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Fuzzy Extension of Coupled Γ-semiring: Some Properties

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Abstract

The concept of coupled Γ-semiring first appeared in [1]. In the present paper assuming M is a Γ-semiring, we introduce concepts of anti-fuzzy prime ideal, anti-fuzzy semi prime ideal, and anti-fuzzy ideal extension, respectively, of $M \times M$. Some properties associated with these new concepts are obtained. The work in this paper takes inspiration from [2].

1. Introduction and Preliminaries

Definition 1.1. [2] A fuzzy subset μ of a Γ-semiring M is called an *anti-fuzzy left ideal* of M, if the following hold:

(a) $\mu(x+y) \leq \max\{\mu(x), \mu(y)\}$

(b) $\mu(x\alpha y) \leq \mu(y)$

for all $x, y \in M$, $\alpha \in \Gamma$.

Remark 1.2. [2] If we replace (b) in the above definition with $\mu(x\alpha y) \leq \mu(x)$, then we say the fuzzy subset μ of the Γ-semiring M is an *anti-fuzzy right ideal* of M.

Remark 1.3. [2] If μ is an anti-fuzzy left (right) ideal of a Γ-semiring M, then $\mu(0) \leq \mu(x)$ for all $x \in M$.

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Definition 1.4. [2] A fuzzy subset μ of a Γ-semiring M is called an *anti fuzzy ideal* of M, if μ is both an anti fuzzy left and anti-fuzzy right ideal of M.

Definition 1.5. [2] A subset A of a Γ-semiring M is a *left* (*right*) *ideal* of M, if A is an additive semigroup of M, and the set

$$M\Gamma A = \{x\alpha y \mid x \in M, \alpha \in \Gamma, y \in A\} \quad (A\Gamma M)$$

is contained in A. If A is both a left and right ideal of M, then A is an *ideal* of M.

Definition 1.6. [2] Let M be a Γ-semiring, and μ be an anti-fuzzy ideal of M. Then μ is an *anti-fuzzy prime ideal* of M, if $\mu(x) \leq \mu(x\alpha x)$ for all $x \in M$, and $\alpha \in \Gamma$.

Definition 1.7. [2] Let M be a Γ-semiring, and μ be a fuzzy subset of M. The fuzzy subset $\langle x, \mu\rangle : M \mapsto [0, 1]$ defined by

$$\langle x, \mu\rangle (y) = \sup_{\alpha\in\Gamma} \mu(x\alpha y)$$

for all $y \in M$ is called an *extension* of μ by x.

Definition 1.8. [2] Let M be a Γ-semiring, and μ be a fuzzy subset of M. Then μ is called an *anti fuzzy prime ideal* of M if

$$\mu(x\alpha y) = \min\{\mu(x), \mu(y)\}$$

for all $x, y \in M$, and $\alpha \in \Gamma$.

Definition 1.9. [2] Let M be a Γ-semiring, and μ be an anti fuzzy ideal of M. For any $t \in [0, 1]$, we define μ_t by the set

$$\{x \in M \mid \mu(x) \leq t\}$$

and call μ_t an *anti level subset*.

Definition 1.10. [2] Let M be a Γ-semiring, $A \subseteq M$, and $x \in M$. We define $\langle x, A\rangle$ by

$$\{y \in M \mid x\alpha y \in A, \text{ for all } \alpha \in \Gamma\}.$$

Definition 1.11. [2] Let M be a Γ-semiring, and μ be an anti-fuzzy ideal of M. We say μ is an *anti fuzzy semi prime ideal* if

$$\mu(x) \leq \mu(x\alpha x)$$

for all $x \in M$.

Definition 1.12. [2] Let M be a Γ-semiring. Then the fuzzy ideal μ of M is called an *anti fuzzy k-ideal* of M if

$$\mu(x) \leq \max\{\mu(x+y), \mu(y)\}$$

for all $x, y \in M$.

2. Main Results

Definition 2.1. Let M be a Γ-semiring. A fuzzy subset μ of $M \times M$ is called an *anti-fuzzy left ideal* of $M \times M$, if the following hold:

(a) $\mu(x+y, m+v) \leq \max\{\mu(x, m), \mu(y, v)\}$

(b) $\mu(x\alpha y, m\alpha v) \leq \mu(y, v)$

for all $(x, m), (y, v) \in M \times M$, $\alpha \in \Gamma$.

Remark 2.2. If we replace (b) in the above definition with $\mu(x\alpha y, m\alpha y) \leq \mu(x, m)$, then we say the fuzzy subset μ of $M \times M$ is an *anti-fuzzy right ideal* of $M \times M$.

Remark 2.3. If μ is an anti-fuzzy left (right) ideal of $M \times M$, then $\mu(0, 0) \leq \mu(x, m)$ for all $(x, m) \in M \times M$.

Definition 2.4. A fuzzy subset μ of $M \times M$ is called an *anti fuzzy ideal* of $M \times M$, if μ is both an anti fuzzy left and anti-fuzzy right ideal of $M \times M$.

Definition 2.5. Let M be a Γ-semiring, and define $M^* := M \times M$. A subset A of M^* is called a *left* (*right*) *ideal* of M^*, if A is an additive semigroup of M^*, and the set

$$M^*\Gamma A = \{(x\alpha y, m\alpha v) | (x, m) \in M^*, \alpha \in \Gamma, (y, v) \in A\} \quad (A\Gamma M^*)$$

is contained in A. If A is both a left and right ideal of M^*, then A is an *ideal* of M^*.

Theorem 2.6. *Let M be a Γ-semiring, and define $M^* := M \times M$. Suppose A is a nonempty subset of M^*, and define fuzzy subset μ in M^* by*

$$\mu(x, m) = \begin{cases} 0 & if \ (x, m) \in A, \\ 1 & if \ (x, m) \notin A. \end{cases}$$

Then μ *is an anti-fuzzy ideal of* M^* *if and only if A is an ideal of* M^*.

Proof. Suppose μ is an anti-fuzzy ideal of M^*. Let $(x, m), (y, v) \in A$. It follows that $\mu(x, m) = \mu(y, m) = 0$. Now observe that

$$\mu(x + y, m + v) \leq \max\{\mu(x, m), \mu(y, v)\} = 0.$$

Hence, $(x + y, z + m) \in A$. Now let $(x, m), (y, v) \in A$ and $\alpha \in \Gamma$, and observe that

$$\mu(x\alpha y, m\alpha v) \leq \min\{\mu(x, m), \mu(y, v)\} = 0.$$

Hence, $(x\alpha y, m\alpha v) \in A$. It now follows that A is an ideal of M^*. For the converse, let $(x, m), (y, v) \in A$, $\alpha \in \Gamma$, and A an ideal of M^*. We consider the following cases.

Case I. $(x, m) \in A$ and $(y, v) \in A$

In this case we know $\mu(x, m) = 0$ and $\mu(y, v) = 0$. Also since A is an ideal of M^*, we know $(x + y, z + m) \in A$ and $(x\alpha y, m\alpha v) \in A$, thus, $\mu(x + y, m + v) = 0$, and $\mu(x\alpha y, m\alpha v) = 0$. Since $\max\{\mu(x, m), \mu(y, v)\} = 0$, and, $\min\{\mu(x, m), \mu(y, v)\} = 0$, we have, $0 = \mu(x + y, m + v) = \max\{\mu(x, m), \mu(y, v)\} = 0$, and $0 = \mu(x\alpha y, m\alpha v) = \min\{\mu(x, m), \mu(y, v)\} = 0$, hence the conclusion.

Case II. $(x, m) \notin A$ and $(y, v) \notin A$

In this case we know $\mu(x, m) = 1$ and $\mu(y, v) = 1$. Also since A is an ideal of M^*, we know $(x + y, z + m) \in A$ and $(x\alpha y, m\alpha v) \in A$, thus, $\mu(x + y, m + v) = 0$, and $\mu(x\alpha y, m\alpha v) = 0$. Since $\max\{\mu(x, m), \mu(y, v)\} = 1$, and, $\min\{\mu(x, m), \mu(y, v)\} = 1$, we have, $0 = \mu(x + y, m + v) < \max\{\mu(x, m), \mu(y, v)\} = 1$, and

$$0 = \mu(x\alpha y, m\alpha v) < \min\{\mu(x, m), \mu(y, v)\} = 1$$

hence the conclusion.

Case III. $(x, m) \in A$ and $(y, v) \notin A$

In this case we know $\mu(x, m) = 0$ and $\mu(y, v) = 1$. Also since A is an ideal of M^*, we know $(x + y, z + m) \in A$ and $(x\alpha y, m\alpha v) \in A$, thus, $\mu(x + y, m + v) = 0$, and $\mu(x\alpha y, m\alpha v) = 0$. Since $\max\{\mu(x, m), \mu(y, v)\} = 1$, and, $\min\{\mu(x, m), \mu(y, v)\} = 0$, we have, $0 = \mu(x + y, m + v) < \max\{\mu(x, m), \mu(y, v)\} = 1$, and

$$0 = \mu(x\alpha y, m\alpha v) = \min\{\mu(x, m), \mu(y, v)\} = 0$$

hence the conclusion.

Case IV. $(x, m) \notin A$ and $(y, v) \in A$

In this case we know $\mu(x, m) = 1$ and $\mu(y, v) = 0$. Also since A is an ideal of M^*, we know $(x + y, z + m) \in A$ and $(x\alpha y, m\alpha v) \in A$, thus, $\mu(x + y, m + v) = 0$, and $\mu(x\alpha y, m\alpha v) = 0$. Since $\max\{\mu(x, m), \mu(y, v)\} = 1$, and, $\min\{\mu(x, m), \mu(y, v)\} = 0$, we have, $0 = \mu(x + y, m + v) < \max\{\mu(x, m), \mu(y, v)\} = 1$, and

$$0 = \mu(x\alpha y, m\alpha v) = \min\{\mu(x, m), \mu(y, v)\} = 0$$

hence the conclusion.

Definition 2.7. Let M be a Γ-semiring, and μ be an anti-fuzzy ideal of $M \times M$. We say μ is an *anti-fuzzy prime ideal* of $M \times M$, if $\mu(x, m) \leq \mu(x\alpha x, m\alpha m)$ for all $(x, m) \in M \times M$, and $\alpha \in \Gamma$.

Theorem 2.8. *Let M be a* Γ*-semiring, and* μ *be an anti-fuzzy ideal of* $M \times M$. *Then the following are equivalent*:

(a) μ *is an anti fuzzy semi prime ideal of* $M \times M$.

(b) $\mu(x, m) = \mu(x\alpha x, m\alpha m)$ *for all* $(x, m) \in M \times M$, *and* $\alpha \in \Gamma$.

Proof. ((b) ⇒ (a)) If (b) holds, then we know two inequalities are satisfied, of which one of them is, $\mu(x, m) \leq \mu(x\alpha x, m\alpha m)$ for all $(x, m) \in M \times M$, and $\alpha \in \Gamma$, hence, μ is an anti fuzzy semi prime ideal of $M \times M$.

((a) ⇒ (b)) If (a) holds, then we know the following inequality holds for all $(x, m) \in M \times M$ and $\alpha \in \Gamma$,

$$\mu(x, m) \leq \mu(x\alpha x, m\alpha m).$$

Since μ is an anti fuzzy ideal of $M \times M$, we may assume μ is an anti fuzzy right ideal of $M \times M$, then we know the following inequality holds for all $(x, m), (x, m) \in M \times M$ and $\alpha \in \Gamma$,

$$\mu(x\alpha x, m\alpha m) \leq \mu(x, m).$$

Thus combining the two inequalities above gives the conclusion.

Definition 2.9. Let M be a Γ-semiring, and μ be a fuzzy subset of $M \times M$. Then the fuzzy subset $\langle (x, m), \mu \rangle : M \times M \mapsto [0, 1]$ defined by

$$\langle (x, m), \mu \rangle (y, v) = \sup_{\alpha \in \Gamma} \mu(x\alpha y, m\alpha v)$$

for all $(y, v) \in M \times M$ is called an *extension* of μ by (x, m).

Theorem 2.10. *Let M be a* Γ*-semiring, and* μ *be an anti fuzzy right ideal of* $M \times M$*. Then* $\langle (x, m), \mu \rangle$ *is an anti fuzzy right ideal of* $M \times M$*.*

Proof. Let $(z, k), (y, v) \in M \times M$, and $\alpha \in \Gamma$. Now observe we have the following:

$$\begin{aligned}
\langle (x, m), \mu \rangle (y + z, v + k) &= \sup_{\alpha \in \Gamma} \mu(x\alpha(y + z), m\alpha(v + k)) \\
&= \sup_{\alpha \in \Gamma} \mu(x\alpha y + x\alpha z, m\alpha v, m\alpha k) \\
&\leq \sup_{\alpha \in \Gamma} \max\{\mu(x\alpha y, m\alpha v), \mu(x\alpha z, m\alpha k)\} \\
&= \max\{\sup_{\alpha \in \Gamma} \mu(x\alpha y, m\alpha v), \sup_{\alpha \in \Gamma} \mu(x\alpha z, m\alpha k)\} \\
&= \max\{\langle (x, m), \mu \rangle (y, v), \langle (x, m), \mu \rangle (z, k)\}
\end{aligned}$$

Also

$$\begin{aligned}
\langle (x, m), \mu \rangle (y\alpha z, v\alpha k) &= \sup_{\beta \in \Gamma} \mu(x\beta(y\alpha z), m\beta(v\alpha k)) \\
&= \sup_{\beta \in \Gamma} \mu((x\beta y)\alpha z, (m\beta v)\alpha k) \\
&\leq \sup_{\beta \in \Gamma} \mu(x\beta y, m\beta v)
\end{aligned}$$

$$= \langle (x, m), \mu\rangle (y, v).$$

Definition 2.11. Let M be a Γ-semiring, and μ be a fuzzy subset of $M \times M$. We say μ is an *anti fuzzy prime ideal* of $M \times M$ if

$$\mu(x\alpha y, m\alpha v) = \min\{\mu(x, m), \mu(y, v)\}$$

for all $(x, m), (y, v) \in M \times M$, and $\alpha \in \Gamma$.

Theorem 2.12. *Let M be a* Γ*-semiring, and* μ *be an anti fuzzy prime ideal of* $M \times M$, *and* $(x, m) \in M \times M$. *Then* $\langle (x, m), \mu\rangle$ *is an anti fuzzy prime ideal of* $M \times M$.

Proof. Let $(x, m), (y, v), (z, k) \in M \times M$, and $\beta \in \Gamma$. Now observe we have the following:

$$\begin{aligned}\langle (x, m), \mu\rangle (y\beta z, v\beta k) &= \sup_{\alpha\in\Gamma} \mu(x\alpha(y\beta z), m\alpha(v\beta k)) \\ &= \sup_{\alpha\in\Gamma} \min\{\mu(x, m), \mu(y\beta z, v\beta k)\} \\ &= \sup_{\alpha\in\Gamma} \min\{\mu(x, m), \min\{\mu(y, v), \mu(z, k)\}\} \\ &= \sup_{\alpha\in\Gamma} \min\{\min\{\mu(x, m), \mu(y, v)\}, \min\{\mu(x, m), \mu(z, k)\}\} \\ &= \sup_{\alpha\in\Gamma} \min\{\mu(x\alpha y, m\alpha v), \mu(x\alpha z, m\alpha k)\} \\ &= \min\{\sup_{\alpha\in\Gamma} \mu(x\alpha y, m\alpha v), \sup_{\alpha\in\Gamma} \mu(x\alpha z, m\alpha k)\} \\ &= \min\{\langle (x, m), \mu\rangle (y, v), \langle (x, m), \mu\rangle (z, k)\}.\end{aligned}$$

Hence the theorem.

Theorem 2.13. *Let M be a commutative* Γ*-semiring, and* μ *be a fuzzy subset of* $M \times M$. *Suppose* $(x, m) \in M \times M$ *such that the extension* $\langle (x, m), \mu\rangle = \mu$ *for every* $(x, m) \in M \times M$. *Then* μ *is a constant function.*

Proof. Let M be a commutative Γ-semiring, and μ be a fuzzy subset of $M \times M$. Suppose $(x, m), (y, v) \in M \times M$, and $\alpha \in \Gamma$. Now observe we have the following:

$$\mu(x, m) = \langle (y, v), \mu\rangle (x, m)$$

$$= \sup_{\alpha\in\Gamma} \mu(y\alpha x, v\alpha m)$$

$$= \sup_{\alpha\in\Gamma} \mu(x\alpha y, m\alpha v)$$

$$= \langle (x, m), \mu\rangle (y, v)$$

$$= \mu(y, v).$$

Hence, $\mu(x, m) = \mu(y, v)$, and the theorem follows. □

Definition 2.14. Let M be a Γ-semiring, and μ be an anti fuzzy ideal of $M \times M$. For any $t \in [0, 1]$, we define μ_t by the set

$$\{(x, m) \in M \times M \mid \mu(x, m) \leq t\}$$

and call μ_t an *anti level subset*.

Definition 2.15. Let M be a Γ-semiring, $A \subseteq M \times M$, and $(x, m) \in M \times M$. We define $\langle (x, m), A\rangle$ by

$$\{(y, v) \in M \times M \mid (x\alpha y, m\alpha v) \in A, \text{ for all } \alpha \in \Gamma\}.$$

Theorem 2.16. *Let M be a commutative Γ-semiring, and μ be a fuzzy subset of* $M \times M$. *Then for every* $t \in \operatorname{Im}(\mu)$, $\langle (x, m), \mu_t\rangle = \langle (x, m), \mu\rangle_t$ *for every* $(x, m) \in M \times M$.

Proof. Observe we have the following

$$(y, v) \in \langle (x, m), \mu\rangle_t$$

$$\Leftrightarrow$$

$$\langle (x, m), \mu\rangle (y, v) \leq t$$

$$\Leftrightarrow$$

$$\sup_{\alpha\in\Gamma} \mu(x\alpha y, m\alpha v)$$

$$\Leftrightarrow$$

$$\mu(x\alpha y,\ m\alpha v) \le t$$

$$\Leftrightarrow$$

$$(x\alpha y,\ m\alpha v) \in \mu_t$$

$$\Leftrightarrow$$

$(y,\ v) \in \langle (x,\ m),\ \mu_t \rangle$ by Definition 2.15.

Hence the theorem. □

Note that Theorem 2.10 holds, if μ is an anti fuzzy left ideal of $M \times M$, thus the following is immediate

Corollary 2.17. *Let M be a commutative* Γ-*semiring and* μ *be an anti fuzzy ideal of* $M \times M$, *and* $(x,\ m) \in M \times M$. *Then the extension*

$$\langle (x,\ m),\ \mu \rangle$$

is an anti fuzzy ideal of $M \times M$.

Definition 2.18. Let *M* be a Γ-semiring, and μ be an anti-fuzzy ideal of $M \times M$. We say μ is an *anti fuzzy semi prime ideal* if

$$\mu(x,\ m) \le \mu(x\alpha x,\ m\alpha m)$$

for all $(x,\ m) \in M \times M$.

Theorem 2.19. *Let M be a commutative* Γ-*semiring,* μ *be an anti-fuzzy semi prime ideal of* $M \times M$, *and* $(x,\ m) \in M \times M$. *Then* $\langle (x,\ m),\ \mu \rangle$ *is an anti-fuzzy semi prime ideal of* $M \times M$.

Proof. Let *M* be a commutative Γ-semiring, μ be an anti-fuzzy semi prime ideal of $M \times M$, $(x,\ m), (y,\ v) \in M \times M$ and $\beta \in \Gamma$. By Corollary 2.17, the extension $\langle (x,\ m),\ \mu \rangle$ is an anti-fuzzy ideal of $M \times M$. Now observe we have the following:

$$\langle (x,\ m),\ \mu \rangle (y,\ v) = \sup_{\alpha \in \Gamma} \mu(x\alpha y,\ m\alpha v)$$

$$\le \sup_{\alpha \in \Gamma} \mu(x\alpha y\beta x\alpha y,\ m\alpha v\beta m\alpha v)$$

$$= \sup_{\alpha \in \Gamma} \mu(x\alpha y\beta y\alpha x,\ m\alpha v\beta v\alpha m)$$

$$\leq \sup_{\alpha\in\Gamma} \mu(x\alpha y\beta y,\ m\alpha v\beta v)$$

$$= \langle (x, m), \mu\rangle (y\beta y,\ v\beta v).$$

Hence the theorem. □

Definition 2.20. Let M be a Γ-semiring. We say the fuzzy ideal μ of $M \times M$ is an *anti fuzzy k-ideal* of $M \times M$ if

$$\mu(x, m) \leq \max\{\mu(x + y,\ m + v),\ \mu(y, v)\}$$

for all $(x, m), (y, v) \in M \times M$.

Theorem 2.21. *Let M be a commutative Γ-semiring, μ be an anti fuzzy k-ideal of $M \times M$, and $(z, k) \in M \times M$. Then $\langle (z, k), \mu\rangle$ is an anti fuzzy k-ideal of $M \times M$.*

Proof. Let M be a commutative Γ-semiring, μ be an anti fuzzy k-ideal of $M \times M$, and $(x, m), (y, v), (z, k) \in M \times M$, and $\alpha \in \Gamma$. Based on the assumption, it follows from Corollary 2.17 that the extension $\langle (z, k), \mu\rangle$ is an anti fuzzy ideal of $M \times M$. Now we know the following for all $(x, m), (y, v) \in M \times M$

$$\mu(x, m) \leq \max\{\mu(x + y,\ m + v),\ \mu(y, v)\}.$$

Thus for all $(x, m), (y, v) \in M \times M$, and $\alpha \in \Gamma$, we deduce the following:

$$\mu(z\alpha x,\ k\alpha m) \leq \max\{\mu(z\alpha x + z\alpha y,\ k\alpha m + k\alpha v),\ \mu(z\alpha y + k\alpha v)\}.$$

Now taking sup on both sides of the above inequality over all $\alpha \in \Gamma$, we deduce the following:

$$\sup_{\alpha\in\Gamma} \mu(z\alpha x,\ k\alpha m) \leq \max\{\sup_{\alpha\in\Gamma} \mu(z\alpha x + z\alpha y,\ k\alpha m + k\alpha v),\ \sup_{\alpha\in\Gamma} \mu(z\alpha y + k\alpha v)\}.$$

By Definition 2.9, the above inequality translates as

$$\langle (z, k), \mu\rangle (x, m) \leq \max\{\langle (z, k), \mu\rangle (x + y,\ m + v),\ \langle (z, k), \mu\rangle (y, v)\}.$$

Hence the theorem. □

Theorem 2.22. *Let M be a commutative Γ-semiring, and μ be an anti fuzzy ideal of $M \times M$. If for $(y, v) \in M \times M$, $\mu(y, v)$ is not minimal in $\mu(M \times M)$, and $\langle (x, m), \mu\rangle = \mu$, then μ is an anti-fuzzy prime ideal of $M \times M$.*

Proof. Let $(a, a'), (b, b') \in M \times M$, and $\alpha \in \Gamma$. We know

$$\sup_{\alpha \in \Gamma} \mu(a\alpha b, a'\alpha b') = \langle (a, a'), \mu \rangle (b, b') = \mu(a, a')$$

and

$$\sup_{\alpha \in \Gamma} \mu(b\alpha a, b'\alpha a') = \langle (b, b'), \mu \rangle (a, a') = \mu(b, b').$$

We also know $\mu(a\alpha b, a'\alpha b') \leq \mu(a, a')$ and $\mu(a\alpha b, a'\alpha b') \leq \mu(b, b')$.

Case I. $\mu(a, a')$ or $\mu(b, b')$ is not minimal in $\mu(M \times M)$

If $\mu(a, a')$ is not minimal, then since

$$\sup_{\alpha \in \Gamma} \mu(b\alpha a, b'\alpha a') = \langle (b, b'), \mu \rangle (a, a') = \mu(b, b')$$

and M is a commutative Γ-semiring, then

$$\mu(a\alpha b, a'\alpha b') = \mu(b\alpha a, b'\alpha a') \geq \sup_{\alpha \in \Gamma} \mu(b\alpha a, b'\alpha a')$$

$$= \langle (b, b'), \mu \rangle (a, a') = \mu(b, b')$$

and since we know $\mu(a\alpha b, a'\alpha b') \leq \mu(b, b')$, hence,

$$\mu(a\alpha b, a'\alpha b') = \mu(b, b') = \min\{\mu(b, b'), \mu(a, a')\}.$$

If $\mu(b, b')$ is not minimal, then in a similar way we can conclude

$$\mu(a\alpha b, a'\alpha b') = \mu(a, a') = \min\{\mu(b, b'), \mu(a, a')\}.$$

Hence the theorem.

Case II. Neither $\mu(a, a')$ nor $\mu(b, b')$ is minimal in $\mu(M \times M)$

Since we know $\mu(a\alpha b, a'\alpha b') \leq \mu(a, a')$ and $\mu(a\alpha b, a'\alpha b') \leq \mu(b, b')$, it follows that

$$\mu(a\alpha b, a'\alpha b') \leq \min\{\mu(a, a'), \mu(b, b')\}.$$

Now from

$$\sup_{\alpha \in \Gamma} \mu(a\alpha b, a'\alpha b') = \sup_{\alpha \in \Gamma} \mu(b\alpha a, b'\alpha a') = \langle (b, b'), \mu \rangle (a, a') = \mu(b, b')$$

and

$$\sup_{\alpha\in\Gamma} \mu(a\alpha b,\ a'\alpha b') = \langle (a,\ a'),\ \mu\rangle (b,\ b') = \mu(a,\ a')$$

we deduce that

$$\mu(a\alpha b,\ a'\alpha b') \geq \mu(a,\ a') \text{ and } \mu(a\alpha b,\ a'\alpha b') \geq \mu(b,\ b')$$

hence

$$\mu(a\alpha b,\ a'\alpha b') \geq \min\{\mu(a,\ a'),\ \mu(b,\ b')\}.$$

Consequently, we have

$$\mu(a\alpha b,\ a'\alpha b') = \min\{\mu(b,\ b'),\ \mu(a,\ a')\}$$

hence the theorem

3. Concluding Remark and Further Direction

Given M is a Γ-semiring, we have introduced concepts of anti-fuzzy prime ideal, anti-fuzzy semi prime ideal, and anti-fuzzy ideal extension, respectively, of $M \times M$; some properties associated with these new concepts have been obtained.

A future interesting problem is introduce some concepts of triple Γ-semiring $(M \times M \times M)$, and study some of their properties.

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