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Some Properties for Strong Differential Subordination of Analytic Functions Associated with Wanas Operator

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Abstract

In this paper, by making use of Wanas operator, we derive some properties related to the strong differential subordinations of analytic functions defined in the open unit disk and closed unit disk of the complex plane.

1. Introduction

Indicate by $\mathcal{H}(U \times \overline{U})$ the family of all analytic functions in $U \times \overline{U}$. Let $U = \{z \in \mathbb{C} : |z| < 1\}$ and $\overline{U} = \{z \in \mathbb{C} : |z| \leq 1\}$ denote the open unit disk and the closed unit disk of the complex plane, respectively. For $n \in \mathbb{N} = \{1, 2, ...\}$ and $a \in \mathbb{C}$, let

$$\mathcal{H}^*[a, n, \zeta] = \{f \in \mathcal{H}(U \times \overline{U}) : f(z, \zeta) = a + a_n(\zeta) z^n + a_{n+1}(\zeta) z^{n+1} + \cdots, \quad z \in U, \zeta \in \overline{U}\},$$

where $a_k(\zeta)$ are holomorphic functions in $\overline{U}$ for $k \geq n$.

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Also, let $\mathcal{A}_{n\zeta}^{*} = \{f \in \mathcal{H}(U \times \overline{U}) : f(z, \zeta) = z + a_{n+1}(\zeta)z^{n+1} + \cdots,\ z \in U,\ \zeta \in \overline{U}\}$, where $a_k(\zeta)$ are holomorphic functions in $\overline{U}$ for $k \geq n+1$.

A function $f \in \mathcal{H}^*[a, n, \zeta]$ is said to be *starlike* in $U \times \overline{U}$ if

$$\operatorname{Re}\left\{\frac{zf'_z(z, \zeta)}{f(z, \zeta)}\right\} > 0, \quad (z \in U,\ \zeta \in \overline{U}).$$

Denote the class of all starlike functions in $U \times \overline{U}$ by S_{ζ}^{*}.

Similar, $f \in \mathcal{H}^*[a, n, \zeta]$ is said to be *convex* in $U \times \overline{U}$ if

$$\operatorname{Re}\left\{\frac{zf''_{z^2}(z, \zeta)}{f'_z(z, \zeta)} + 1\right\} > 0, \quad (z \in U,\ \zeta \in \overline{U}).$$

Denote the class of all convex functions in $U \times \overline{U}$ by K_{ζ}^{*}.

Definition 1.1 [9]**.** Let $f(z, \zeta), g(z, \zeta)$ be analytic in $U \times \overline{U}$. The function $f(z, \zeta)$ is said to be *strongly subordinate* to $g(z, \zeta)$, written $f(z, \zeta) \prec\prec F(z, \zeta)$, $z \in U$, $\zeta \in \overline{U}$, if there exists an analytic function *w* in *U* with $w(0) = 0$ and $|w(z)| < 1,\ z \in U$ such that $f(z, \zeta) = g(w(z), \zeta)$ for all $\zeta \in \overline{U}$.

Remark 1.1 [9]**.**

(1) Since $f(z, \zeta)$ is analytic in $U \times \overline{U}$, for all $\zeta \in \overline{U}$ and univalent in *U*, for all $\zeta \in \overline{U}$, Definition 1.1 is equivalent to $f(0, \zeta) = g(0, \zeta)$ for all $\zeta \in \overline{U}$ and $f(U \times \overline{U}) \subset g(U \times \overline{U})$.

(2) If $f(z, \zeta) = f(z)$ and $g(z, \zeta) = g(z)$, then the strong subordination becomes the usual notion of subordination.

Let $\mathcal{A}_{\zeta}^{*}$ denote the subclass of the functions $f(z, \zeta) \in \mathcal{H}(U \times \overline{U})$ of the form:

$$f(z, \zeta) = z + \sum_{k=2}^{\infty} a_k(\zeta)z^k, \quad z \in U,\ \zeta \in \overline{U} \tag{1.1}$$

which are analytic and univalent in $U \times \overline{U}$.

For $\alpha \in \mathbb{R}$, $\beta \geq 0$ with $\alpha + \beta > 0$, $l \in \mathbb{N}$, $m, \lambda \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and $f \in \mathcal{A}$, the Wanas operator $W_{\alpha,\beta}^{l,\lambda} : \mathcal{A}_\zeta^* \to \mathcal{A}_\zeta^*$, (see [12]) is defined by

$$W_{\alpha,\beta}^{l,\lambda} f(z, \zeta) = z + \sum_{k=2}^{\infty}\left[\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\frac{\alpha^m + n\beta^m}{\alpha^m + \beta^m}\right)\right]^{\lambda} a_k(\zeta) z^n. \tag{1.2}$$

It is easily verified from (1.2) that

$$z(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))' = \left[\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^m + 1\right)\right] W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)$$

$$-\left[\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\frac{\alpha}{\beta}\right)^m\right] W_{\alpha,\beta}^{l,\lambda} f(z, \zeta). \tag{1.3}$$

Some of the special cases of the operator defined by (1.2) can be found in [1, 2, 4, 10, 11].

In recent years, many authors obtained various interesting results associated with strong differential subordination and superordination for example (see [3, 5, 6, 13, 14]).

In order to derive our main results, we need the following lemmas.

Lemma 1.1 [8]**.** *Let* $h(z, \zeta)$ *be a convex function with* $h(0, \zeta) = a$, *for every* $\zeta \in \overline{U}$ *and let* $\gamma \in \mathbb{C}^* = \mathbb{C}\backslash\{0\}$ *with* $\operatorname{Re}(\gamma) \geq 0$. *If* $p \in \mathcal{H}^*[a, n, \zeta]$ *and*

$$p(z, \zeta) + \frac{1}{\gamma} z p_z'(z, \zeta) \prec\prec h(z, \zeta), \quad (z \in U, \zeta \in \overline{U}), \tag{1.4}$$

then

$$p(z, \zeta) \prec\prec q(z, \zeta) \prec\prec h(z, \zeta), \quad (z \in U, \zeta \in \overline{U}),$$

where $q(z, \zeta) = \dfrac{\gamma}{nz^{\frac{\gamma}{n}}} \int_0^z t^{\frac{\gamma}{n}-1} h(t, \zeta)\, dt$ *is convex and it is the best dominant of* (1.4).

Lemma 1.2 [7]**.** *Let* $q(z, \zeta)$ *be a convex function in* $U \times \overline{U}$ *for all* $\zeta \in \overline{U}$ *and let* $h(z, \zeta) = q(z, \zeta) + n\delta z q_z'(z, \zeta)$, $z \in U$, $\zeta \in \overline{U}$, *where* $\delta > 0$ *and n is a positive integer.*

If

$$p(z, \zeta) = q(0, \zeta) + p_n(\zeta) z^n + p_{n+1}(\zeta) z^{n+1} + \cdots,$$

is analytic in $U \times \overline{U}$ *and*

$$p(z, \zeta) + \delta z p'_z(z, \zeta) \prec\prec h(z, \zeta), \quad (z \in U, \zeta \in \overline{U}),$$

then

$$p(z, \zeta) \prec\prec q(z, \zeta), \quad (z \in U, \zeta \in \overline{U}),$$

and this result is sharp.

2. Main Results

Theorem 2.1. *Let* $h(z, \zeta)$ *be a convex function such that* $h(0, \zeta) = 1$. *If* $f \in \mathcal{A}_\zeta^*$ *satisfies the strong differential subordination*:

$$(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z \prec\prec h(z, \zeta), \tag{2.1}$$

then

$$\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} \prec\prec q(z, \zeta) \prec\prec h(z, \zeta),$$

where $q(z, \zeta) = \frac{1}{z} \int_0^z h(t, \zeta) dt$ *is convex and it is the best dominant.*

Proof. Suppose that

$$p(z, \zeta) = \frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z}, \quad z \in U, \zeta \in \overline{U}. \tag{2.2}$$

Then the function $p(z, \zeta)$ is analytic in $U \times \overline{U}$ and $p(0, \zeta) = 1$.

Simple computations from (2.2), we get

$$p(z, \zeta) + z p'_z(z, \zeta) = (W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z. \tag{2.3}$$

Using (2.3), (2.1) becomes

$$p(z, \zeta) + zp'_z(z, \zeta) \prec\prec h(z, \zeta).$$

An application of Lemma 1.1 with $n = 1, \gamma = 1$ yields

$$\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} \prec\prec q(z, \zeta) = \frac{1}{z}\int_0^z h(t, \zeta)dt \prec\prec h(z, \zeta).$$

By taking $h(z, \zeta) = \dfrac{\zeta + (2\rho - \zeta)z}{1 + z}$, $0 \le \rho < 1$ in Theorem 2.1, we obtain the following corollary:

Corollary 2.1. *If* $f \in \mathcal{A}_\zeta^*$ *satisfies the strong differential subordination*:

$$(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z \prec\prec \frac{\zeta + (2\rho - \zeta)z}{1 + z},$$

then

$$\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} \prec\prec \frac{1}{z}\int_0^z \frac{\zeta + (2\rho - \zeta)t}{1 + t}dt = 2\rho - \zeta + \frac{2(\zeta - \rho)}{z}\ln(1 + z).$$

Theorem 2.2. *Let* $h(z, \zeta)$ *be a convex function such that* $h(0, \zeta) = 1$. *If* $0 \le \sigma < p$, $\theta \in \mathbb{C}$ *and* $f \in \mathcal{A}_\zeta^*$ *satisfies the strong differential subordination*:

$$\frac{1 - \theta}{1 - \sigma}\left(\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} - \sigma\right) + \frac{\theta}{1 - \sigma}((W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z - \sigma) \prec\prec h(z, \zeta), \qquad (2.4)$$

then

$$\frac{1}{1 - \sigma}\left(\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} - \sigma\right) \prec\prec q(z, \zeta) \prec\prec h(z, \zeta),$$

where $q(z, \zeta) = \dfrac{1}{\theta} z^{-\frac{1}{\theta}} \int_0^z t^{\frac{1}{\theta} - 1} h(t, \zeta)dt$ *is convex and it is the best dominant*.

Proof. Suppose that

$$p(z, \zeta) = \frac{1}{1-\sigma}\left(\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} - \sigma\right), \quad z \in U, \zeta \in \overline{U}. \tag{2.5}$$

Then the function $p(z, \zeta)$ is analytic in $U \times \overline{U}$ and $p(0, \zeta) = 1$.

Differentiating both sides of (2.5) with respect to z, we have

$$p(z, \zeta) + \theta z p_z'(z, \zeta) = \frac{1-\theta}{1-\sigma}\left(\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} - \sigma\right) + \frac{\theta}{1-\sigma}\left(\left(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)\right)_z' - \sigma\right). \tag{2.6}$$

From (2.4) and (2.6), we get

$$p(z, \zeta) + \theta z p_z'(z, \zeta) \prec\prec h(z, \zeta).$$

An application of Lemma 1.1 with $n = 1$, $\gamma = \frac{1}{\theta}$ yields

$$\frac{1}{1-\sigma}\left(\frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}{z} - \sigma\right) \prec\prec q(z, \zeta) = \frac{1}{\theta} z^{-\frac{1}{\theta}} \int_0^z t^{\frac{1}{\theta}-1} h(t, \zeta)\, dt \prec\prec h(z, \zeta).$$

Theorem 2.3. *Let* $q(z, \zeta)$ *be a convex function such that* $q(0, \zeta) = 1$ *and let* h *be the function* $h(z, \zeta) = q(z, \zeta) + z q_z'(z, \zeta)$. *If* $f \in \mathcal{A}_\zeta^*$ *satisfies the strong differential subordination*:

$$\left(\frac{z W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}\right)_z' \prec\prec h(z, \zeta), \tag{2.7}$$

then

$$\frac{W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)} \prec\prec q(z, \zeta).$$

Proof. Suppose that

$$p(z, \zeta) = \frac{W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}, \quad z \in U, \ \zeta \in \overline{U}. \tag{2.8}$$

Then the function $p(z, \zeta)$ is analytic in $U \times \overline{U}$ and $p(0, \zeta) = 1$.

Differentiating both sides of (2.8) with respect to z and using (2.7), we have

$$p(z, \zeta) + zp'_z(z, \zeta)$$

$$= \frac{W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)}$$

$$+ \frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)(W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta))'_z - W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z}{[W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)]^2}$$

$$= \frac{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)(zW_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta))'_z - zW_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)(W_{\alpha,\beta}^{l,\lambda} f(z, \zeta))'_z}{[W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)]^2}$$

$$= \left(\frac{zW_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)} \right)'_z \prec\prec h(z, \zeta).$$

An application of Lemma 1.2, we obtain

$$\frac{W_{\alpha,\beta}^{l,\lambda+1} f(z, \zeta)}{W_{\alpha,\beta}^{l,\lambda} f(z, \zeta)} \prec\prec q(z, \zeta).$$

Theorem 2.4. *Let $q(z, \zeta)$ be a convex function such that $q(0, \zeta) = 1$ and let h be the function*

$$h(z, \zeta) = q(z, \zeta) + \frac{1}{1 + \sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right)} zq'_z(z, \zeta),$$

where $\sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) > 0$. *Suppose that*

$$F(z, \zeta) = \frac{1 + \sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right)}{z^{\sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right)}}$$

$$\times \int_0^z t^{\sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) - 1} f(t, \zeta) dt. \quad (2.9)$$

If $f \in \mathcal{A}_\zeta^*$ *satisfies the strong differential subordination*

$$(W_{\alpha, \beta}^{l, \lambda} f(z, \zeta))_z' \prec\prec h(z, \zeta), \quad (2.10)$$

then

$$(W_{\alpha, \beta}^{l, \lambda} F(z, \zeta))_z' \prec\prec q(z, \zeta).$$

Proof. Suppose that

$$p(z, \zeta) = (W_{\alpha, \beta}^{l, \lambda} F(z, \zeta))_z', \quad z \in U, \zeta \in \overline{U}. \quad (2.11)$$

Then the function $p(z, \zeta)$ is analytic in $U \times \overline{U}$ and $p(0, \zeta) = 1$.

From (2.9), we have

$$z^{\sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right)} F(z, \zeta)$$

$$= \left(1 + \sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) \right) \int_0^z t^{\sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) - 1} f(t, \zeta) dt. \quad (2.12)$$

Differentiating both sides of (2.12) with respect to z, we get

$$\left(1 + \sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) \right) f(z, \zeta)$$

$$= \sum_{m=1}^{l} \binom{l}{m} (-1)^{m+1} \left(\left(\frac{\alpha}{\beta} \right)^m + 1 \right) F(z, \zeta) + z F_z'(z, \zeta)$$

and

$$\left(1+\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)\right)W_{\alpha,\beta}^{l,\lambda}f(z,\zeta)$$

$$=\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)W_{\alpha,\beta}^{l,\lambda}F(z,\zeta)+z(W_{\alpha,\beta}^{l,\lambda}F(z,\zeta))_{z}'.$$

So

$$(W_{\alpha,\beta}^{l,\lambda}f(z,\zeta))_{z}'=(W_{\alpha,\beta}^{l,\lambda}F(z,\zeta))_{z}'+\frac{z(W_{\alpha,\beta}^{l,\lambda}F(z,\zeta))_{z^2}''}{1+\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)}. \qquad (2.13)$$

From (2.11) and (2.13), we obtain

$$p(z,\zeta)=\frac{1}{1+\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)}zp_{z}'(z,\zeta)=(W_{\alpha,\beta}^{l,\lambda}f(z,\zeta))_{z}'. \qquad (2.14)$$

Using (2.14), (2.10) becomes

$$p(z,\zeta)=\frac{1}{1+\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)}zp_{z}'(z,\zeta)$$

$$\prec q(z,\zeta)+\frac{1}{1+\sum_{m=1}^{l}\binom{l}{m}(-1)^{m+1}\left(\left(\frac{\alpha}{\beta}\right)^{m}+1\right)}zq_{z}'(z,\zeta).$$

An application of Lemma 1.2 yields $p(z,\zeta)\prec\prec q(z,\zeta)$. By using (2.10), we obtain

$$(W_{\alpha,\beta}^{l,\lambda}F(z,\zeta))_{z}'\prec\prec q(z,\zeta).$$

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