefficients of powers of p in (2.29) gives

$$\left. \begin{array}{l} p^0 : e_0' = 0 \\ p^1 : e_1' - \tau b_0 + Q_2 e_0 + \beta\theta - \beta = 0 \\ p^2 : e_2' - \tau b_1 + Q_2 e_1 = 0 \end{array} \right\}. \tag{2.30}$$

Applying HPM to (2.13) gives

$$(1-p)\frac{dD}{dt} + p\left[\frac{dD}{dt} - \alpha S + \alpha\phi_1 S + Q_3 D - \omega T\right] = 0. \tag{2.31}$$

Substituting (2.21) to (2.25) into (2.31) and simplify gives

$$(1-p)[f_0' + pf_1' + p^2 f_2' + \cdots] + p \begin{bmatrix} f_0' + pf_1' + p^2 f_2' + \cdots \\ -\alpha(e_0 + pe_1 + p^2 e_2 + \cdots) \\ +\alpha\phi_1(e_0 + pe_1 + p^2 e_2 + \cdots) \\ +Q_3(f_0 + pf_1 + p^2 f_2 + \cdots) \\ -\omega(j_0 + pj_1 + p^2 j_2 + \cdots) \end{bmatrix} = 0,$$

$$f_0' + pf_1' + p^2 f_2' + \cdots - p\alpha(e_0 + pe_1 + p^2 e_2 + \cdots) + p\alpha\phi_1(e_0 + pe_1 + p^2 e_2 + \cdots)$$

$$+ pQ_3(f_0 + pf_1 + p^2 f_2 + \cdots) - p\omega(j_0 + pj_1 + p^2 j_2 + \cdots) = 0. \tag{2.32}$$

Collecting the coefficients of the powers of p in (2.32) gives

$$\left.\begin{aligned} p^0 &: f_0' = 0 \\ p^1 &: f_1' - \alpha e_0 + \alpha\phi_1 e_0 + Q_3 f_0 - \omega j_0 = 0 \\ p^2 &: f_2' - \alpha e_1 + \alpha\phi_1 e_1 + Q_3 f_1 - \omega j_1 = 0 \end{aligned}\right\}. \tag{2.33}$$

Applying HPM to (2.14) gives

$$(1-p)\frac{dC}{dt} + p\left[\frac{dC}{dt} - \lambda D + \lambda\phi_2 D + Q_4 C\right] = 0. \tag{2.34}$$

Substituting (2.21) to (2.25) into (2.34) and simplify gives

$$(1-p)[g_0' + pg_1' + p^2 g_2' + \cdots] + p \begin{bmatrix} g_0' + pg_1' + p^2 g_2' + \cdots \\ -\lambda(f_0 + pf_1 + p^2 f_2 + \cdots) \\ +\lambda\phi_2(f_0 + pf_1 + p^2 f_2 + \cdots) \\ +Q_4(g_0 + pg_1 + p^2 g_2 + \cdots) \end{bmatrix} = 0,$$

$$g_0' + pg_1' + p^2 g_2' + \cdots - p\lambda(f_0 + pf_1 + p^2 f_2 + \cdots)$$

$$+ p\lambda\phi_2(f_0 + pf_1 + p^2 f_2 + \cdots) + pQ_4(g_0 + pg_1 + p^2 g_2 + \cdots) = 0. \tag{2.35}$$

Collecting the coefficients of the powers of p in (2.35) gives

$$\left.\begin{aligned} p^0 &: g_0' = 0 \\ p^1 &: g_1' - \lambda f_0 + \lambda\phi_2 f_0 + Q_4 g_0 = 0 \\ p^2 &: g_2' - \lambda f_1 + \lambda\phi_2 f_1 + Q_4 g_1 = 0 \end{aligned}\right\}. \tag{2.36}$$

Applying HPM to (2.15) gives

$$(1 - p)\frac{dT}{dt} + p\left[\frac{dT}{dt} - \gamma D + Q_3 T\right] = 0. \tag{2.37}$$

Substituting (2.21) to (2.25) into (2.37) and simplify gives

$$(1 - p)[j_0' + pj_1' + p^2 j_2' + \cdots] + p\left[\begin{array}{c} j_0' + pj_1' + p^2 j_2' + \cdots \\ -\gamma(g_0 + pg_1 + p^2 g_2 + \cdots) \\ + Q_5(j_0 + pj_1 + p^2 j_2 + \cdots) \end{array}\right] = 0,$$

$$j_0' + pj_1' + p^2 j_2' + \cdots - p\gamma(g_0 + pg_1 + p^2 g_2 + \cdots)$$

$$+ pk_5(j_0 + pj_1 + p^2 j_2 + \cdots) = 0. \tag{2.38}$$

Collecting the coefficients of powers of p in (2.38) gives

$$\left.\begin{array}{l} p^0 : j_0' = 0 \\ p^1 : j_1' - \gamma g_0 + Q_5 j_0 = 0 \\ p^2 : j_2' - \gamma g_1 + Q_5 j_1 = 0 \end{array}\right\}. \tag{2.39}$$

Solving the first equations of (2.27), (2.30), (2.33), (2.36) and (2.39) gives

$$\left.\begin{array}{l} b_0 = H_0 \\ e_0 = S_0 \\ f_0 = D_0 \\ g_0 = C_0 \\ j_0 = T_0 \end{array}\right\}. \tag{2.40}$$

Substituting (2.40) into second equations of (2.27), (2.30), (2.33), (2.36) and (2.39) gives

$$\left.\begin{array}{l} b_1' + Q_1 H_0 - \sigma S_0 - \beta\theta = 0 \\ e_1' - \tau H_0 + Q_2 S_0 + \beta\theta - \beta = 0 \\ f_1' - \alpha S_0 + \alpha\phi_1 S_0 + Q_3 D_0 - \omega T_0 = 0 \\ g_1' - \lambda D_0 + \lambda\phi_2 D_0 + Q_4 C_0 = 0 \\ j_1 - \gamma C_0 + Q_3 T_0 = 0 \end{array}\right\}. \tag{2.41}$$

Solving (2.41) gives

$$b_1 = (\beta\theta - Q_1 H_0 + \sigma S_0)t$$
$$e_1 = (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t$$
$$\left. f_1 = (\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0)t \right\} . \tag{2.42}$$
$$g_1 = (\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0)t$$
$$j_1 = (\gamma C_0 - Q_3 T_0)t$$

Substituting (2.42) into the third equations of (2.27), (2.30), (2.33), (2.36) and (2.39) gives

$$\int b_2' dt = -Q_1 \int (\beta\theta - Q_1 H_0 + \sigma S_0)t\,dt + \sigma \int (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t\,dt$$

$$b_2 = [\sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) - Q_1(\beta\theta - Q_1 H_0 + \sigma S_0)]\frac{t^2}{2} + \cdots \tag{2.43}$$

$$\int e_2' dt = \tau \int (\beta\theta - Q_1 H_0 + \sigma S_0)t\,dt - Q_2 \int (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t\,dt$$

$$e_2 = [\tau(\beta\theta - Q_1 H_0 + \sigma S_0) - Q_2(\tau H_0 - Q_2 S_0 - \beta\theta + \beta)]\frac{t^2}{2} + \cdots \tag{2.44}$$

$$\int f_2' dt = \alpha \int (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t\,dt - Q_3 \int (\alpha S_0 - Q_3 D_0 + \omega T_0)t\,dt$$

$$+ \omega \int (\gamma C_0 - Q_5 T_0)t\,dt$$

$$f_2 = \begin{bmatrix} \sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) - \alpha\phi_1(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_3(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) + \omega(\gamma C_0 - Q_5 T_0) \end{bmatrix}\frac{t^2}{2} + \cdots \tag{2.45}$$

$$\int g_2' dt = \lambda \int (\alpha S_0 - Q_3 D_0 + \omega T_0)t\,dt - \lambda\phi \int (\alpha S_0 - Q_3 D_0 + \omega T_0)t\,dt$$

$$- Q_4 \int (\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0)t\,dt$$

$$g_2 = \begin{bmatrix} \lambda(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - \lambda\phi_2(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - Q_4(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \end{bmatrix}\frac{t^2}{2} + \cdots \tag{2.46}$$

$$\int j_2' dt = \gamma \int (\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0)t\,dt - Q_5 \int (\gamma C_0 - Q_5 T_0)t\,dt$$

$$j_2 = [\gamma(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) - Q_5(\gamma C_0 - Q_5 T_0)]\frac{t^2}{2} + \cdots \tag{2.47}$$

$$
\left.\begin{aligned}
b_2 &= [\sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) - Q_1(\beta\theta - Q_1 H_0 + \sigma S_0)]\frac{t^2}{2} + \cdots \\
e_2 &= [\tau(\beta\theta - Q_1 H_0 + \sigma S_0) - Q_2(\tau H_0 - Q_2 S_0 - \beta\theta + \beta)]\frac{t^2}{2} + \cdots \\
f_2 &= \begin{bmatrix} \sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) - \alpha\phi_1(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_3(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) + \omega(\gamma C_0 - Q_5 T_0) \end{bmatrix}\frac{t^2}{2} + \cdots \\
g_2 &= \begin{bmatrix} \lambda(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - \lambda\phi_2(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - Q_4(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \end{bmatrix}\frac{t^2}{2} + \cdots \\
j_2 &= [\gamma(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) - Q_5(\gamma C_0 - Q_5 T_0)]\frac{t^2}{2} + \cdots
\end{aligned}\right\}. \tag{2.48}
$$

Substituting (2.40), (2.42) and (2.48) into (2.21) gives

$$
H(t) = H_0 + p[(\beta\theta - Q_1 H_0 + \sigma S_0)t] + p^2 \begin{bmatrix} \sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_1(\beta\theta - Q_1 H_0 + \sigma S_0) \end{bmatrix}\frac{t^2}{2} + \cdots. \tag{2.49}
$$

Setting $p = 1$ of (2.49) becomes

$$
H(t) = \lim_{p \to 1} b = b_0 + b_1 + b_2. \tag{2.50}
$$

Hence,

$$
H(t) = H_0 + (\beta\theta - Q_1 H_0 + \sigma S_0)t + \begin{bmatrix} \sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_1(\beta\theta - Q_1 H_0 + \sigma S_0) \end{bmatrix}\frac{t^2}{2} + \cdots. \tag{2.51}
$$

Substituting (2.40), (2.42) and (2.48) into (2.22) gives

$$
S(t) = S_0 + p[(\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t] + p^2 \begin{bmatrix} \tau(\beta\theta - Q_1 H_0 + \sigma S_0) \\ - Q_2(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \end{bmatrix}\frac{t^2}{2} + \cdots. \tag{2.52}
$$

Setting $p = 1$ of (2.52) becomes

$$
S(t) = \lim_{p \to 1} e = e_0 + e_1 + e_2. \tag{2.53}
$$

Hence,

$$
S(t) = S_0 + (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t + \begin{bmatrix} \tau(\beta\theta - Q_1 H_0 + \sigma S_0) \\ - Q_2(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \end{bmatrix}\frac{t^2}{2} + \cdots. \tag{2.54}
$$

Substituting (2.40), (2.42) and (2.48) into (2.23) gives

$$D(t) = D_0 + p[(\alpha S_0 - \alpha\phi_1 S_0 - D_3 D_0 + \omega T_0)t]$$

$$+ p^2 \begin{bmatrix} \alpha(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - \alpha\phi_1(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_3(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ + \omega(\gamma C_0 - Q_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.55)$$

Setting $p = 1$ of (2.55) becomes

$$D(t) = \lim_{p \to 1} f = f_0 + f_1 + f_2. \quad (2.56)$$

Hence,

$$D(t) = D_0 + (\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0)t$$

$$+ \begin{bmatrix} \alpha(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - \alpha\phi_1(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_3(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ + \omega(\gamma C_0 - Q_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.57)$$

Substituting (2.40), (2.42) and (2.48) into (2.24) gives

$$C(t) = C_0 + p[(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0)t]$$

$$+ p^2 \begin{bmatrix} \lambda(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - \lambda\phi_2(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - Q_4(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.58)$$

Setting $p = 1$ of (2.58) becomes

$$C(t) = \lim_{p \to 1} g = g_0 + g_1 + g_2. \quad (2.59)$$

Hence,

$$C(t) = C_0 + (\lambda D_0 - \lambda\phi_2 D_0 - k_4 C_0)t$$

$$+ \begin{bmatrix} \lambda(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - \lambda\phi_2(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - Q_4(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.60)$$

Substituting (2.40), (2.42) and (2.48) into (2.25) gives

$$T(t) = T_0 + p[(\gamma C_0 - Q_5 T_0)t] + p^2 \begin{bmatrix} \gamma(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \\ - Q_5(\gamma C_0 - Q_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.61)$$

Setting $p = 1$ of (2.61) becomes

$$T(t) = \lim_{p \to 1} j = j_0 + j_1 + j_2. \quad (2.62)$$

Hence,

$$T(t) = T_0 + (\gamma C_0 - Q_5 T_0)t + \begin{bmatrix} \gamma(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \\ - Q_5(\gamma C_0 - k_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots. \quad (2.63)$$

Equations (2.51), (2.54), (2.57), (2.60) and (2.63) are the solutions of the system of equations (2.1) to (2.5).

$$\left. \begin{aligned}
H(t) &= H_0 + (\beta\theta - Q_1 H_0 + \sigma S_0)t + \begin{bmatrix} \sigma(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_1(\beta\theta - Q_1 H_0 + \sigma S_0) \end{bmatrix} \frac{t^2}{2} + \cdots \\
S(t) &= S_0 + (\tau H_0 - Q_2 S_0 - \beta\theta + \beta)t + \begin{bmatrix} \tau(\beta\theta - Q_1 H_0 + \sigma S_0) \\ - Q_2(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \end{bmatrix} \frac{t^2}{2} + \cdots \\
D(t) &= D_0 + (\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0)t \\
&\quad + \begin{bmatrix} \alpha(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - \alpha\phi_1(\tau H_0 - Q_2 S_0 - \beta\theta + \beta) \\ - Q_3(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ + \omega(\gamma C_0 - Q_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots \\
C(t) &= C_0 + (\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0)t \\
&\quad + \begin{bmatrix} \lambda(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - \lambda\phi_2(\alpha S_0 - \alpha\phi_1 S_0 - Q_3 D_0 + \omega T_0) \\ - Q_4(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \end{bmatrix} \frac{t^2}{2} + \cdots \\
T(t) &= T_0 + (\gamma C_0 - Q_5 T_0)t + \begin{bmatrix} \gamma(\lambda D_0 - \lambda\phi_2 D_0 - Q_4 C_0) \\ - Q_5(\gamma C_0 - Q_5 T_0) \end{bmatrix} \frac{t^2}{2} + \cdots
\end{aligned} \right\}.$$

$$(2.64)$$

2.4. Variables and estimation of parameter values

Variables and parameter values were estimated based on the available data from [33], [25], [27], [38], [12], [3] and [2].

Table 2.3. Values of variables used for numerical simulation.

Variables	Values
$H(0)$	198,195,839
$S(0)$	101,535,728
$D(0)$	940,000
$C(0)$	3,760,000
$T(0)$	1,193,250

Table 2.4. Values of parameters used for numerical simulation.

Parameters	Values	Source
α	0.02	[27]
β	0.038	[33]
γ	0.08	[10]
λ	0.05	[30]
μ	0.118	[33]
σ	0.08	[7]
θ	0.923	[31]
ϕ_1	0.5	Assumed
ϕ_2	0.5	Assumed
ω	0.08	[30]
δ	0.02	[7]
τ	0.04	[30]
$1 - \theta$	0.077	[31]

3. Numerical Simulation and Result

Numerical simulation of the results obtained in Section 2.3 was carried out using mathematical software (MAPLE 18) and the graphical profiles of the system responses are presented below.
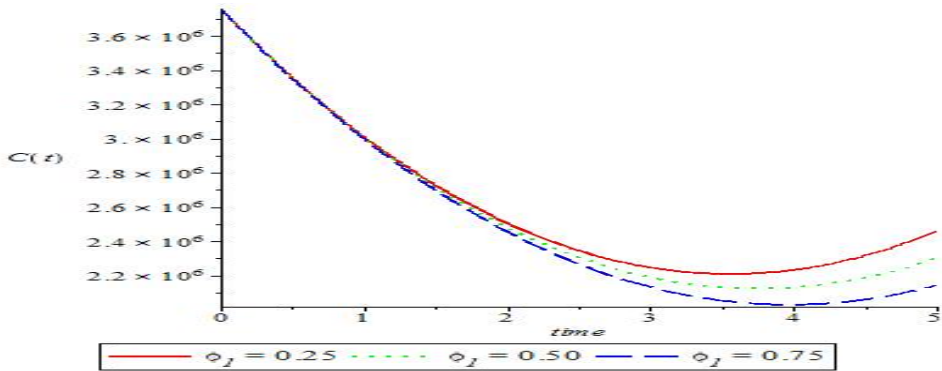

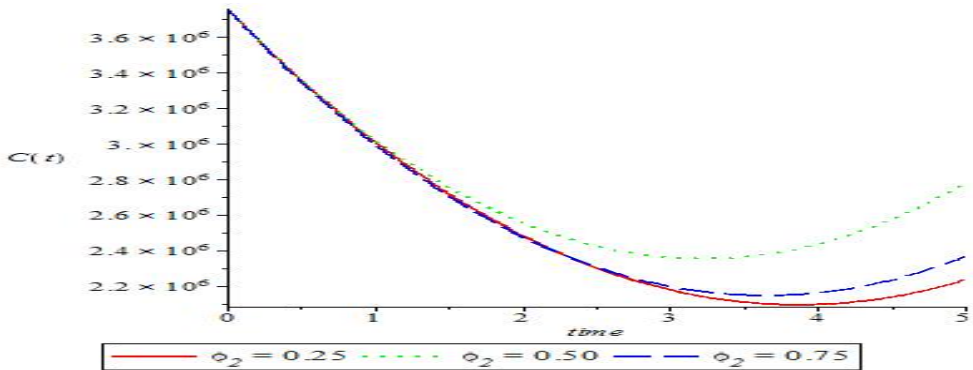**Figure 3.1.** Effect of control measure ϕ_1 on diabetics with complications class.

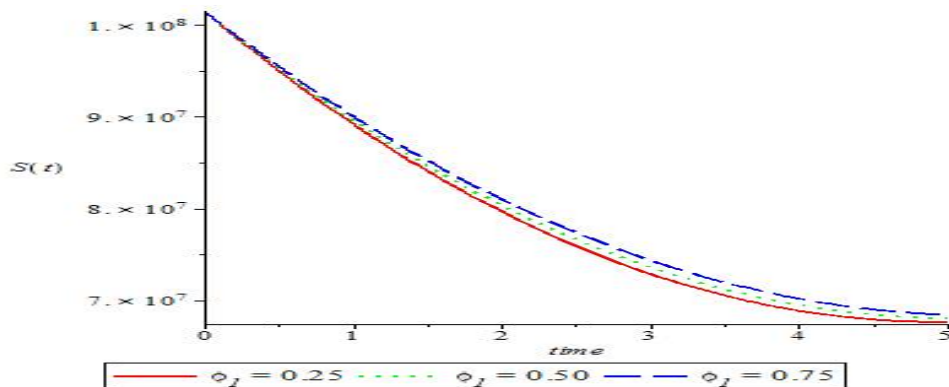**Figure 3.2.** Effect of control measure ϕ_2 on diabetics with complications class.

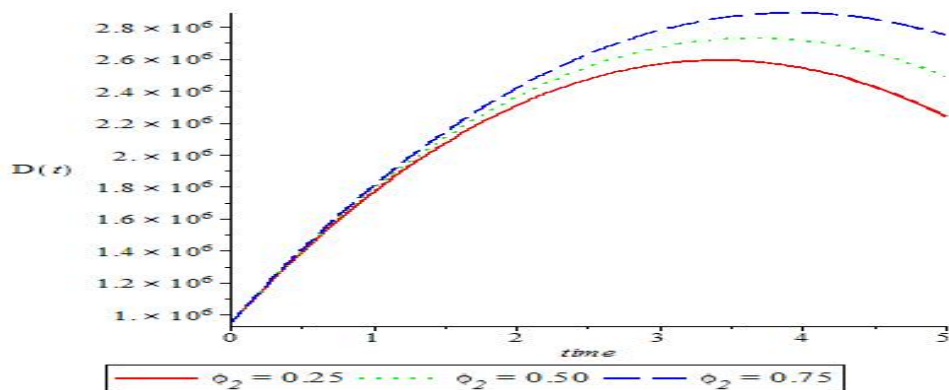**Figure 3.3.** Effect of control measure ϕ_1 on susceptible class.

Figure 3.4. Effect of control measure ϕ_2 on diabetics without complications class.

4. Analysis of the Result

Figure 3.1 shows that, as control rate increases, diabetics with complications decreases faster. This shows that the more people adopt positive lifestyle such as good dieting and regular physical exercises, the less cases of diabetics with complications. It is observed that as the control rate increases, the diabetics with complications is almost zero. However, effort to increase control measure will go a long way to reduce complications arising from diabetes. Figure 3.2 shows that, as control rate increases, diabetics with complications decreases. This shows that high rate of effective management of diabetes condition results to less cases of diabetics with complications.

Figure 3.3 shows that, as control rate increases, susceptible class increases. This can be attributed to high rate of positive lifestyle. It pointed to the fact that the more people adopt positive lifestyle such as good dieting and regular physical exercises, the less they are susceptible to diabetes while Figure 3.4 shows that as control rate increases, diabetics without complications class increases. This is attributed to high rate of effective management of diabetes condition which helps in controlling the transition from diabetics without complications state to diabetics with complications state. It shows that the more effective management of diabetes condition, the less cases of complications.

5. Conclusion

The study presented a modified deterministic model for controlling the incidence of diabetes and its complications in a population. Two control parameters were imposed on the model equations and the effect of the control measures on the dynamics of diabetes are analysed. The model equations were solved analytically using homotopy perturbation method and solutions were obtained. The result showed that if control parameters rate could be increased, the transition rate from susceptible class to diabetics without complications class and transition rate from diabetics without complications to diabetics with complications would be drastically reduced. This will significantly reduce the incidence of diabetes and occurrence of complications in a population and the number of deaths attributable to diabetes and its complications would be minimised.

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