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Coupled Anti Q-Fuzzy Subgroups using t-Conorms

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Abstract

In this paper we study coupled anti Q-fuzzy subgroups of G with respect to t-conorm C. We also discuss the union, normal and direct product of them. Moreover, the homomorphic image and pre-image of them is investigated under group homomorphisms and anti homomorphisms.

1 Some Old and New Notions and Notations

Remark 1.1. (a) In this paper we assume G is a group as defined in [1].

(b) We assume the notions of homomorphism and anti-homomorphism as defined in [1].

(c) We assume $C : [0,1] \times [0,1] \mapsto [0,1]$ is a t-conorm as defined in [1].

(d) By idempotent, we refer to t-conorm C as defined in [1].

Definition 1.2. Let G be an arbitrary group with multiplicative binary operation and identity e. By a fuzzy subset of $G \times G$ we mean a function from $G \times G$ into $[0,1]$. The set of all fuzzy subsets of $G \times G$ will be called the $[0,1]$-power set of $G \times G$, and will be denoted by $[0,1]^{G\times G}$.

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Definition 1.3. Let $\alpha : A \mapsto B$ be a function such that $\mu \in [0,1]^{(A\times A)\times Q}$ and $\beta \in [0,1]^{(B\times B)\times Q}$. The fuzzy image, $\alpha(\mu)$ of μ under α, if $\alpha^{-1}(y), \alpha^{-1}(v) \neq \emptyset$, is defined as

$$\alpha(\mu)(y,v,q) = \inf\{\mu(x,m,q) | (x,m,q) \in (A\times A)\times Q, \alpha(x) = y, \alpha(m) = v\}$$

and if $\alpha^{-1}(y), \alpha^{-1}(v) = \emptyset$, then

$$\alpha(\mu)(y,v,q) = 0$$

and the fuzzy pre-image (or fuzzy inverse image) of β under α is

$$\alpha^{-1}(\beta)(x,m,q) = \beta(\alpha(x), \alpha(m), q)$$

for all $(x,m,q) \in (A\times A)\times Q$.

Definition 1.4. Let $(G,\cdot)$ be a group and Q be a nonempty set. Then $\mu \in [0,1]^{(G\times G)\times Q}$ will be called an anti Q-fuzzy subgroup of $G\times G$ with respect to t-conorm C if the following conditions are satisfied

(a) $\mu(xy, mv, q) \leq C[\mu(x,m,q), \mu(y,v,q)]$

(b) $\mu(x^{-1}, m^{-1}, q) \leq \mu(x,m,q)$

for all $(x,m), (y,v) \in G\times G$, and $q \in Q$.

Notation 1.5. The set of all anti Q-fuzzy subgroups of $G\times G$ with respect to t-conorm C will be denoted by $AQFSC(G\times G)$.

Proposition 1.6. *(Compare with Proposition 2.3[1]) Let G be a group, and H be a nonempty subset of $G\times G$. Then the following are equivalent*

(a) *H is a subgroup of $G\times G$*

(b) *$(x,m), (y,v) \in H$ implies $(xy^{-1}, mv^{-1}) \in H$ for all $(x,m), (y,v)$.*

Definition 1.7. Let $\mu, \beta \in [0,1]^{(G\times G)\times Q}$. The union of μ and β is defined by

$$(\mu \cup \beta)(x, m, q) = C[\mu(x, m, q), \beta(x, m, q)]$$

for all $(x, m) \in G \times G$, and $q \in Q$.

Definition 1.8. We say that $\mu \in AQFSC(G \times G)$ is normal if $\mu(xyx^{-1}, mvm^{-1}, q) = \mu(y, v, q)$ for all $(x, m), (y, v) \in G \times G$ and $q \in Q$.

Notation 1.9. $NAQFSC(G \times G)$ will denote the set of all normal anti Q-fuzzy subgroups of $G \times G$ with respect to t-conorm C.

2 Some Properties

Proposition 2.1. *Let $\mu \in AQFSC(G \times G)$, and C be idempotent t-conorm. Then, $\mu(e_G, e_G, q) \leq \mu(x, m, q)$ for all $(x, m) \in G \times G$, and $q \in Q$.*

Proof.

$$\begin{aligned}
\mu(e_G, e_G, q) &= \mu(xx^{-1}, mm^{-1}, q) \\
&\leq C[\mu(x, m, q), \mu(x^{-1}, m^{-1}, q)] \\
&= C[\mu(x, m, q), \mu(x, m, q)] \\
&= \mu(x, m, q)
\end{aligned}$$

for all $(x, m) \in G \times G$, and $q \in Q$. □

Proposition 2.2. *Let $\mu \in AQFSC(G \times G)$, and C be idempotent t-conorm. If $\mu(xy^{-1}, mv^{-1}, q) = \mu(e_G, e_G, q)$, then $\mu(x, m, q) = \mu(y, v, q)$ for all $(x, m), (y, v) \in G \times G$, and $q \in Q$.*

Proof.

$$\begin{aligned}
\mu(x,m,q) &= \mu(xy^{-1}y, mv^{-1}v, q) \\
&\le C[\mu(xy^{-1}, mv^{-1}, q), \mu(y,v,q)] \\
&= C[\mu(e_G, e_G, q), \mu(y,v,q)] \\
&\le C[\mu(y,v,q), \mu(y,v,q)] \\
&= \mu(y,v,q) \\
&= \mu(yx^{-1}x, vm^{-1}m, q) \\
&\le C[\mu(yx^{-1}, vm^{-1}, q), \mu(x,m,q)] \\
&= C[\mu((xy^{-1})^{-1}, (mv^{-1})^{-1}, q), \mu(x,m,q)] \\
&\le C[\mu(xy^{-1}, mv^{-1}, q), \mu(x,m,q)] \\
&= C[\mu(e_G, e_G, q), \mu(x,m,q)] \\
&\le C[\mu(x,m,q), \mu(x,m,q)] \\
&= \mu(x,m,q).
\end{aligned}$$

Thus, $\mu(x,m,q) = \mu(y,v,q)$. □

Proposition 2.3. *Let C be idempotent t-conorm. Then $\mu \in AQFSC(G \times G)$ iff $\mu(xy^{-1}, mv^{-1}, q) \le C[\mu(x,m,q), \mu(y,v,q)]$ for all $(x,m),(y,v) \in G \times G$, and $q \in Q$.*

Proof. Let $\mu \in AQFSC(G \times G)$, $(x,m),(y,v) \in G \times G$, $q \in Q$. Then,

$$\begin{aligned}
\mu(xy^{-1}, mv^{-1}, q) &\le C[\mu(x,m,q), \mu(y^{-1}, v^{-1}, q)] \\
&\le C[\mu(x,m,q), \mu(y,v,q)].
\end{aligned}$$

Conversely, let $\mu(xy^{-1}, mv^{-1}, q) \le C[\mu(x,m,q), \mu(y,v,q)]$, then

$$\begin{aligned}
\mu(x^{-1}, m^{-1}, q) &= \mu(e_G x^{-1}, e_G m^{-1}, q) \\
&\le C[\mu(e_G, e_G, q), \mu(x^{-1}, m^{-1}, q)] \\
&= C[\mu(e_G, e_G, q), \mu(x,m,q)] \\
&\le C[\mu(x,m,q), \mu(x,m,q)] \\
&= \mu(x,m,q).
\end{aligned}$$

Thus, $\mu(x^{-1}, m^{-1}, q) \leq \mu(x, m, q)$. Also

$$\begin{aligned}\mu(xy, mv, q) &= \mu(x(y^{-1})^{-1}, m(v^{-1})^{-1}, q) \\ &\leq C[\mu(x, m, q), \mu(y^{-1}, v^{-1}, q)] \\ &\leq C[\mu(x, m, q), \mu(y, v, q)].\end{aligned}$$

Thus, $\mu \in AQFSC(G \times G)$. □

Proposition 2.4. *Let $\mu \in [0,1]^{(G\times G)\times Q}$ be such that $\mu(e_G, e_G, q) = 0$ and $\mu(xy^{-1}, mv^{-1}, q) \leq C[\mu(x, m, q), \mu(y, v, q)]$ for all $(x, m), (y, v) \in G \times G$, and $q \in Q$. Then $\mu \in AQFSC(G \times G)$.*

Proof. Let $(x, m), (y, v) \in G \times G$, and $q \in Q$. Observe

$$\begin{aligned}\mu(x^{-1}, m^{-1}, q) &= \mu(e_G x^{-1}, e_G m^{-1}, q) \\ &\leq C[\mu(e_G, e_G, q), \mu(x^{-1}, m^{-1}, q)] \\ &= C[\mu(e_G, e_G, q), \mu(x, m, q)] \\ &= C[0, \mu(x, m, q)] \\ &= \mu(x, m, q).\end{aligned}$$

Now

$$\begin{aligned}\mu(xy, mv, q) &= \mu[x(y^{-1})^{-1}, m(v^{-1})^{-1}, q] \\ &\leq C[\mu(x, m, q), \mu(y^{-1}, v^{-1}, q)] \\ &\leq C[\mu(x, m, q), \mu(y, v, q)].\end{aligned}$$

Thus, $\mu \in AQFSC(G \times G)$. □

Proposition 2.5. *If $\mu \in AQFSC(G \times G)$, then $H = \{(x, m) \in G \times G | \mu(x, m, q) = 0\}$ is a subgroup of $G \times G$.*

Proof. Let $(x,m),(y,v) \in H$, and $q \in Q$. Since $\mu \in AQFSC(G \times G)$,

$$\begin{aligned}\mu(xy^{-1},mv^{-1},q) &\leq C[\mu(x,m,q),\mu(y^{-1},v^{-1},q)]\\ &\leq C[\mu(x,m,q),\mu(y,v,q)]\\ &= C[0,0]\\ &= 0.\end{aligned}$$

Thus $(xy^{-1},mv^{-1}) \in H$, and by Proposition 1.6, H is a subgroup of $G \times G$. □

Proposition 2.6. *Let $\mu \in AQFSC(G \times G)$ and C be idempotent t-conorm. Then*

$$H = \{(x,m) \in G | \mu(x,m,q) = \mu(e_G,e_G,q)\}$$

is a subgroup of $G \times G$.

Proof. Let $(x,m),(y,v) \in H$, $q \in Q$, and $\mu \in AQFSC(G \times G)$. Then,

$$\begin{aligned}\mu(xy^{-1},mv^{-1},q) &\leq C[\mu(x,m,q),\mu(y^{-1},v^{-1},q)]\\ &= C[\mu(x,m,q),\mu(y,v,q)]\\ &= C[\mu(e_G,e_G,q),\mu(e_G,e_G,q)]\\ &= \mu(e_G,e_G,q)\\ &\leq \mu(xy^{-1},mv^{-1},q).\end{aligned}$$

Thus $\mu(xy^{-1},mv^{-1},q) = \mu(e_G,e_G,q)$, which implies $(xy^{-1},mv^{-1}) \in H$. So by Proposition 1.6, H is a subgroup of $G \times G$. □

Proposition 2.7. *Let $\mu \in AQFSC(G \times G)$, and $\mu(xy^{-1},mv^{-1},q) = 0$. Then $\mu(x,m,q) = \mu(y,v,q)$ for all $(x,m),(y,v) \in G \times G$, and $q \in Q$.*

Proof. Let $\mu \in AQFSC(G \times G)$, $(x, m), (y, v) \in G \times G$, $q \in Q$. Then

$$\begin{aligned}
\mu(x, m, q) &= \mu(xy^{-1}y, mv^{-1}v, q) \\
&\leq C[\mu(xy^{-1}, mv^{-1}, q), \mu(y, v, q)] \\
&= C[0, \mu(y, v, q)] \\
&= \mu(y, v, q) \\
&= \mu(y^{-1}, v^{-1}, q) \\
&= \mu(x^{-1}xy^{-1}, m^{-1}mv^{-1}, q) \\
&\leq C[\mu(x^{-1}, m^{-1}, q), \mu(xy^{-1}, mv^{-1}, q)] \\
&= C[\mu(x^{-1}, m^{-1}, q), 0] \\
&= \mu(x^{-1}, m^{-1}, q) \\
&= \mu(x, m, q).
\end{aligned}$$

Hence $\mu(x, m, q) = \mu(y, v, q)$. □

Proposition 2.8. *Let $\mu \in AQFSC(G \times G)$. Then $\mu(xy, mv, q) = \mu(yx, vm, q)$ iff $\mu(x, m, q) = \mu(y^{-1}xy, v^{-1}mv, q)$ for all $(x, m), (y, v) \in G \times G$, and $q \in Q$.*

Proof. Let $(x, m), (y, v) \in G \times G$, $q \in Q$, and $\mu(xy, mv, q) = \mu(yx, vm, q)$. Then

$$\begin{aligned}
\mu(y^{-1}xy, v^{-1}mv, q) &= \mu(y^{-1}(xy), v^{-1}(mv), q) \\
&= \mu(xyy^{-1}, mvv^{-1}, q) \\
&= \mu(xe_G, me_G, q) \\
&= \mu(x, m, q).
\end{aligned}$$

Conversely, let $\mu(x, m, q) = \mu(y^{-1}xy, v^{-1}mv)$, then we obtain

$$\begin{aligned}
\mu(xy, mv, q) &= \mu(x(yx)x^{-1}, m(vm)m^{-1}, q) \\
&= \mu(yx, vm, q).
\end{aligned}$$

□

Proposition 2.9. *Let $\mu \in AQFSC(G \times G)$. If $\mu(xy^{-1}, mv^{-1}, q) = 1$, then either $\mu(x, m, q) = 1$ or $\mu(y, v, q) = 1$ for all $(x, m), (y, v) \in G \times G$, and $q \in Q$.*

Proof. As $\mu \in AQFSC(G \times G)$. So for all $(x, m), (y, v) \in G \times G$ and $q \in Q$, we have

$$1 = \mu(xy^{-1}, mv^{-1}, q) \leq C[\mu(x, m, q), \mu(y, v, q)].$$

So either $\mu(x, m, q) = 1$ or $\mu(y, v, q) = 1$. □

Proposition 2.10. *Let $\mu, \beta \in AQFSC(G \times G)$. Then $\mu \cup \beta \in AQFSC(G \times G)$.*

Proof.

$$\begin{aligned}(\mu \cup \beta)(xy, mv, q) &= C[\mu(xy, mv, q), \beta(xy, mv, q)] \\ &\leq C[C[\mu(x, m, q), \mu(y, v, q)], C[\beta(x, m, q), \beta(y, v, q)]] \\ &= C[C[\mu(x, m, q), \beta(x, m, q)], C[\mu(y, v, q), \beta(y, v, q)]] \\ &= C[(\mu \cup \beta)(x, m, q), (\mu \cup \beta)(y, v, q)].\end{aligned}$$

Also

$$\begin{aligned}(\mu \cup \beta)(x^{-1}, m^{-1}, q) &= C[\mu(x^{-1}, m^{-1}, q), \beta(x^{-1}, m^{-1}, q)] \\ &\leq C[\mu(x, m, q), \beta(x, m, q)] \\ &= (\mu \cup \beta)(x, m, q).\end{aligned}$$

Hence, $\mu \cup \beta \in AQFSC(G \times G)$. □

Proposition 2.11. *Let $\mu \in AQFSC(G \times G)$ and $(x, m), (y, v) \in G \times G$, $q \in Q$. If C is idempotent t-conorm and $\mu(x, m, q) \neq \mu(y, v, q)$, then*

$$\mu(xy, mv, q) = C[\mu(x, m, q), \mu(y, v, q)].$$

Proof. Let $\mu(x, m, q) < \mu(y, v, q)$ for all $(x, m), (y, v) \in G \times G$, and $q \in Q$, then $\mu(x, m, q) < \mu(xy, mv, q)$, and so $\mu(y, v, q) = C[\mu(x, m, q), \mu(y, v, q)]$ and

$\mu(xy, mv, q) = C[\mu(x, m, q), \mu(xy, mv, q)]$. Now

$$\begin{aligned}\mu(xy, mv, q) &\leq C[\mu(x, m, q), \mu(y, v, q)] \\ &= \mu(y, v, q) \\ &= \mu(x^{-1}xy, m^{-1}mv, q) \\ &\leq C[\mu(x^{-1}, m^{-1}, q), \mu(xy, mv, q)] \\ &= C[\mu(x, m, q), \mu(xy, mv, q)] \\ &= \mu(xy, mv, q).\end{aligned}$$

So, $\mu(xy, mv, q) = \mu(y, v, q) = C[\mu(x, m, q), \mu(y, v, q)]$. □

Proposition 2.12. *Let $\mu_1, \mu_2 \in NAQFSC(G \times G)$. Then $\mu_1 \cup \mu_2 \in NAQFSC(G \times G)$.*

Proof.

$$\begin{aligned}(\mu_1 \cup \mu_2)(xyx^{-1}, mvm^{-1}, q) &= C[\mu_1(xyx^{-1}, mvm^{-1}, q), \mu_2(xyx^{-1}, mvm^{-1}, q)] \\ &= C[\mu_1(y, v, q), \mu_2(y, v, q)] \\ &= (\mu_1 \cup \mu_2)(y, v, q).\end{aligned}$$

□

Proposition 2.13. *Let α be an epimorphism from group G into group H. If $\mu \in AQFSC(G \times G)$, then $\alpha(\mu) \in AQFSC(H \times H)$.*

Proof. Let $(h_1, h_1'), (h_2, h_2') \in H \times H$ and $q \in Q$. Then,

$$\begin{aligned}\alpha(\mu)(h_1h_2, h_1'h_2', q) &= \inf\{\mu(g_1g_2, g_1'g_2', q) | g_1, g_2, g_1', g_2' \in G, \alpha(g_1) = h_1, \alpha(g_1') = h_1', \\ &\qquad \alpha(g_2) = h_2, \alpha(g_2') = h_2'\} \\ &\leq \inf\{C[\mu(g_1, g_1', q), \mu(g_2, g_2', q)] | g_1, g_2, g_1', g_2' \in G, \alpha(g_1) = h_1, \\ &\qquad \alpha(g_1') = h_1', \alpha(g_2) = h_2, \alpha(g_2') = h_2'\} \\ &= C[(\inf\{\mu(g_1, g_1', q)) | g_1, g_1' \in G, \alpha(g_1) = h_1, \alpha(g_1') = h_1'\}), \\ &\qquad (\inf\{\mu(g_2, g_2', q)) | g_2, g_2' \in G, \alpha(g_2) = h_2, \alpha(g_2') = h_2'\})] \\ &= C[\alpha(\mu)(h_1, h_1', q), \alpha(\mu)(h_2, h_2', q)].\end{aligned}$$

Also

$$\begin{aligned}\alpha(\mu)(h_1^{-1},(h_1')^{-1},q) &= \inf\{\mu(g_1^{-1},(g_1')^{-1},q)|g_1,g_1'\in G,\alpha(g_1^{-1})=h_1^{-1},\alpha((g_1')^{-1})\\ &\qquad = (h_1')^{-1}\}\\ &\le \inf\{\mu(g_1,g_1',q)|g_1,g_1'\in G,\alpha(g_1,g_1',q)=h_1,\alpha(g_1',g_1,q)=h_2\}\\ &= \alpha(\mu)(h_1,h_2,q).\end{aligned}$$

Therefore $\alpha(\mu)\in AQFSC(H\times H)$. □

Proposition 2.14. *Let α be a homomorphism from group G into group H. If $\beta\in AQFSC(H\times H)$, then $\alpha^{-1}(\beta)\in AQFSC(G\times G)$.*

Proof. Let $(x,m),(y,v)\in G\times G$, and $q\in Q$. Then

$$\begin{aligned}\alpha^{-1}(\beta)(xy,mv,q) &= \beta(\alpha(xy),\alpha(mv),q)\\ &= \beta(\alpha(x)\alpha(y),\alpha(m)\alpha(v),q)\\ &\le C[\beta(\alpha(x),\alpha(m),q),\beta(\alpha(y),\alpha(v),q)]\\ &= C[\alpha^{-1}(\beta)(x,m,q),\alpha^{-1}(\beta)(y,v,q)].\end{aligned}$$

Also,

$$\begin{aligned}\alpha^{-1}(\beta)(x^{-1},m^{-1},q) &= \beta(\alpha(x^{-1}),\alpha(m^{-1}),q)\\ &\le \beta(\alpha(x),\alpha(m),q)\\ &= \alpha^{-1}(\beta)(x,m,q).\end{aligned}$$

Thus, $\alpha^{-1}(\beta)\in AQFSC(G\times G)$. □

Proposition 2.15. *Let α be an anti-homomorphism from group G into group H. If $\beta\in AQFSC(H\times H)$, then $\alpha^{-1}(\beta)\in AQFSC(G\times G)$.*

Proof. Let $(x,m),(y,v) \in G \times G$ and $q \in Q$. Then

$$\begin{aligned}
\alpha^{-1}(\beta)(xy, mv, q) &= \beta(\alpha(xy), \alpha(mv), q) \\
&= \beta(\alpha(y)\alpha(x), \alpha(v)\alpha(m), q) \\
&\leq C[\beta(\alpha(y), \alpha(v), q), \beta(\alpha(x), \alpha(m), q)] \\
&= C[\alpha^{-1}(\beta)(y, v, q), \alpha^{-1}(\beta)(x, m, q)] \\
&= C[\alpha^{-1}(\beta)(x, m, q), \alpha^{-1}(\beta)(y, v, q)].
\end{aligned}$$

Also

$$\begin{aligned}
\alpha^{-1}(\beta)(x^{-1}, m^{-1}, q) &= \beta(\alpha(x^{-1}), \alpha(m^{-1}), q) \\
&= \beta(\alpha^{-1}(x), \alpha^{-1}(m), q) \\
&\leq \beta(\alpha(x), \alpha(m), q) \\
&= \alpha^{-1}(\beta)(x, m, q).
\end{aligned}$$

Thus, $\alpha^{-1}(\beta) \in AQFSC(G \times G)$. □

Proposition 2.16. *Let $\mu \in NAQFSC(G \times G)$ and H be a group. Suppose that α is an epimorphism of G into H. Then $\alpha(\mu) \in NAQFSC(H \times H)$.*

Proof. By Proposition 2.13, we have $\alpha(\mu) \in AQFSC(H \times H)$. Let $(x,m),(y,v) \in H \times H$ and $q \in Q$. Since α is a surjection, $\alpha(u) = x$ and $\alpha(u') = m$ for some $u, u' \in G$. Now,

$$\begin{aligned}
\alpha(\mu)(xyx^{-1}, mvm^{-1}, q) &= \inf\{\mu(w, w', q) | w, w' \in G, \alpha(w) = xyx^{-1}, \alpha(w') = mvm^{-1}\} \\
&= \inf\{\mu(u^{-1}wu, (u')^{-1}w'u', q) | w, w' \in G, \alpha(u^{-1}wu) = y, \\
&\qquad \alpha((u')^{-1}w'u' = v\} \\
&= \inf\{\mu(w, w', q) | w, w' \in G, \alpha(w) = y, \alpha(w') = v\} \\
&= \alpha(\mu)(y, v, q).
\end{aligned}$$

and the proof is finished. □

Proposition 2.17. *Let H be a group and $\beta \in NAQFSC(H \times H)$. Suppose that α is a homomorphism of G into H. Then $\alpha^{-1}(\beta) \in NAQFSC(G \times G)$.*

Proof. By Proposition 2.14, we obtain that $\alpha^{-1}(\beta) \in AQFSC(G \times G)$. Now for any $(x, m), (y, v) \in G \times G$, and $q \in Q$, we obtain that

$$\begin{aligned}
\alpha^{-1}(\beta)(xyx^{-1}, mvm^{-1}, q) &= \beta(\alpha(xyx^{-1}), \alpha(mvm^{-1}), q) \\
&= \beta[\alpha(x)\alpha(y)\alpha(x^{-1}), \alpha(m)\alpha(v)\alpha(m^{-1}), q] \\
&= beta[\alpha(x)\alpha(y)\alpha^{-1}(x), \alpha(m)\alpha(v)\alpha^{-1}(m), q] \\
&= \beta(\alpha(y), \alpha(v), q) \\
&= \alpha^{-1}(\beta)(y, v, q).
\end{aligned}$$

Therefore, $\alpha^{-1}(\beta) \in NAQFSC(G \times G)$. □

3 Open Problems

We begin with the following

Definition 3.1. Let $(G, \cdot)$ and $(H, \cdot)$ be any two groups such that $\mu \in AQFSC(G \times G)$ and $\beta \in AQFSC(H \times H)$. The product of μ and β, denoted by $\mu \times \beta \in [0, 1]^{((G \times G) \times (H \times H)) \times Q}$ is defined as

$$(\mu \times \beta)[((x, m), (y, v)), q] = C[\mu(x, m, q), \beta(y, v, q)]$$

for all $(x, m) \in G \times G$, $(y, v) \in H \times H$, $q \in Q$.

Conjecture 3.2. *If $\mu \in AQFSC(G \times G)$ and $\beta \in AQFSC(H \times H)$, then $\mu \times \beta \in AQFSC(G^2 \times H^2)$.*

Conjecture 3.3. *Let $\mu \in [0, 1]^{(G \times G) \times Q}$ and $\beta \in [0, 1]^{(H \times H) \times Q}$. If C is idempotent t-conorm and $\mu \times \beta \in AQFSC(G^2 \times H^2)$, then at least one of the following two statements must hold*

(a) *$\beta(e_H, e_H, q) \leq \mu(x, m, q)$ for all $(x, m) \in G \times G$, and $q \in Q$.*

(b) $\mu(e_H, e_H, q) \leq \beta(y, v, q)$ *for all* $(y, v) \in H \times H$, *and* $q \in Q$.

Conjecture 3.4. *Let* $\mu \in [0,1]^{(G\times G)\times Q}$ *and* $\beta \in [0,1]^{(H\times H)\times Q}$. *If* C *is idempotent* t*-conorm and* $\mu \times \beta \in AQFSC(G^2 \times H^2)$, *then we obtain the following statements*

(a) *If* $\mu(x, m, q) \geq \beta(e_H, e_H, q)$, *then* $\mu \in AQFSC(G \times G)$ *for all* $(x, m) \in G \times G$ *and* $q \in Q$.

(b) *If* $\beta(x, m, q) \geq \mu(e_G, e_G, q)$, *then* $\beta \in AQFSC(H \times H)$ *for all* $(x, m) \in H$ *and* $q \in Q$.

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