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Degree Subtraction Adjacency Polynomial and Energy of Graphs obtained from Complete Graph

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Abstract

The degree subtraction adjacency matrix of a graph G is a square matrix $DSA(G) = [d_{ij}]$, in which $d_{ij} = d(v_i) - d(v_j)$, if the vertices v_i and v_j are adjacent and $d_{ij} = 0$, otherwise, where $d(u)$ is the degree of a vertex u. The DSA energy of a graph is the sum of the absolute values of the eigenvalues of DSA matrix. In this paper, we obtain the characteristic polynomial of the DSA matrix of graphs obtained from the complete graph. Further we study the DSA energy of these graphs.

1. Introduction

Let G be a simple, undirected graph with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E(G) = \{e_1, e_2, ..., e_m\}$. The degree of a vertex v_i denoted by $d(v_i)$ is the number of edges incident to it. If all vertices have same degree equal to r, then G is called an *r-regular graph.*

The matrices of graphs based on sum of degrees and subtraction of degrees are studied in [5, 6, 9, 10, 11]. In [8] the degree subtraction adjacency (DSA) matrix is

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defined as $DSA(G) = [d_{ij}]$, where

$$d_{ij} = \begin{cases} d(v_i) - d(v_j), & \text{if } v_i \text{ is adjacent to } v_j \\ 0, & \text{otherwise.} \end{cases}$$

The characteristic polynomial of $DSA(G)$ is called the *DSA-polynomial* and is denoted by $\psi(G : \xi)$. Thus $\psi(G : \xi) = \det(\xi I - DSA(G))$, where I is an identity matrix of order n.

For any regular graph of order n, $\psi(G : \xi) = \xi^n$.

The eigenvalues of $DSA(G)$, denoted by $\xi_1, \xi_2, ..., \xi_n$ are called *DSA-eigenvalues* of G. Since $DSA(G)$ is a skew-symmetric matrix, its eigenvalues are purely imaginary or zero.

The *DSA-energy* of a graph G is defined as [8]

$$DSAE(G) = \sum_{i=1}^{n} |\xi_i|. \tag{1}$$

The Eq. (1) is analogous to the ordinary energy of a graph defined as [2]

$$E_A(G) = \sum_{i=1}^{n} |\lambda_i|,$$

where $\lambda_1, \lambda_2, ..., \lambda_n$ are the eigenvalues of the adjacency matrix of G.

The floor function $\lfloor x \rfloor$ is the greatest integer less than or equal to x and the ceiling function $\lceil x \rceil$ is the least integer greater than or equal to x.

In [8] the DSA-polynomial and DSA-energy of a path, complete bipartite graph, wheel, windmill graph and corona graph have been obtained, where as in [7] the DSA polynomial and DSA energy of graphs obtained from regular graphs are reported. Gutman and Pavlović [3] studied the ordinary energy of graphs obtained from complete graph. We obtain the DSA-polynomial and DSA energy of graphs obtained from complete graph. Further we find extremum values of DSA-energy within the class of graphs.

2. DSA-polynomial of Graphs obtained from Complete Graph

Definition 2.1 [3]. Let e_i, $i = 1, 2, \ldots, k$, $1 \leq k \leq n-1$ be the distinct edges of a complete graph K_n, $n \geq 3$, all being incident to a single vertex. The graph $Ka_n(k)$ is obtained by deleting e_i, $i = 1, 2, \ldots, k$ from K_n. In addition $Ka_n(0) \cong K_n$.

Definition 2.2 [3]. Let f_i, $i = 1, 2, \ldots, k$, $1 \leq k \leq \lfloor n/2 \rfloor$ be independent edges of the complete graph K_n, $n \geq 3$. The graph $Kb_n(k)$ is obtained by deleting f_i, $i = 1, 2, \ldots, k$ from K_n. In addition $Kb_n(0) \cong K_n$.

Definition 2.3 [3]. Let V_k be a k-element subset of the vertex set of the complete graph K_n, $2 \leq k \leq n$, $n \geq 3$. The graph $Kc_n(k)$ is obtained by deleting from K_n all the edges connecting pairs of vertices from V_k. In addition $Kc_n(0) \cong Kc_n(1) \cong K_n$.

Definition 2.4 [3]. Let $3 \leq k \leq n$, $n \geq 3$. The graph $Kd_n(k)$ is obtained by deleting from the complete graph K_n, the edges belonging to a k-membered cycle.

We need following lemma.

Lemma 2.5 [1]. *If M is a non-singular matrix, then*

$$\begin{vmatrix} M & N \\ P & Q \end{vmatrix} = |M||Q - PM^{-1}N|.$$

Theorem 2.6. *For* $n \geq 2$, $0 \leq k \leq n-1$, *the DSA-polynomial of* $Ka_n(k)$ *is*

$$\psi(Ka_n(k), \xi) = \xi^{n-2}[\xi^2 + k(k+1)(n-k-1)]. \tag{2}$$

Proof. Let $V(K_n) = \{v_1, v_2, \ldots, v_n\}$. Without loss of generality, for $1 \leq k \leq n-1$, let $e_i = v_1 v_{i+1}$, $i = 1, 2, \ldots, k$ be the edges incident to v_1. Therefore DSA-polynomial of $Ka_n(k)$ is

$$\psi(Ka_n(k) : \xi) = |\xi I - DSA(Ka_n(k))|$$

$$= \begin{vmatrix} \xi & 0 & \cdots & 0 & k & k & \cdots & k \\ 0 & \xi & \cdots & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & 1 & 1 & \cdots & 1 \\ -k & -1 & \cdots & -1 & \xi & 0 & \cdots & 0 \\ -k & -1 & \cdots & -1 & 0 & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -k & -1 & \cdots & -1 & 0 & 0 & \cdots & \xi \end{vmatrix}. \tag{3}$$

Subtract $(k+1)$-th row from the rows $k+2, k+3, ..., n$ to get (4).

$$\begin{vmatrix} \xi & 0 & \cdots & 0 & k & k & \cdots & k \\ 0 & \xi & \cdots & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & 1 & 1 & \cdots & 1 \\ -k & -1 & \cdots & -1 & \xi & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & -\xi & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & -\xi & 0 & \cdots & \xi \end{vmatrix}. \tag{4}$$

Adding columns $k+2, k+3, ..., n$ to the column $k+1$ in (4) we get (5).

$$\begin{vmatrix} \xi & 0 & \cdots & 0 & (n-k-1)k & k & \cdots & k \\ 0 & \xi & \cdots & 0 & (n-k-1) & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & (n-k-1) & 1 & \cdots & 1 \\ -k & -1 & \cdots & -1 & \xi & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \xi \end{vmatrix}. \tag{5}$$

Equation (5) reduces to (6).

$$\xi^{n-k-2}\begin{vmatrix} \xi & 0 & \cdots & 0 & (n-k-1)k \\ 0 & \xi & \cdots & 0 & (n-k-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \xi & (n-k-1) \\ -k & -1 & \cdots & -1 & \xi \end{vmatrix}. \tag{6}$$

By Lemma 2.5, the Eq. (6) reduces to

$$\xi^{n-k-2}\xi^{k+1}\left| \xi I + \frac{I}{\xi}(k^2+k)(n-k-1) \right|_{1\times 1}$$

$$= \xi^{n-2}[\xi^2 + k(k+1)(n-k-1)]. \qquad \square$$

Theorem 2.7. *For* $n \geq 2,\ 0 \leq k \leq \lfloor n/2 \rfloor,$ *the DSA-polynomial of* $Kb_n(k)$ *is*

$$\psi(Kb_n(k):\xi) = \xi^{n-2}[\xi^2 + 2k(n-2k)]. \tag{7}$$

Proof. Let $V(K_n) = \{v_1, v_2, ..., v_n\}$. Without loss of generality, for $1 \leq k \leq \lfloor n/2 \rfloor$, let $e_i = v_{2i-1}v_{2i},\ i = 1, 2, ..., k$ be the independent edges of K_n. Therefore DSA-polynomial of $Kb_n(k)$ is

$$\psi(Kb_n(k):\xi) = |\,\xi I - DSA(Kb_n(k))\,|$$

$$= \begin{vmatrix} \xi & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 0 & \xi & \cdots & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & 1 & 1 & \cdots & 1 \\ -1 & -1 & \cdots & -1 & \xi & 0 & \cdots & 0 \\ -1 & -1 & \cdots & -1 & 0 & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & -1 & 0 & 0 & \cdots & \xi \end{vmatrix}$$

$$= \begin{vmatrix} \xi I_{2k\times 2k} & J_{2k\times(n-2k)} \\ -J_{(n-2k)\times 2k} & \xi I_{(n-2k)\times(n-2k)} \end{vmatrix}, \tag{8}$$

where *I* is an identity matrix and *J* is a matrix whose all entries are equal to 1. By Lemma 2.5, the Eq. (8) reduces to

$$\xi^{2k}\left|\xi I + \frac{2k}{\xi} J\right|_{(n-2k)\times(n-2k)}$$

$$= \xi^{4k-n}|\xi^2 I + 2kJ|$$

$$= \xi^{4k-n}\begin{vmatrix} \xi^2 + 2k & 2k & \cdots & 2k \\ 2k & \xi^2 + 2k & \cdots & 2k \\ \vdots & \vdots & \ddots & \vdots \\ 2k & 2k & \cdots & \xi^2 + 2k \end{vmatrix}. \quad (9)$$

Subtract first row from the rows $2, 3, ..., n - 2k$ to get (10).

$$\xi^{4k-n}\begin{vmatrix} \xi^2 + 2k & 2k & \cdots & 2k \\ -\xi^2 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\xi^2 & 0 & \cdots & \xi^2 \end{vmatrix}. \quad (10)$$

Adding columns $2, 3, ..., n - 2k$ to the first column in (10) we get

$$\xi^{4k-n}\begin{vmatrix} \xi^2 + 2k(n-2k) & 2k & \cdots & 2k \\ 0 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi^2 \end{vmatrix}$$

$$= \xi^{4k-n}[\xi^2 + 2k(n-2k)](\xi^2)^{n-2k-1}$$

$$= \xi^{n-2}[\xi^2 + 2k(n-2k)].$$

Theorem 2.8. *For $n \geq 3$, $0 \leq k \leq n$, the DSA-polynomial of $Kc_n(k)$ is*

$$\psi(Kc_n(k) : \xi) = \xi^{n-2}[\xi^2 + k(k-1)^2(n-k)]. \quad (11)$$

Proof. Let $V(K_n) = \{v_1, v_2, ..., v_n\}$. Without loss of generality, for $1 \leq k \leq n$, let $V_k = \{v_1, v_2, ..., v_k\}$. Therefore DSA-polynomial of $Kc_n(k)$ is

$$\psi(Kc_n(k) : \xi) = |\xi I - DSA(Kc_n(k))|$$

$$= \begin{vmatrix} \xi & 0 & \cdots & 0 & k-1 & \cdots & k-1 \\ 0 & \xi & \cdots & 0 & k-1 & \cdots & k-1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & k-1 & \cdots & k-1 \\ -(k-1) & -(k-1) & \cdots & -(k-1) & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -(k-1) & -(k-1) & \cdots & -(k-1) & 0 & \cdots & \xi \end{vmatrix}$$

$$= \begin{vmatrix} \xi I_{k\times k} & (k-1)J_{k\times(n-k)} \\ -(k-1)J_{(n-k)\times k} & \xi I_{(n-k)\times(n-k)} \end{vmatrix}, \tag{12}$$

where I is an identity matrix and J is a matrix whose all entries are equal to 1. By Lemma 2.5, Eq. (12) reduces to

$$\xi^k \left| \xi I + \frac{k(k-1)^2}{\xi} J \right|_{(n-k)\times(n-k)}$$

$$= \xi^{2k-n} |\xi^2 I + k(k-1)^2 J|$$

$$= \xi^{2k-n} \begin{vmatrix} \xi^2 + k(k-1)^2 & k(k-1)^2 & \cdots & k(k-1)^2 \\ k(k-1)^2 & \xi^2 + k(k-1)^2 & \cdots & k(k-1)^2 \\ \vdots & \vdots & \ddots & \vdots \\ k(k-1)^2 & k(k-1)^2 & \cdots & \xi^2 + k(k-1)^2 \end{vmatrix}. \tag{13}$$

Subtract first row from the rows $2, 3, ..., n-k$ to get (14).

$$\xi^{2k-n} \begin{vmatrix} \xi^2 + k(k-1)^2 & k(k-1)^2 & \cdots & k(k-1)^2 \\ -\xi^2 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\xi^2 & 0 & \cdots & \xi^2 \end{vmatrix}. \tag{14}$$

Adding columns $2, 3, ..., n-k$ to the first column in (14) we get

$$\xi^{2k-n} \begin{vmatrix} \xi^2 + k(k-1)^2(n-k) & k(k-1)^2 & \cdots & k(k-1)^2 \\ 0 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi^2 \end{vmatrix}$$

$$= \xi^{2k-n}[\xi^2 + k(k-1)^2(n-k)](\xi^2)^{n-k-1}$$

$$= \xi^{n-2}[\xi^2 + k(k-1)^2(n-k)].$$

Theorem 2.9. *For $n \geq 3$, $0 \leq k \leq n$, the DSA-polynomial of $Kd_n(k)$ is*

$$\psi(Kd_n(k) : \xi) = \xi^{n-2}[\xi^2 + 4k(n-k)]. \tag{15}$$

Proof. Let $V(K_n) = \{v_1, v_2, ..., v_n\}$. Without loss of generality, for $3 \leq k \leq n$, let $v_1, v_2, ..., v_k$ be the vertices of k-membered cycle of K_n. Therefore DSA-polynomial of $Kd_n(k)$ is

$$\psi(Kd_n(k) : \xi) = |\xi I - DSA(Kd_n(k))|$$

$$= \begin{vmatrix} \xi & 0 & \cdots & 0 & 2 & \cdots & 2 \\ 0 & \xi & \cdots & 0 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi & 2 & \cdots & 2 \\ -2 & -2 & \cdots & -2 & \xi & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ -2 & -2 & \cdots & -2 & 0 & \cdots & \xi \end{vmatrix}$$

$$= \begin{vmatrix} \xi I_{k\times k} & 2J_{k\times(n-k)} \\ -2J_{(n-k)\times k} & \xi I_{(n-k)\times(n-k)} \end{vmatrix}, \tag{16}$$

where I is an identity matrix and J is a matrix whose all entries are equal to 1. By Lemma 2.5, the Eq. (16) reduces to

$$\xi^k \left| \xi I + \frac{4k}{\xi} J \right|_{(n-k)\times(n-k)}$$

$$= \xi^{2k-n}|\xi^2 I + 4kJ|$$

$$= \xi^{2k-n} \begin{vmatrix} \xi^2 + 4k & 4k & \cdots & 4k \\ 4k & \xi^2 + 4k & \cdots & 4k \\ \vdots & \vdots & \ddots & \vdots \\ 4k & 4k & \cdots & \xi^2 + 4k \end{vmatrix}. \tag{17}$$

Subtract first row from the rows $2, 3, \ldots, n-k$ to get (18).

$$\xi^{2k-n}\begin{vmatrix} \xi^2+4k & 4k & \cdots & 4k \\ -\xi^2 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\xi^2 & 0 & \cdots & \xi^2 \end{vmatrix}. \tag{18}$$

Adding columns $2, 3, \ldots, n-k$ to the first column in (18) we get

$$\xi^{2k-n}\begin{vmatrix} \xi^2+4k(n-k) & 4k & \cdots & 4k \\ 0 & \xi^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \xi^2 \end{vmatrix}$$

$$= \xi^{2k-n}[\xi^2+4k(n-k)](\xi^2)^{n-k-1}$$

$$= \xi^{n-2}[\xi^2+4k(n-k)].$$

3. DSA-energy

By Eqs. (2), (7), (11) and (15) and by the definition of DSA energy via Eq. (1), we get following propositions.

Proposition 3.1. *For* $n \geq 2$ *and* $0 \leq k \leq n-1,$

$$DSAE(Ka_n(k)) = 2\sqrt{k(k+1)(n-k-1)}.$$

Proposition 3.2. *For* $n \geq 2$ *and* $0 \leq k \leq \lfloor n/2 \rfloor,$

$$DSAE(Kb_n(k)) = 2\sqrt{2k(n-2k)}.$$

Proposition 3.3. *For* $n \geq 3$ *and* $0 \leq k \leq n,$

$$DSAE(Kc_n(k)) = 2(k-1)\sqrt{k(n-k)}.$$

Proposition 3.4. *For* $n \geq 3$ *and* $0 \leq k \leq n,$

$$DSAE(Kd_n(k)) = 4\sqrt{k(n-k)}.$$

From Propositions 3.3 and 3.4 we have:

Corollary 3.5. *For* $n \geq 3$ *and* $0 \leq k \leq n$,

$$2(DSAE(Kc_n(k))) = (k-1)(DSAE(Kd_n(k))).$$

Corollary 3.6 follows from Propositions 3.2 and 3.4.

Corollary 3.6. *For* $n \geq 3$ *and* $0 \leq k \leq \lfloor n/2 \rfloor$,

$$DSAE(Kb_n(k)) < DSAE(Kd_n(k)).$$

Theorem 3.7. *For* $n \geq 2$,

$$DSAE(Ka_n(0)) < DSAE(Ka_n(1)) < \cdots < DSAE(Ka_n(\lfloor p \rfloor))$$

and

$$DSAE(Ka_n(\lceil p \rceil)) > DSAE(Ka_n(\lceil p \rceil + 1)) > \cdots > DSAE(Ka_n(n-1)),$$

where $p = \dfrac{n - 2 + \sqrt{n^2 - n + 1}}{3}$.

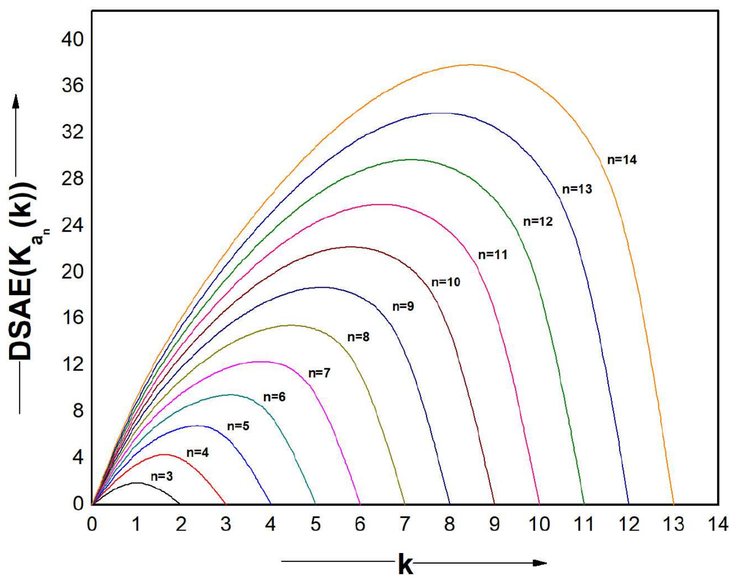

Figure 1. DSA energy of $Ka_n(k)$ for $n = 3, 4, ..., 14$.

Proof. By Proposition 3.1, $DSAE(Ka_n(k)) = 2\sqrt{k(k+1)(n-k-1)}$.

Consider $f_a(x) = x(x+1)(n-x-1)$

$f_a'(x) = -3x^2 + 2(n-2)x + n - 1 = 0$ gives that

$$x = \frac{n-2 \pm \sqrt{n^2 - n + 1}}{3}.$$

The function $f_a(x)$ is increasing in the interval $\left[0, \left\lfloor \frac{n-2+\sqrt{n^2-n+1}}{3} \right\rfloor\right]$ and decreasing in the interval $\left[\left\lceil \frac{n-2+\sqrt{n^2-n+1}}{3} \right\rceil, n-1\right]$. Therefore result follows. □

Theorem 3.8. *For* $n \geq 2$.

$$DSAE(Kb_n(0)) < DSAE(Kb_n(1)) < \cdots < DSAE(Kb_n(\lfloor n/4 \rfloor))$$

and

$$DSAE(Kb_n(\lceil n/4 \rceil)) > DSAE(Kb_n(\lceil n/4 \rceil + 1)) > \cdots > DSAE(Kb_n(\lfloor n/2 \rfloor)).$$

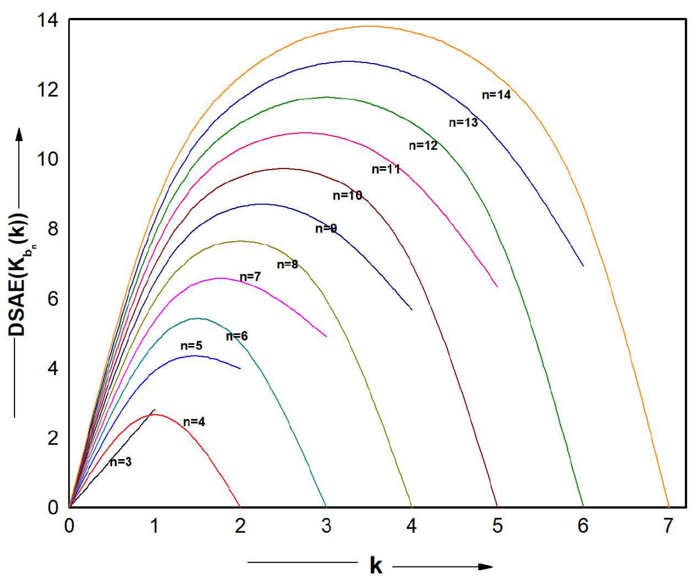

Figure 2. DSA energy of $Kb_n(k)$ for $n = 3, 4, ..., 14$.

Proof. By Proposition 3.2, $DSAE(Kb_n(k)) = 2\sqrt{2k(n-2k)}$.

Consider $f_b(x) = 2x(n-2x)$

$f_b'(x) = 2n - 8x = 0$ gives that $x = \frac{n}{4}$.

The function $f_b(x)$ is increasing in the interval $\left[0, \left\lfloor \frac{n}{4} \right\rfloor\right]$ and decreasing in the interval $\left[\left\lceil \frac{n}{4} \right\rceil, \left\lfloor \frac{n}{2} \right\rfloor\right]$. Therefore result follows. □

Theorem 3.9. *For* $n \geq 3$,

$$DSAE(Kc_n(1)) < DSAE(Kc_n(2)) < \cdots < DSAE(Kc_n(\lfloor q \rfloor))$$

and

$$DSAE(Kc_n(\lceil q \rceil)) > DSAE(Kc_n(\lceil q \rceil + 1)) > \cdots > DSAE(Kc_n(n)),$$

where $q = \frac{3n + 2 + \sqrt{9n^2 - 4n + 4}}{8}$.

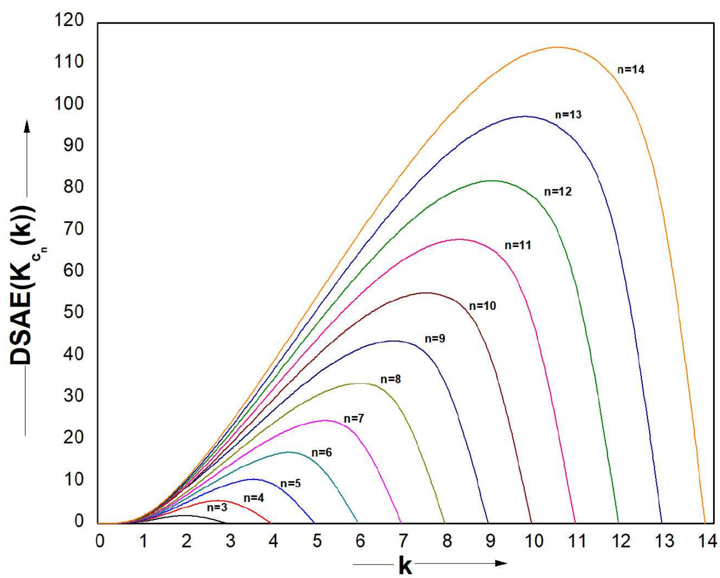

Figure 3. DSA energy of $Kc_n(k)$ for $n = 3, 4, ..., 14$.

Proof. By Proposition 3.3, $DSAE(Kc_n(k)) = 2(k-1)\sqrt{k(n-k)}$.

Consider $f_c(x) = (x-1)^2 x(n-x)$

$f_c'(x) = (x-1)(-4x^2 + (3n+2)x - n) = 0$ gives that $x = 1$ and

$$x = \frac{3n + 2 \pm \sqrt{9n^2 - 4n + 4}}{8}.$$

The function $f_c(x)$ is increasing in the interval $\left[1, \left\lfloor \frac{3n+2+\sqrt{9n^2-4n+4}}{8} \right\rfloor\right]$ and decreasing in the interval $\left[\left\lfloor \frac{3n+2+\sqrt{9n^2-4n+4}}{8} \right\rfloor, n\right]$. Therefore result follows. □

Theorem 3.10. *For* $n \geq 2$,

$$DSAE(Kc_d(0)) < DSAE(Kd_n(1)) < \cdots < DSAE(Kd_n(\lfloor n/2 \rfloor))$$

and

$$DSAE(Kd_n(\lceil n/2 \rceil)) > DSAE(Kd_n(\lceil n/2 \rceil + 1)) > \cdots > DSAE(Kd_n(n)).$$

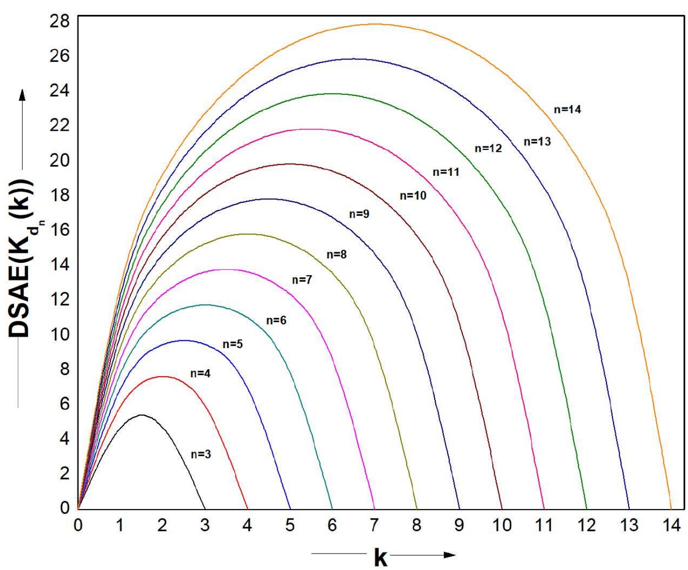

Figure 4. DSA energy of $Kd_n(k)$ for $n = 3, 4, \ldots, 14$.

Proof. By Proposition 3.4, $DSAE(Kd_n(k)) = 4\sqrt{k(n-k)}$.

Consider $f_d(x) = x(n-x)$

$f_d'(x) = n - 2x = 0$ gives that $x = \frac{n}{2}$.

The function $f_d(x)$ is increasing in the interval $\left[0, \left\lfloor \frac{n}{2} \right\rfloor\right]$ and decreasing in the interval $\left[\left\lceil \frac{n}{2} \right\rceil, n\right]$. Therefore result follows. □

4. Conclusion

The DSA spectra and DSA energy of graphs $Ka_n(k)$, $Kb_n(k)$, $Kc_n(k)$ and $Kd_n(k)$ are obtained. Figures 1, 2, 3 and 4 clearly show that the DSA energy of these graphs is increasing first and then decreasing as the number of removal of edges increases.

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