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Coupled Anti Multigroups: Some Properties

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Abstract

Motivated by [1], the authors in [2] extended the notion of anti fuzzy groups to the multigroup context and studied some of their properties. In this paper we extend the work in a new direction termed *coupled multigroup* and obtain some new properties in this context. A conjecture concludes the paper.

1 Some New Notions and Notations

Definition 1.1. Let X be a set. A coupled multiset A over $X \times X$ will be a pair $< X \times X, C_A >$, where $C_A : X \times X \mapsto \mathbb{N} \cup \{0\}$ is a function such that for $(x, y) \in X \times X$ implies $A(x, y)$ is cardinal, and $A(x, y) = C_A(x, y) > 0$, where $C_A(x, y)$ denotes the number of times an object (x, y) occur in A, whenever $C_A(x, y) = 0$ implies $(x, y) \notin X \times X$.

Let B be a subset of $X \times X$ and define $1_B : X \times X \mapsto \{0, 1\}$ by

$$1_B(x, y) = \begin{cases} 1 & (x, y) \in B \\ 0 & (x, y) \notin B. \end{cases}$$

It follows that B is a coupled multiset $< B, 1_B >$, where 1_B is its characteristic function.

Remark 1.2. We call $X \times X$ the ground or generic set of the class of all coupled multisets containing objects from $X \times X$.

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Definition 1.3. Let $X \times X$ be the set from which coupled multisets are constructed. By $(X \times X)^n$ we mean the set of all coupled multisets of $X \times X$ such that no element occurs more than n times.

Definition 1.4. Let $X \times X$ be the set from which coupled multisets are constructed. By $(X \times X)^\infty$ we mean the set of all coupled multisets of $X \times X$ such that there is no limit on the number of occurences of an element.

Notation 1.5. $CMS(X \times X)$ will denote the set of all coupled multisets over $X \times X$.

Remark 1.6. In this paper we focus on $CMS(X \times X)$ contained in $(X \times X)^n$.

Example 1.7. Let $X = \{a, b, c\}$ and

$$X \times X = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}.$$

Then

$$A = \{(a,a)^2,(a,b)^2,(a,c)^2,(b,a)^2,(b,b)^2,(b,c)^2,(c,a)^3,(c,b)^3,(c,c)^3\},$$

where $(x,y)^n$ means (x,y) repeated n times, is a coupled multiset of $X \times X$.

Motivated by [3], we introduce the following

Definition 1.8. Let X be a nonempty set, and $(X \times X)^n$ be the coupled multiset space defined over $X \times X$. For any

$$A \in CMS(X \times X) \subseteq (X \times X)^n$$

we define the complement of A in $(X \times X)^n$, denoted A^c, by

$$C_{A^c}(x,y) = n - C_A(x,y)$$

for every $(x,y) \in X \times X$.

Remark 1.9. From now on $CMS(X \times X)$ will mean the set of all coupled multisets over $X \times X$ drawn from the coupled multiset space $(X \times X)^n$.

Motivated by [4], we introduce the following

Definition 1.10. Let $A, B \in CMS(X \times X)$. We say A is a *coupled submultiset* of B written $A \subseteq B$, if

$$C_A(x, y) \leq C_B(x, y)$$

for all $(x, y) \in X \times X$. Also if $A \subseteq B$ and $A \neq B$, then we say A is a *proper coupled submultiset* of B written $A \subset B$. Moreover, we say a coupled multiset is the *parent* in relation to its coupled submultiset.

Motivated by [5], we introduce the following

Definition 1.11. Let $A, B \in CMS(X \times X)$, $\wedge$ and $\vee$ denote minimum and maximum, respectively, and let (x, y) be any object in $X \times X$.

(a) The intersection of A and B, $A \cap B$, will be defined as

$$C_{A \cap B}(x, y) = C_A(x, y) \wedge C_B(x, y).$$

(b) The union of A and B, $A \cup B$, will be defined as

$$C_{A \cup B}(x, y) = C_A(x, y) \vee C_B(x, y).$$

(c) The sum of A and B, $A + B$, will be defined as

$$C_{A+B}(x, y) = C_A(x, y) + C_B(x, y).$$

Motivated by [5], we introduce the following

Definition 1.12. Let $A, B \in CMS(X \times X)$. We say A and B are comparable to each other if and only if $B \subseteq A$ or $A \subseteq B$. Moreover, we say $A = B$ if and only if $C_A(x, y) = C_B(x, y)$ for all $(x, y) \in X \times X$.

Motivated by [6], we introduce the following

Definition 1.13. Let X be a group. A coupled multiset A over $X \times X$ will be called a *coupled multigroupoid* of $X \times X$ if for all $(x, m), (y, v) \in X \times X$ we have

$$C_A(xy, mv) \geq C_A(x, m) \wedge C_A(y, v),$$

where C_A denotes count function of A from $X \times X$ into $\mathbb{N}$.

Motivated by [6] and [7] we introduce the following

Definition 1.14. Let X be a group. A coupled multiset A of $X \times X$ will be called a *coupled multigroup* of $X \times X$ if it satisfies the following two conditions

(a) A is a coupled multigroupoid of $X \times X$.

(b) $C_A(x^{-1}, m^{-1}) = C_A(x, m)$ for all $(x, m) \in X \times X$.

Moreover, $CMG(X \times X)$ denotes the set of all coupled multigroups of $X \times X$

Motivated by [7], we have the following alternate characterization of coupled multigroups

Definition 1.15. Let X be a group, and A be a multiset over $X \times X$. If

$$C_A(xy^{-1}, mv^{-1}) \geq C_A(x, m) \wedge C_A(y, m)$$

for all $(x, m), (y, v) \in X \times X$, then A is a coupled multigroup of $X \times X$.

Motivated by [6], we introduce the following

Definition 1.16. Let $A \in CMG(X \times X)$. A coupled submultiset B of A will be called a *coupled submultigroup* of A denoted by $B \sqsubseteq A$ if B is a coupled multigroup. A coupled submultigroup B of A will be called a *proper coupled submultigroup* denoted by $B \sqsubset A$, if $B \sqsubseteq A$ and $A \neq B$.

Motivated by [8], we introduce the following

Definition 1.17. Let $A \in CMG(X \times X)$. By the strong upper cut of A, we mean the set

$$A_{[n]} = \{(x,m) \in X \times X | C_A(x,m) \geq n, n \in \mathbb{N}\}.$$

By the weak upper cut of A, we mean the set

$$A_{(n)} = \{(x,m) \in X \times X | C_A(x,m) \geq n, n \in \mathbb{N}\}.$$

Motivated by [7], we introduce the following

Definition 1.18. The inverse of an element $(x,m) \in X \times X$ in a coupled multigroup A of $X \times X$ will be given by

$$C_A(x^{-1}, m^{-1}) = C_{A^{-1}}(x,m)$$

for all $(x,m) \in X \times X$.

In Section 3 of [2], the author presents anti-multigroup as a multigroup in reverse order. From now on we refine some of these concepts to the coupled anti multigroup setting and obtain some related properties in the next section.

Definition 1.19. Let X be a groupoid. The coupled multiset A of $X \times X$ will be called a *coupled anti multigroupoid* of $X \times X$ if

$$C_A(xy, mv) \leq C_A(x,m) \vee C_A(y,v)$$

for all $(x,m),(y,v) \in X \times X$.

Definition 1.20. A coupled multiset A of $X \times X$ will be called a *coupled anti multigroup* of $X \times X$ if the following conditions hold:

(a) $C_A(xy, mv) \leq C_A(x,m) \vee C_A(y,v)$ for all $(x,m),(y,v) \in X \times X$.

(b) $C_A(x^{-1}, m^{-1}) \leq C_A(x,m)$ for all $(x,m) \in X \times X$.

Notation 1.21. The set of all coupled anti multigroups of $X \times X$ will be denoted by $CAMG(X \times X)$.

Conjecture 1.22. *Let $X = \{e, a, b, c\}$ be a group. Let the elements of $X \times X$ be such that*

$$(a, e) = (a, a) = (a, b) = (a, c)$$

$$(b, e) = (b, a) = (b, b) = (b, c)$$

$$(c, e) = (c, a) = (c, b) = (c, c)$$

$$(e, e) = (e, a) = (e, b) = (e, c).$$

Assume we have the following

$$(a^2, e^2) = (b^2, e^2) = (c^2, e^2) = (e, e)$$

$$(ab, e^2) = (c, e)$$

$$(ac, e^2) = (b, e)$$

$$(bc, e^2) = (a, e).$$

Then the coupled multiset $A = \{(e^2, e^2), (a^5, e^5), (b^4, e^4), (c^5, e^5)\}$ is a coupled anti multigroup of $X \times X$.

In what follows we introduce a concept of *cuts* for coupled anti multigroups.

Definition 1.23. Let $A \in CAMG(X \times X)$. Then the set $\mathbb{A}_{[n]}$ for $n \in \mathbb{N}$ defined by

$$\mathbb{A}_{[n]} = \{(x, m) \in X \times X | C_A(x, m) \leq n\}$$

will be called the *cut* of A.

2 Some Properties

Proposition 2.1. *Let X be a nonempty set. If A is a coupled multigroup of $X \times X$, then the following holds:*

(a) $C_A(x^{-1}, m^{-1}) = C_A(x, m)$ *for all* $(x, m) \in X \times X$.

(*b*) $C_A(e, e') \leq C_A(x, m)$ *for all* $(x, m) \in X \times X$*, where* (e, e') *is the identity element of* $X \times X$.

(*c*) $C_A(x^n, m^n) \leq C_A(x, m)$ *for all* $(x, m) \in X \times X$ *and* $n \in \mathbb{N}$.

Proof. For (a): By Definition 1.20,

$$C_A(x^{-1}, m^{-1}) \leq C_A(x, m)$$

for all $(x, m) \in X \times X$. Also we have

$$C_A(x, m) \leq C_A((x^{-1})^{-1}, (m^{-1})^{-1}) \leq C_A(x^{-1}, m^{-1}).$$

Combining the above two inequalities completes the proof.

For (b): Let $(x, m) \in X \times X$. Note that $xx^{-1} = e$ and $mm^{-1} = e'$. It now follows that

$$\begin{aligned} C_A(e, e') &= C_A(xx^{-1}, mm^{-1}) \\ &\leq C_A(x, m) \vee C_A(x, m) \\ &= C_A(x, m) \end{aligned}$$

and (*b*) follows.

For (c): We have the following for all $n \in \mathbb{N}$,

$$\begin{aligned} C_A(x^n, m^n) &\leq C_A(x^{n-1}, m^{n-1}) \vee C_A(x, m) \\ &\leq C_A(x^{n-2}, m^{n-2}) \vee C_A(x, m) \vee C_A(x, m) \\ &\vdots \\ &\leq C_A(x, m) \vee \cdots \vee C_A(x, m) \\ &= C_A(x, m). \end{aligned}$$

□

Proposition 2.2. *If A and B are coupled anti multigroups of $X \times X$, then $A \cap B$ is a coupled anti multigroup of $X \times X$.*

Proof. Let $(x, m), (y, v) \in X \times X$. Observe we have the following

$$\begin{aligned} C_{A\cap B}(xy^{-1}, mv^{-1}) &= C_A(xy^{-1}, mv^{-1}) \wedge C_B(xy^{-1}, mv^{-1}) \\ &\le [C_A(x, m) \vee C_A(y, v)] \wedge [C_B(x, m) \vee C_B(y, v)] \\ &= [C_A(x, m) \wedge C_B(x, m)] \vee [C_A(y, v) \wedge C_B(y, v)] \\ &= C_{A\cap B}(x, m) \vee C_{A\cap B}(y, v). \end{aligned}$$

Hence the conclusion. □

Proposition 2.3. *If A and B are coupled anti multigroups of $X \times X$, then the sum of A and B is a coupled multigroup of $X \times X$.*

Proof. Let $(x, m), (y, v) \in X \times X$. Observe we have the following

$$\begin{aligned} C_{A+B}(xy^{-1}, mv^{-1}) &= C_A(xy^{-1}, mv^{-1}) + C_B(xy^{-1}, mv^{-1}) \\ &\le [C_A(x, m) \vee C_A(y, v)] + [C_B(x, m) \vee C_B(y, v)] \\ &= [C_A(x, m) + C_B(x, m)] \vee [C_A(y, v) + C_B(y, v)] \\ &= C_{A+B}(x, m) \vee C_{A+B}(y, v). \end{aligned}$$

Hence the conclusion. □

Proposition 2.4. *A coupled multiset A is a coupled multigroup of $X \times X$ iff*

$$C_A(xy^{-1}, mv^{-1}) \le C_A(x, m) \vee C_A(y, v)$$

for all $(x, m), (y, v) \in X \times X$.

Proof. Assume A is a coupled anti multigroup of $X \times X$. Then we know

$$C_A(xy, mv) \le C_A(x, m) \vee C_A(y, v)$$

for all $(x, m), (y, v) \in X \times X$ and

$$C_A(x^{-1}, m^{-1}) \le C_A(x, m)$$

for all $(x, m) \in X \times X$. By these conditions we have

$$C_A(xy^{-1}, mv^{-1}) \leq C_A(x, m) \vee C_A(y, v)$$

for all $(x, m), (y, v) \in X \times X$. Conversely, suppose the given condition is satisfied. By the following facts

$$C_A(e, e') \leq C_A(x, m)$$
$$C_A(x^{-1}, m^{-1}) = C_A(x, m)$$

for all $(x, m) \in X \times X$, and

$$\begin{aligned} C_A(xy, mv) &\leq C_A[x(y^{-1})^{-1}, m(v^{-1})^{-1}] \\ &\leq C_A(x, m) \vee C_A(y^{-1}, v^{-1}) \\ &= C_A(x, m) \vee C_A(y, v) \end{aligned}$$

for all $(x, m), (y, v) \in X \times X$. It follows that A is a coupled anti multigroup of $X \times X$. □

Theorem 2.5. *Let X be a finite group, and A be a coupled anti multigroupoid of $X \times X$. Then A is a coupled anti multigroup.*

Proof. Let $(x, m) \in X \times X$, $(x, m) \neq (e, e')$. Since X is finite, x and m have finite order. Thus

$$(x^n, m^n) = (e, e') \Longrightarrow (x^{-1}, m^{-1}) = (x^{n-1}, m^{n-1}).$$

By repeated application of the definition of coupled anti multigroupoid, we deduce the following

$$\begin{aligned} C_A(x^{-1}, m^{-1}) &= C_A(x^{n-1}, m^{n-1}) \\ &= C_A(x^{n-2}x, m^{n-2}m) \\ &\leq C_A(x^{n-2}, m^{n-2}) \vee C_A(x, m) \\ &\vdots \\ &\leq C_A(x, m) \vee \cdots \vee C_A(x, m) \\ &= C_A(x, m). \end{aligned}$$

Hence the conclusion. □

Theorem 2.6. *Let A be a multiset of X. Then $A \in CMG(X \times X)$ iff $A^c \in CAMG(X \times X)$.*

Proof. Suppose $A \in CMG(X \times X)$. Then for all $(x, m), (y, v) \in X \times X$, we have

$$C_A(xy^{-1}, mv^{-1}) \geq C_A(x, m) \wedge C_A(y, v)$$

$\Longrightarrow$

$$C_{(A^c)^c}(xy^{-1}, mv^{-1}) \geq C_{(A^c)^c}(x, m) \wedge C_{(A^c)^c}(y, v)$$

$\Longrightarrow$

$$1 - C_{A^c}(xy^{-1}, mv^{-1}) \geq (1 - C_{A^c}(x, m)) \wedge (1 - C_{A^c}(y, v))$$

$\Longrightarrow$

$$-C_{A^c}(xy^{-1}, mv^{-1}) \geq -1 + [(1 - C_{A^c}(x, m)) \wedge (1 - C_{A^c}(y, v))]$$

$\Longrightarrow$

$$C_{A^c}(xy^{-1}, mv^{-1}) \leq 1 - [(1 - C_{A^c}(x, m)) \wedge (1 - C_{A^c}(y, v))]$$

$\Longrightarrow$

$$C_{A^c}(xy^{-1}, mv^{-1}) \leq C_{A^c}(x, m)) \vee C_{A^c}(y, v))$$

$\Longrightarrow$

$$A^c \in CAMG(X \times X).$$

Conversely suppose A^c is a coupled anti multigroup of $X \times X$, then we have

$$C_{A^c}(xy^{-1}, mv^{-1}) \leq C_{A^c}(x, m) \vee C_{A^C}(y, v)$$

$\Longrightarrow$

$$1 - C_A(xy^{-1}, mv^{-1}) \leq (1 - C_A(x, m) \vee (1 - C_A(y, v))$$

$\Longrightarrow$

$$-C_A(xy^{-1}, mv^{-1}) \leq -1 + [(1 - C_A(x, m) \vee (1 - C_A(y, v))]$$

$\Longrightarrow$

$$C_A(xy^{-1}, mv^{-1}) \geq 1 - [(1 - C_A(x, m) \vee (1 - C_A(y, v))]$$

$\implies$

$$C_A(xy^{-1}, mv^{-1}) \geq C_A(x, m) \wedge C_A(y, v)$$

$\implies$

$$A \in CMG(X \times X).$$

$\square$

Proposition 2.7. *Let $A \in CAMG(X \times X)$. If $C_A(x, m) > C_A(y, v)$ for some*

$$(x, m), (y, v) \in X \times X,$$

then

$$C_A(xy, mv) = C_A(x, m) = C_A(yx, vm).$$

Proof. Suppose $C_A(x, m) > C_A(y, v)$ for some $(x, m), (y, v) \in X \times X$. Observe

$$C_A(xy, mv) \leq C_A(x, m) \vee C_A(y, v) = C_A(x, m).$$

Similarly

$$\begin{aligned} C_A(x, m) &= C_A(xyy^{-1}, mvv^{-1}) \\ &\leq C_A(xy, mv) \vee C_A(y, v) \\ &= C_A(xy, mv). \end{aligned}$$

Thus, $C_A(xy, mv) = C_A(x, m)$. In a similar way, we have $C_A(yx, vm) = C_A(x, m)$. Hence, the conclusion. $\square$

Proposition 2.8. *Let $A \in CAMG(X \times X)$. Then*

$$C_A(xy^{-1}, mv^{-1}) = C_A(e, e')$$

iff $C_A(x, m) = C_A(y, v)$.

Proof. Assume $C_A(xy^{-1}, mv^{-1}) = C_A(e, e')$ for all $(x, m), (y, v) \in X \times X$, where (e, e') is the identity of $X \times X$. Now observe we have the following

$$\begin{aligned}C_A(x,m) &= C_A(x(y^{-1}y), m(v^{-1}v)) \\ &= C_A((xy^{-1})y, (mv^{-1})v) \\ &\leq C_A(xy^{-1}, mv^{-1}) \vee C_A(y,v) \\ &= C_A(y,v).\end{aligned}$$

Also we have

$$\begin{aligned}C_A(y,v) &= C_A[(x^{-1}x)y^{-1}, (m^{-1}m)v^{-1}] \\ &= C_A[x^{-1}(xy^{-1}), m^{-1}(mv^{-1})] \\ &\leq C_A(x,m) \vee C_A(xy^{-1}, mv^{-1}) \\ &\leq C_A(x,m).\end{aligned}$$

Thus, $C_A(x,m) = C_A(y,m)$. For the converse, assume $C_A(x,m) = C_A(y,v)$ for all $(x,m),(y,v) \in X \times X$. Then we have

$$C_A(xy^{-1}, mv^{-1}) = C_A(yy^{-1}, mm^{-1})$$

$\Longrightarrow$

$$C_A(xy^{-1}, mv^{-1}) = C_A(e, e').$$

□

Proposition 2.9. *Let $A \in CAMG(X \times X)$. Then*

$$C_A(xy, mv) \leq C_A(y,v)$$

for all $(x,m),(y,v) \in X \times X$ iff $C_A(x,m) = C_A(e,e')$.

Proof. Suppose $C_A(xy, mv) = C_A(y,v)$ for all $(x,m),(y,v) \in X \times X$. By letting $y = e$ and $v = e'$, we obtain that

$$C_A(x,m) = C_A(e,e')$$

for all $(x, m) \in X \times X$. For the converse, suppose that $C_A(x, m) = C_A(e, e')$. Then $C_A(y, v) \geq C_A(x, m)$ and so

$$C_A(xy, mv) \leq C_A(x, m) \vee C_A(y, v) = C_A(y, v).$$

Also

$$\begin{aligned} C_A(y, v) &= C_A(x^{-1}xy, m^{-1}mv) \\ &\leq C_A(x, m) \vee C_A(xy, mv) \\ &= C_A(xy, mv). \end{aligned}$$

It follows that $C_A(xy, mv) = C_A(y, v)$ for all $(x, m), (y, v) \in X \times X$, finishing the proof. □

Theorem 2.10. *Let $A \in CAMG(X \times X)$. If $(x, m), (y, v) \in X \times X$ with $C_A(x, m) \neq C_A(y, v)$, then*

$$C_A(xy, mv) = C_A(yx, vm) = C_A(x, m) \vee C_A(y, v).$$

Proof. Let $(x, m), (y, v) \in X \times X$. Since $C_A(x, m) \neq C_A(y, v)$, $C_A(x, m) < C_A(y, v)$ or $C_A(y, v) < C_A(x, m)$. Suppose $C_A(x, m) < C_A(y, v)$, then $C_A(xy, mv) \leq C_A(y, v)$ and

$$\begin{aligned} C_A(y, v) &= C_A(x^{-1}xy, m^{-1}mv) \\ &\leq C_A(x^{-1}, m^{-1}) \vee C_A(xy, mv) \\ &= C_A(x, m) \vee C_A(xy, mv) \\ &= C_A(xy, mv). \end{aligned}$$

Thus

$$\begin{aligned} C_A(y, v) &\leq C_A(xy, mv) \\ &\leq C_A(x, m) \vee C_A(y, v) \\ &= C_A(y, v). \end{aligned}$$

From here, we have

$$C_A(xy, mv) \leq C_A(x, m) \vee C_A(y, v)$$

and

$$C_A(x, m) \vee C_A(y, v) \leq C_A(xy, mv)$$

which implies

$$C_A(xy, mv) = C_A(x, m) \vee C_A(y, v).$$

Similarly, suppose $C_A(y, v) < C_A(x, m)$, then $C_A(yx, vm) \leq C_A(x, m)$ and

$$\begin{aligned} C_A(x, m) &= C_A(y^{-1}yx, v^{-1}vm) \\ &\leq C_A(y^{-1}, v^{-1}) \vee C_A(yx, vm) \\ &= C_A(y, v) \vee C_A(yx, vm) \\ &= C_A(yx, vm). \end{aligned}$$

It now follows that

$$\begin{aligned} C_A(x, m) &\leq C_A(yx, vm) \\ &\leq C_A(y, v) \vee C_A(x, m) \\ &= C_A(x, m). \end{aligned}$$

Therefore we have

$$C_A(yx, vm) = CA(y, v) \vee C_A(x, m).$$

Hence the conclusion. □

Corollary 2.11. *If A is a coupled anti multigroup of $X \times X$, then*

$$C_A(xy, mv) = C_A(x, m) \vee C_A(y, v)$$

for all $(x, m), (y, v) \in X \times X$ with

$$C_A(x, m) \neq C_A(y, v).$$

Proof. Let $(x, m), (y, v) \in X \times X$. Assume that $C_A(x, m) < C_A(y, m)$. Then

$$C_A(xy, mv) \leq C_A(x, m) \vee C_A(y, v) = C_A(y, v)$$

for all $(x, m), (y, v) \in X \times X$. Similarly,

$$\begin{aligned} C_A(x, m) \vee C_A(y, v) &= C_A(x^{-1}xy, m^{-1}mv) \\ &\leq C_A(x^{-1}, m^{-1}) \vee C_A(xy, mv) \\ &= C_A(x, m) \vee C_A(xy, mv) \\ &= C_A(xy, mv). \end{aligned}$$

Hence

$$C_A(xy, mv) = C_A(x, m) \vee C_A(y, v).$$

□

Proposition 2.12. *Let A be a coupled multigroup of $X \times X$. Then for any $n \in \mathbb{N}$ such that $n \geq C_A(e, e')$, $\mathbb{A}_{[n]}$ is a subgroup of $X \times X$.*

Proof. For all $(x, m), (y, v) \in \mathbb{A}_{[n]}$, we have

$$C_A(xy^{-1}, mv^{-1}) \leq [C_A(x, m) \vee C_A(y, v)] \leq n.$$

Hence the result. □

Proposition 2.13. *Let A be a coupled multiset of $X \times X$ such that $\mathbb{A}_{[n]}$ is a subgroup of $X \times X$ for all $n \in \mathbb{N}$ with*

$$n \geq C_A(e, e').$$

Then A is a coupled anti multigroup of $X \times X$.

Proof. Let $(x, m), (y, v) \in X \times X$ and $C_A(x, m) = n_1$, $C_A(y, v) = n_2$. Suppose $n_2 \geq n_1$. Then $(x, m), (y, v) \in \mathbb{A}_{[n]}$, so that $(xy^{-1}, mv^{-1}) \in \mathbb{A}_{[n]}$. It follows that

$$C_A(xy^{-1}, mv^{-1}) \leq n_2 = n_1 \vee n_2 = C_A(x, m) \vee C_A(y, v).$$

Hence the result. □

3 Open Problem

We conjecture Theorem 3.14 of [9] can be proved in the setting of this paper. First we introduce the following

Definition 3.1. Let $A, B \in CMG(X \times X)$. If there exists $(y, v), (z, z') \in X \times X$ such that $x = yz$ and $m = vz'$, then the product $A \circ B$ of A and B will be defined by

$$C_{A\circ B}(x, m) = \bigvee_{x=yz, m=vz'} (C_A(y, v) \wedge C_B(z, z'))$$

otherwise

$$C_{A\circ B}(x, m) = 0.$$

In the setting of this paper, we claim Theorem 3.14 of [9] can be interpreted as follows

Conjecture 3.2. *Let X be a group. Suppose A and B are coupled multigroups of $X \times X$. Then*

(a) $A \subseteq A \circ B$, *if* $C_A(e, e') \leq C_B(e, e')$.

(b) $A \subseteq A \circ B$ *and* $B \subseteq A \circ B$, *if* $C_A(e, e') = C_B(e, e')$.

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