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A Semi-Markov Model for the Analysis of Crude Oil Spills in Niger Delta Region, Nigeria

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Abstract

This paper developed a three-state Semi-Markov model with Weibull waiting time distribution to analyse crude oil spill occurrences in the Niger Delta region of Nigeria. Oil spills were categorized into minor, medium, and major categories based on the volume of spill. An empirical data from the National Oil Spill Detection and Response Agency (NOSDRA), Nigeria from January, 2006 to March, 2020 with particular consideration of crude oil spill incidences due to CHEVRON Nigeria Ltd and Nigeria Agip Oil Company (NAOC) was used in the analysis. In the case of NAOC, the stationary distribution of the Semi-Markov process was obtained as 0.6954, 0.2461 and 0.0585 for minor, medium and major oil spills respectively, with corresponding mean waiting times before transition of 2.5438, 2.7429 and 3.6204 days. For CHEVRON Nigeria Ltd, the stationary distribution of the Semi-Markov process was found to be 0.9373, 0.0393 and 0.0234 for minor, medium and major oil spills respectively with corresponding mean waiting times of 6.4609, 6.0408 and 8.7861 days. Further results reveal that crude oil spills due to NAOC occurred once in every 3 days while that of CHEVRON Nigeria Ltd was once in every 6 days. The study recommends further investigation on the associated risk factors to oil spills of all categories in the Niger Delta region.

1 Introduction

The oil and gas sector accounted for over 80% of Nigeria's total export revenue in the first quarter of 2024 [1]. The Niger Delta region possesses Africa's biggest crude oil reserve and hence it is the main crude oil producer in Nigeria [2]. Crude oil exploration in the Niger Delta has been characterized by incessant spills [3]. According to Akinpelu [4], 881 oil spill incidences were recorded in 12 States of Nigeria from January 2019 to April 2021 with a corresponding volume of 42,565.42 barrels. Out of this, the Niger Delta Region accounted for 90.12% of the oil spill incidences with a total volume of 40,618.14 barrels. An

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earlier study by Udoumoh et al. [5] revealed that a total of 6,288 oil spill incidences, which amounted to 211,377.17 barrels were recorded between January 2006 and May 2014 in the Niger Delta Region of Nigeria. Another independent assessment conducted by the United Nations Environment Programme (UNEP) on the environmental and public health impacts of crude oil contamination in Ogoni land presented 67 contaminated oil spill sites [6]. Suffice it to say that oil spills and their attendant effects on the ecosystem is a common theme which characterize the Niger Delta region of Nigeria.

This development has generated several researches in the field of environmental studies [7–14]. While some studies have provided information on incidences and causes of oil spills, others have handled the economic and environmental impacts assessment of oil spills as well as security implications [15–19]. Wizor and Wali [20] stated that less than 2% of oil spills are due to production failure/crude transfer operations, while 28% of oil spills are due to sabotage/oil theft. Reliability theory has been adopted by some researchers to analyze crude oil spills in the Niger Delta and also to identify risk factors associated with it [5, 21]. In Udoumoh [22], crude oil spills were modelled as a renewal process where the event of oil spills were considered to be a counting process. Using the data from CHEVRON Nigeria Ltd with the assumption of exponential inter-event time distribution, the mean time to spill was estimated to be 6 days. Mba et al. [23] used a two-state Markov model to predict the occurrence of oil spills in the Niger Delta Region. The model simply classified the states of the Markov chain as "spills" and "no-spills", ignoring the volume of spills and also the holding time distribution before each transition. Such a model is no doubt limited in effectiveness, information and application. It is believed that a model which further classifies oil spills based on volume of spills, taking into consideration the varying length of time spent on each state before transition would provide a wealth of resource for a better understanding and analysis of crude oil spill scenario. In general, Markov models provide robust approach for the analysis of multi-state stochastic processes [24–27].

This paper therefore presents a Semi-Markov model for the analysis of crude oil spills using volume of spills for state classification. The model presents a descriptive as well as an analytical procedure for a robust understanding of oil spill incidences in selected oil exploration companies in Niger Delta, Nigeria.

2 Methods

2.1 Model Assumptions and Notations

Assumptions

The following assumptions hold for the model:

1. The events of crude oil spills are random, transiting between the three states of the process.

2. The increasing and decreasing properties of a future transition of oil spills depend only on the last transition and the waiting time distribution.

3. The time between two successive transitions is termed the waiting time and is measured in days.

4. The Semi-Markov chain is irreducible.

Notations

$N =$ number of states in the Semi-Markov process (SMP).

$Z_t =$ integer value that denotes the state of the SMP at time t.

$F_{ij}(t) =$ CDF of the waiting time distribution from state i to state j.

$f_{ij}(t) =$ PDF of the waiting time distribution from state i to state j.

$f_i(t) =$ CDF of the waiting time of the process in state i before making any transition at time t.

$X_t =$ a stochastic process at time t.

$P_{ij} =$ probability that the next state in the process is j given that the previous state was i $(i, j = 1, 2, 3)$.

$n_{ij} =$ number of spills in state j at time $t+1$ given that the process was in state i at time t.

$T_{ij} =$ the time that the process takes in state i before making a transition to state j, and is called the holding time in state i.

$h_{ij}(.) =$ the Holding time distribution function for a transition from state i to state j.

$F_i(t) =$ the cumulative probability distribution of a random variable T_i, where T_i is the time being spent in state i irrespective of the successor state.

$\phi_{ij}(t) =$ probability that the process that start in initial state i at instant $t = 0$ will be in state j at instant t.

2.2 The Semi-Markov Trajectory

The Semi-Markov process considers the transition probabilities and the period of stay (holding time) in a state before making a transition to the next state. Figure 1 presents a portion of a possible Semi-Markov trajectory of the process. Figure 1 shows a process which enters state B at time 0, remained in state B for a time period of 2 days before transiting to state A. It again remained in state A for 1 day and then transits to state C and was held for a time period of 2 days as at the last observation. We shall start model implementation with the transition probability matrix which describes the transition of the spills among the three states (See Section 2.3).

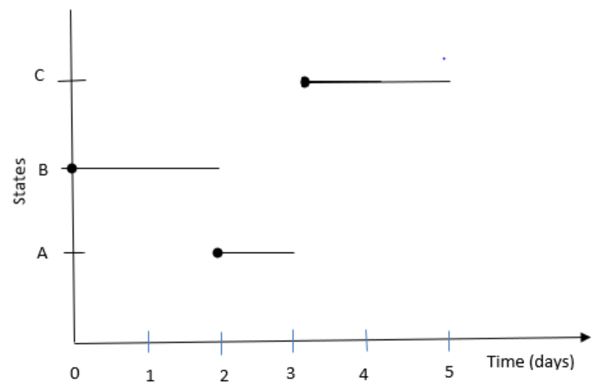

Figure 1: A Possible Semi-Markov Trajectory.

2.3 Markov Chain and Transition Probabilities

A discrete-time stochastic process $X = \{X_t,\ t \geq 0\}$ is called a Markov chain if, for any sequence $\{X_0, X_1, \ldots, X_t\}$ of states, the next state depends only on the current state and not on the sequence of events that preceded it. This is called the Markov property. Mathematically,

$$P(X_{t+1} = j \mid X_0 = i_0, X_1 = i_1, \ldots, X_t = i) = P(X_{t+1} = j \mid X_t = i) = P_{ij}. \tag{1}$$

P_{ij} must satisfy transition probabilities conditions as follows :

$$0 \leq P_{ij} \leq 1, \quad 1 \leq i, j \leq N, \qquad \text{and} \qquad \sum_{j=1}^{N} P_{ij} = 1, \quad 1 \leq i, j \leq N, \tag{2}$$

where N is the number of states in the system. Any matrix satisfying the above two equations is a transition probability matrix of a Markov chain [28].

2.4 The Semi-Markov Process

The Semi-Markov model and its derivation as presented in this study is due to Howard [29].

Definition 2.1. *Let $\{X_t; t \geq 0\}$ be a stochastic process that visits states in discrete time intervals*

$$S = \{s_1, s_2, \ldots, s_n\}$$

such that whenever it enters state i, $i \geq 0$, the next state it will enter is state j, $j \geq 0$, with probability P_{ij} $(i, j \in S)$. Given that the next state to be entered is state j, the time until the transition from i to j occurs has the distribution $F_{ij}(t)$.

If $Z(t)$ denotes the state of the process at time t, then $\{Z(t); t \geq 0\}$ is a semi-Markov process. Hence, a semi-Markov process $\{Z(t); t \geq 0\}$ is a stochastic process that changes states in accordance with a Markov *chain but takes a random amount of time between changes. A Markov chain is therefore a semi-Markov process with*

$$F_{ij}(t) = \begin{cases} 0, & \text{if } t < 1, \\ 1, & \text{if } t \geq 1. \end{cases} \tag{3}$$

A schematic diagram of the three state Semi-Markov process is presented in Figure 2.

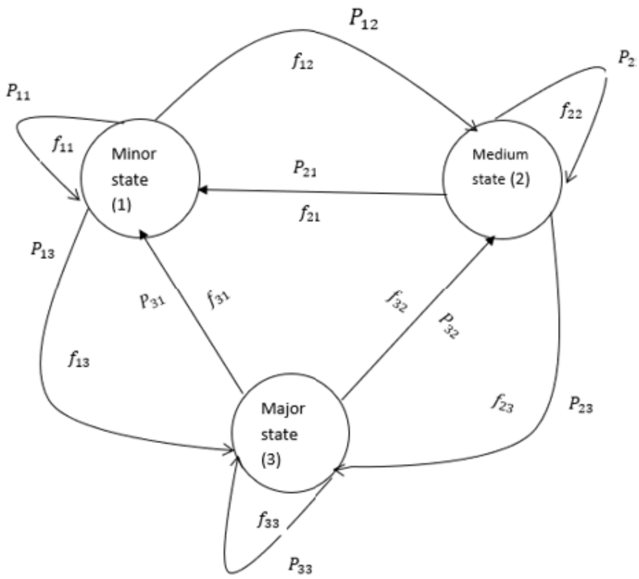

Figure 2: Schematic Presentation of the Semi-Markov Process.

P_{ij} are computed for all pairs i, j $(i, j = 1, 2, 3)$ using Equation (4)

$$P_{ij} = \frac{n_{ij}}{n_{ij} + n_{ij} + n_{ij}}. \tag{4}$$

The frequency matrix is presented in Equation (5).

$$\mathbb{N}_{ij} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}. \tag{5}$$

Thus, the transition probability matrix for the embedded Markov Chain is presented in Equation (6).

$$P = (p_{ij}) = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}. \tag{6}$$

2.5 Derivation of Semi-Markov Process Model

Let $X_n \colon \Omega \to S$ be a stochastic process with state space $S = \{s_1, s_2, \ldots, s_n\}$ and $T_n \colon \Omega \to R_+$ be the time of the n^{th} transition, with Ω domain of the process and R_+ set of positive real numbers. The kernel,

$Q_{ij}(t)$ associated with the process and the transition probability P_{ij} of the embedded Markov Chain is defined as follows:

$$Q_{ij}(t) = P[X_{n+1} = j; T_{n+1} - T_n \leq t / X_n = i], \tag{7}$$

$$P_{ij} = \lim_{t \to \infty} Q_{ij}(t). \tag{8}$$

Equation (8) defines the one-step transition probabilities for the embedded Markov chain.

Let $F_i(t)$, which denote the probability that the process will leave state i in a time t be defined as Equation (9).

$$F_i(t) = P[T_{n+1} - T_n \leq t / X_n = i] = \sum_{j=1}^{m} Q_{ij}(t), \tag{9}$$

$$f_i(t) = \frac{d}{dt} F_i(t).$$

Actually $F_i(t)$ is the cumulative probability distribution of a random variable T_i, where T_i is the time being spent in state i irrespective of the successor state.

The cumulative density function of the waiting time from state i to state j is

$$F_{ij}(t) = P[T_{n+1} - T_n \leq t / X_n = i, X_{n+1} = j] \tag{10}$$

which can be computed as

$$F_{ij}(t) = \begin{cases} Q_{ij}(t)/P_{ij} & \text{if } P_{ij} \neq 0 \\ 1 & \text{if } P_{ij} = 0 \end{cases}. \tag{11}$$

From equation (8) the Semi-Markov kernel is obtained

$$Q_{ij}(t) = P_{ij} F_{ij}(t). \tag{12}$$

Substituting Equation (12) into Equation (9) yields Equation (13).

$$F_i(t) = \sum_{j=1}^{m} P_{ij} F_{ij}(t). \tag{13}$$

Noting that

$$f_i(t) = \sum_{j=1}^{m} P_{ij} f_{ij}(t). \tag{14}$$

2.6 Stationary Distribution of the Semi-Markov Process

In order to obtain the stationary distribution of the Semi-Markov process, the theorem due to Ross [28] is adopted without proof.

Theorem 1.

Let $\{Z(t); t \geq 0\}$ be a Semi-markov process with discrete state space S and continuous kernel $\boldsymbol{Q}(t) = [Q_{ij}(t); ij \in S]$. If the embedded Markov chain $\{X_n; n = 0, 1, \dots\}$ contains one positive recurrent class C, such that for each state $i \in S, j \in C$, $f_{ij} = 1$ and $0 < E(T_i) < \infty, i \in S$, then

$$\phi_i = \frac{\pi_i E(T_i)}{\sum_j \pi_j E(T_j)}, \tag{15}$$

where $\pi_j = [\pi_j, j \in S]$ is the unique stationary distribution of the embedded Markov chain that satisfy the following system of Equation (16)

$$\sum_{i \in S} \pi_i p_{ij} = \pi_j, \quad \sum_{i \in S} \pi_i = 1. \tag{16}$$

$E(T_j)$ is the expected (mean) time spent in state j per transition is defined in Equation (17);

$$\mu_i = E(T_i) = \int_0^\infty t dF_i(t) \tag{17}$$

and the mean time between transitions is given as Equation (18);

$$\mu = \sum_j \pi_j E(T_j). \tag{18}$$

2.7 Waiting Time Distributions

It should be noted that for the Semi-Markov model that is considered here, the Weibull waiting time distribution was selected for all pairs of transitions as the best fit distribution. The probability density function of a Weibull waiting time distribution for all pairs of transitions $(i, j \in S)$ is given as Equation (19):

$$f_{ij}(t) = \frac{\alpha_{ij}}{\beta_{ij}} \left(\frac{t}{\beta_{ij}}\right)^{\alpha_{ij}-1} e^{-\left(\frac{t}{\beta_{ij}}\right)^{\alpha_{ij}}}; \quad t \geq 0, \tag{19}$$

where β_{ij} and α_{ij} are the scale and shape parameters respectively.

β_{ij} denote the spread of the distribution from state i to j and

α_{ij} denote the shape of the distribution from state i to j.

The probability distribution function of the waiting time distribution for all pairs of transitions $(i, j \in S)$ is given in Equation (20).

$$F_{ij}(t) = 1 - e^{-\left(\frac{t}{\beta_{ij}}\right)^{\alpha_{ij}}}; \quad t \geq 0. \tag{20}$$

The mean waiting time for all pairs of transitions $(i, j \in S)$ is defined in Equation (21):

$$\mu_{ij} = \beta_{ij} \Gamma\left(1 + \frac{1}{\alpha_{ij}}\right). \tag{21}$$

2.8 Goodness of Fit Test

In order to select a parametric distribution that suits the waiting time, the Anderson-Darling (AD) test was adopted to carry out goodness of fit test. The Anderson-Darling test is a distance test which is suitable for both small and large samples as presented by the waiting time data. The Weibull version of the AD test statistic is given as:

$$AD = \left[\sum_{i=1}^{n} \frac{1-2i}{n} \left\{ \ln\left(1 - \exp[-Z_{(i)}]\right) - Z_{(n+1-i)} \right\} - n \right], \tag{22}$$

with adjusted statistic

$$AD^{*} = \left(1 + \frac{0.2}{\sqrt{n}}\right) AD, \tag{23}$$

where $Z_{(i)} = \left[\frac{X_{(i)}}{\theta^{*}}\right]^{\beta^{*}}$ and the asterisks $(^{*})$ represent the estimators of the Weibull parameters [30]. The null hypothesis H_0, that the waiting time follows a Weibull distribution, is rejected if the p-value is less than 0.05 or if the test statistic is greater than the critical value at significance level α.

3 Application and Results

3.1 Data Source and Description

The data for this study were taken from the records of the National Oil Spill Detection and Response Agency (NOSDRA), Nigeria (https://nosdra.oilspillmonitor.ng/). Oil spill data due to CHEVRON Nigeria Limited and Nigeria Agip Oil Company (NAOC) exploration were considered. The data consist of documented crude oil spills and associated volumes during exploration by the oil companies under review from January 2006 to March 2020. With reference to Egbe and Thompson [3], the following oil spill categorizations were considered: oil spills with volume less than 25 barrels per unit time were classified as minor spills (State 1); oil spills with volume from 25 to 250 barrels per unit time were classified as medium spills (State 2); and oil spills with volume greater than 250 barrels per unit time were classified as major spills (State 3).

3.2 Goodness of Fit Test

Goodness of fit tests were conducted to select the parametric distribution which fit the waiting times for all pairs $(i, j) \in S$ of transitions in the Semi-Markov process. Results in Table 1 show that the Weibull distribution makes a good fit of the waiting times for all pairs $(i, j) \in S$ in the Semi-Markov process because the Anderson–Darling test statistics are all less than the critical value at the 5% level of significance. In other words, the null hypotheses (H_0), which state that the waiting time follows the Weibull distribution, cannot be rejected at $\alpha = 0.05$.

Table 1: Goodness of Fit Test

Oil Company	Transition (i, j)	Test Statistic	Critical Value	Decision: Reject H_0?
CHEVRON	(1,1)	1.5368	2.5018	No
	(1,2)	0.8297	2.5018	No
	(1,3)	0.3547	2.5018	No
	(2,1)	0.4537	2.5018	No
	(2,2)	N/A	N/A	–
	(2,3)	N/A	N/A	–
	(3,1)	0.4755	2.5018	No
	(3,2)	N/A	N/A	–
	(3,3)	N/A	N/A	–
NAOC	(1,1)	2.3570	2.5018	No
	(1,2)	1.0140	2.5018	No
	(1,3)	2.4580	2.5018	No
	(2,1)	1.2535	2.5018	No
	(2,2)	2.3253	2.5018	No
	(2,3)	0.6216	2.5018	No
	(3,1)	2.0974	2.5018	No
	(3,2)	0.7150	2.5018	No
	(3,3)	1.5368	2.5018	No

N/A = Not Applicable.

3.3 Semi-Markov Results

3.3.1 Transition Counts and Probabilities

Table 2 presents the transition counts and probabilities of the embedded Markov chain for all pairs of transitions $(i, j) \in S$. Matrix $\mathbb{N}_{ij}^{NAOC}$ represents the transition counts for NAOC while matrix $\mathbb{N}_{ij}^{CHEVRON}$ represents the transition counts for CHEVRON. Matrices P_{ij}^{NAOC} and $P_{ij}^{CHEVRON}$ present the transition probabilities for NAOC and CHEVRON respectively. For instance, $P_{ij}^{CHEVRON}$ shows that the probability of transitioning from state 1 to state 1 is 0.7344, from state 1 to state 2 is 0.2209, and from state 1 to state 3 is 0.0447. This implies that the probabilities of having a minor spill after a minor spill, medium spill after a minor spill and a major spill after a minor spill are 0.7344, 0.2209 and 0.0447 respectively. Observe that the probabilities of transiting from state 3 to state 1, from state 3 to state 2 and from state 3 to state 3 are quite low with values 0.0447, 0.0356, and 0.0435 respectively. This means that the probability of recording a major spill from NAOC exploration is low irrespective of the immediate preceding category of oil spill. Similarly, matrix $P_{ij}^{CHEVRON}$ shows high probabilities of having a minor spill after a minor spill, a minor spill after a medium spill and a minor spill after a major spill with values 0.9420, 0.8824, and 1.000 respectively. These indicate that the probability of recording a minor spill from CHEVRON exploration is high irrespective of the immediate preceding category of oil spill. $P_{ij}^{CHEVRON}$ further records zero (0) probabilities of having a medium spill after a major spill and a major spill after a major spill.

Table 2: Transition Counts and Probabilities

Transition Counts	**Transition Probabilities**
$\mathbb{N}_{ij}^{\text{NAOC}} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 575 & 173 & 35 \\ 171 & 73 & 9 \\ 34 & 10 & 2 \end{pmatrix} \end{array}$	$P_{ij}^{\text{NAOC}} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.7344 & 0.2209 & 0.0447 \\ 0.6759 & 0.2885 & 0.0356 \\ 0.7391 & 0.2174 & 0.0435 \end{pmatrix} \end{array}$
$\mathbb{N}_{ij}^{\text{CHEVRON}} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 357 & 16 & 6 \\ 15 & 1 & 1 \\ 7 & 0 & 0 \end{pmatrix} \end{array}$	$P_{ij}^{\text{CHEVRON}} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.9420 & 0.0422 & 0.0158 \\ 0.8824 & 0.0588 & 0.0588 \\ 1 & 0 & 0 \end{pmatrix} \end{array}$

3.3.2 Waiting Time Distribution of the Semi-Markov Process

Table 3 presents the scale (β_{ij}) parameter, shape (α_{ij}) parameter and mean (μ_{ij}) of the waiting time distribution of the Semi-Markov process for NAOC and CHEVRON for all pairs of transitions $(i, j) \in S$. It should be observed that the mean (μ_{ij}) values for CHEVRON (where it exists) are generally higher than that of NAOC except for the transition from State 2 to State 2. These indicate a longer waiting time distribution in the case of CHEVRON as compared to NAOC exploration.

Table 3: Parameter Estimates of the Waiting Time Distribution

NAOC	**CHEVRON**
$\beta_{ij}^{\text{NAOC}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2.8178 & 2.7081 & 3.1147 \\ 3.0444 & 2.9983 & 2.6950 \\ 2.8517 & 2.2126 & 27.5900 \end{pmatrix} \end{array}$	$\beta_{ij}^{\text{CHEVRON}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 6.7493 & 6.1840 & 8.4022 \\ 6.1230 & - & - \\ 8.1190 & - & - \end{pmatrix} \end{array}$
$\alpha_{ij}^{\text{NAOC}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1.4325 & 1.6067 & 1.3267 \\ 1.4666 & 1.3779 & 1.2324 \\ 1.2827 & 2.0256 & 0.9285 \end{pmatrix} \end{array}$	$\alpha_{ij}^{\text{CHEVRON}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1.1408 & 1.2507 & 0.7839 \\ 1.0315 & - & - \\ 0.8574 & - & - \end{pmatrix} \end{array}$
$\mu_{ij}^{\text{NAOC}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2.5593 & 2.4271 & 2.8652 \\ 2.7561 & 2.7397 & 2.5184 \\ 2.6410 & 1.9605 & 28.5570 \end{pmatrix} \end{array}$	$\mu_{ij}^{\text{CHEVRON}} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 6.4386 & 5.7589 & 9.6628 \\ 6.0462 & 1 & 11 \\ 8.7861 & - & - \end{pmatrix} \end{array}$

3.3.3 The Semi-Markov Kernel

The Semi-Markov kernel for all pairs of transitions $(i, j) \in S$ were obtained using Equation (9) and the graphs of the Semi-Markov kernels are presented in Figure 3, for all pairs of transitions $(i, j) \in S$. Graphs of the Semi-Markov kernels for NAOC depict increasing functions which converge as t increases. Apart from the transition from state 3 to state 3 ($S_3 \to S_3$), all Semi-Markov kernels for NAOC approach a limit at about $t = 10$. Graphs of the kernels for CHEVRON are presented in Figure 4. Again, the Semi-Markov kernels depict increasing functions which converge as t increases. However, the Semi-Markov kernels of CHEVRON converge to a limit at a slower rate compared to that of NAOC.

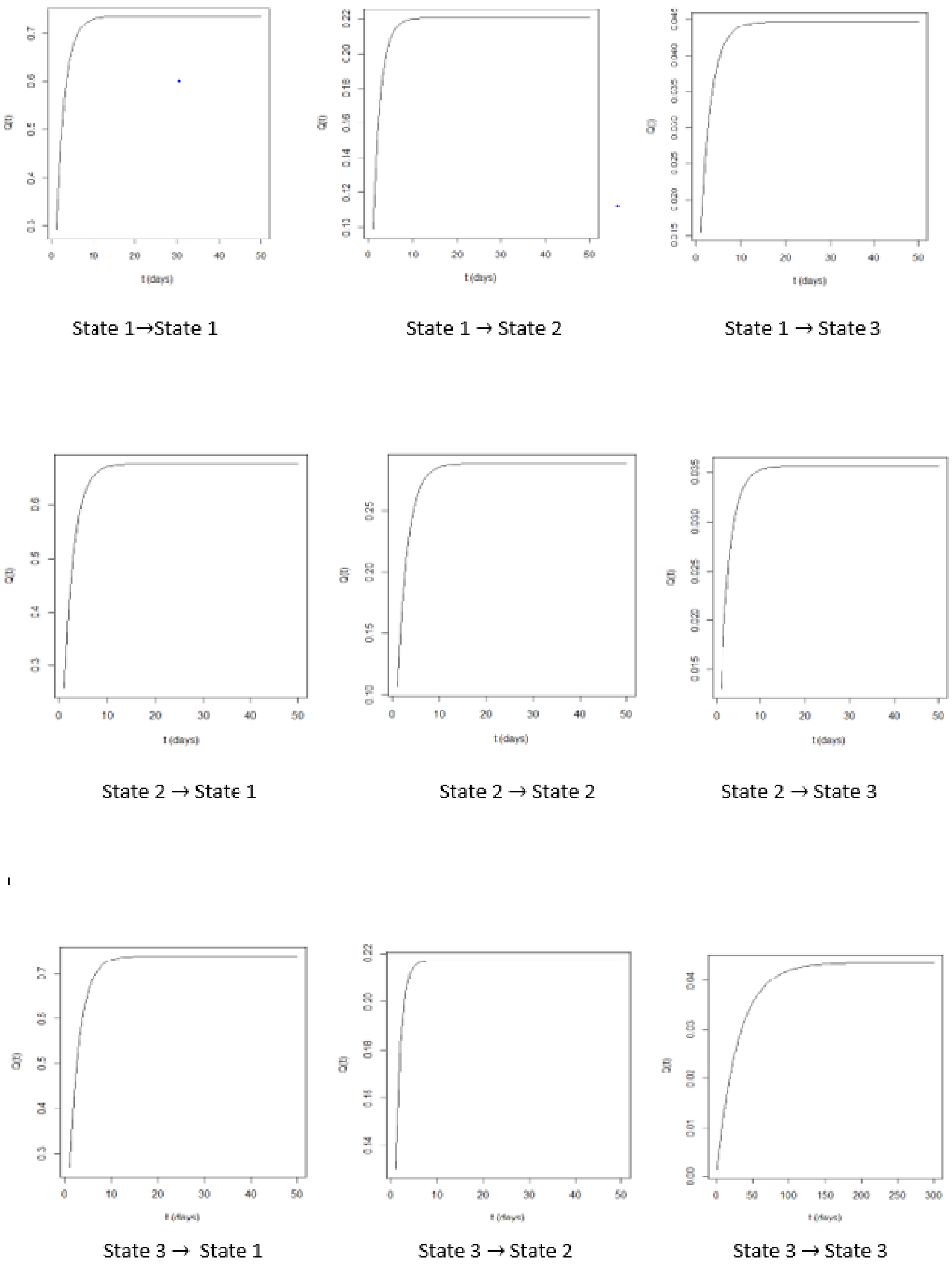

Figure 3: Graphs of Semi-Markov Kernel for NAOC.

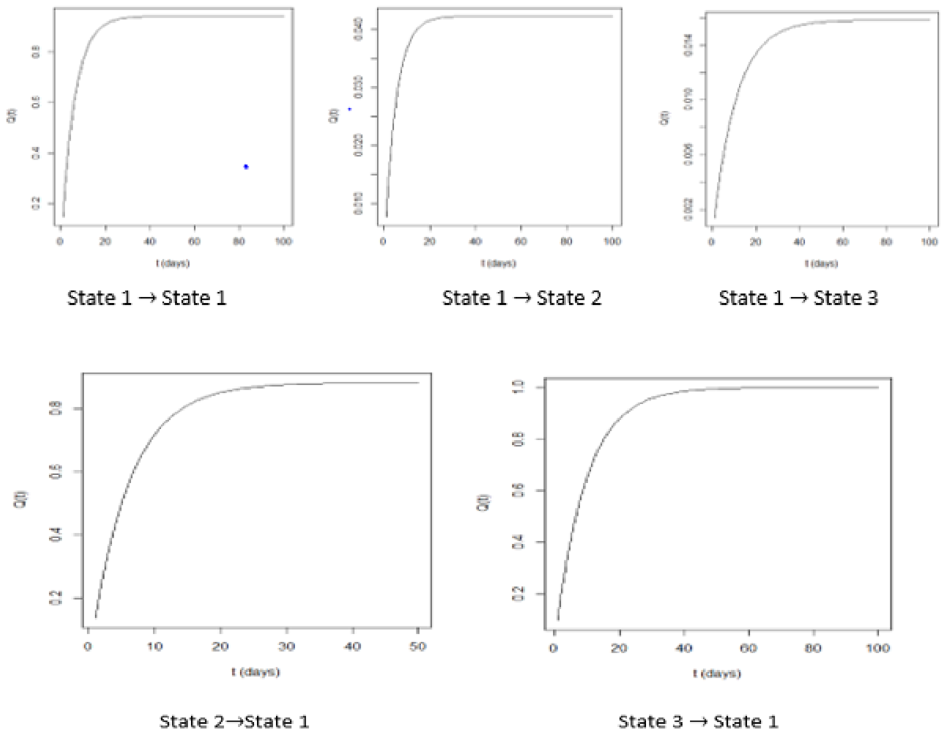

Figure 4: Graphs of Semi-Markov Kernel for CHEVRON.

3.3.4 Stationary Distribution of the Semi-Markov Process

Table 4 presents results on the stationary distribution of the Semi-Markov process for NAOC and CHEVRON. From Table 4, the probabilities of recording minor, medium, and major oil spills by NAOC are 0.6954, 0.2461, and 0.0585 respectively. This result reveals that NAOC exploration was characterized by minor and medium spills. The probability of recording minor, medium, and major oil spills by CHEVRON are 0.9373, 0.0393 and 0.0234 respectively. Hence, CHEVRON exploration was characterized by minor spills.

Table 4: Stationary Distribution of the Semi-Markov Process

	NAOC	CHEVRON
ϕ_1	0.6954	0.9373
ϕ_2	0.2461	0.0393
ϕ_3	0.0585	0.0234

3.3.5 The Mean Waiting Time (MWT) Distribution

Table 5 presents the mean waiting time distribution of the Semi-Markov process for NAOC and CHEVRON Nigeria Ltd. MWT describes the length of time that it takes for the process to transit into various states of the Semi-Markov process. The mean waiting time before a record of a minor, medium and major spills for the case of NAOC were obtained as $\mu_1 = 2.5438$, $\mu_2 = 2.7429$, $\mu_3 = 3.6204$. This implies that it takes approximately 3 days, 3 days and 4 days to record a minor, medium and major spills respectively due to NAOC exploration. The mean waiting time before a record of minor, medium and major spills for the case of CHEVRON were obtained as $\mu_1 = 6.4609$, $\mu_2 = 6.0408$, $\mu_3 = 8.7861$. This implies that it takes approximately 6 days, 6 days, and 9 days to record a minor, medium and major spills respectively due to CHEVRON exploration.

Furthermore, the length of time that it takes to record a spill of any category from NAOC and CHEVRON were obtained using Equation (18). The mean time to oil spills due to NAOC exploration was found to be $\mu = 2.6368$; meaning that crude oil spills due to NAOC occurred approximately once every 3 days. The mean time to oil spills due to CHEVRON Nigeria Ltd exploration was obtained as $\mu = 6.4834$; meaning that crude oil spills due to CHEVRON Nigeria Ltd exploration occurred approximately once every 6 days. This latter result agrees with that of Udoumoh [22] who estimated the mean time to spill by CHEVRON Nigeria Ltd to be 6 days.

Table 5: Mean Waiting Time (MWT) Distribution of the Semi-Markov Process

	MWT for NAOC (in days)	MWT for CHEVRON (in days)
μ_1	2.5438	6.4609
μ_2	2.7429	6.0408
μ_3	3.6204	8.7861
μ	2.6368	6.4834

4 Conclusion

In this study, a three-state Semi-Markov model for the analysis of crude oil spills was developed with Weibull waiting time distributions. The model provided an extension on the commonly used Markov models by incorporating waiting time distributions for a more realistic application. An empirical data of crude oil exploration from Nigeria Agip Oil Company (NAOC) and CHEVERON Nigeria Limited was considered for model implementation. It was established that crude oil spills due to CHEVRON Nigeria Ltd were mostly minor while that of NAOC were minor and medium. Further study is recommended to identify risk factors associated with the various categories of oil spills in the Niger Delta region.

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References

[1] National Bureau of Statistics. (2024). *Foreign trade in goods statistics Q1 2024*. https://www.nigerianstat.gov.ng/elibrary/read/1241511

[2] Mohamadi, B., Liu, F., & Xie, Z. (2016). Oil spill influence on vegetation in Nigeria and its determinants. *Polish Journal of Environmental Studies*, *25*(6), 2533–2540. https://doi.org/10.15244/pjoes/63666

[3] Egbe, R. E., & Thompson, D. (2010). Environmental challenges of oil spillage for families in oil producing communities of the Niger Delta region. *Journal of Home Economics Research*, *13*, 24–34.

[4] Akinpelu, Y. (2021, May 15). Analysis: 77% of oil spills in Nigeria occurred in only three states. *Premium Times*.

[5] Udoumoh, E. F., Ekpo, A., & Samuel, A. A. (2016). Reliability analysis of crude oil exploration in Niger Delta, Nigeria. *International Journal of Science, Environment and Technology*, *5*(3), 4560–4570.

[6] United Nations Environment Programme. (2016). *Environmental assessment of Ogoniland*. http://postconflict.unep.ch/publications/dea/unep-oea.pdf

[7] Raimi, M. O., & Sabinus, C. E. (2017). An assessment of trace elements in surface and ground water quality in the Ebocha-Obrikom oil and gas producing area of Rivers State, Nigeria. *International Journal of Scientific and Engineering Research*, *8*(6), 1102–1115. https://ssrn.com/abstract=3155472

[8] Raimi, M. O., Pigha, T. K., & Owobi, O. E. (2017). Water-related problems and health conditions in the oil producing communities in Central Senatorial District of Bayelsa State. *Imperial Journal of Interdisciplinary Research*, *3*(6), 1888–1905. https://ssrn.com/abstract=3108032

[9] Olalekan, M. R., Omidiji, O. A., Nimisngha, D., Odipe, O. E., & Olalekan, A. S. (2018). Health risk assessment on heavy metals ingestion through ground water drinking pathway for residents in the oil and gas producing area of Rivers State, Nigeria. *Open Journal of Yangtze Gas and Oil*, *3*(3), 191–206. https://doi.org/10.4236/ojogas.2018.33017

[10] Sawyer, O. H., Odipe, O. E., & Olalekan, R. M. (2018). Assessment of cyanide and some heavy metal concentration in consumable cassava flour "lafun" across Osogbo metropolis, Nigeria. *Journal of Environmental Science, Toxicology and Food Technology*, *3*(6), 369–372. https://ssrn.com/abstract=3294152

[11] Raimi, M. O., Omidiji, A. O., Adeolu, T. A., Odipe, O. E., & Babatunde, A. (2019). An analysis of Bayelsa State water challenges on the rise and its possible solutions. *Acta Scientific Agriculture*, *3*(4), 110–125. https://doi.org/10.31080/ASAG.2019.03.0572

[12] Raimi, M. O., Omidiji, A. O., Adedipe, A. A., Anu, B., Odipe, O. E., & Amakama, N. J. (2019). Digging deeper evidence on water crisis and its solution in Nigeria for Bayelsa State: A study of current scenario. *International Journal of Hydrology*, *3*(4), 244–257. https://doi.org/10.2139/ssrn.3422483

[13] Okoyen, E., Omidiji, O. A., Raimi, M. O., & Ebuete, A. W. (2020). Governing the environmental impact of dredging: Consequences for marine biodiversity in the Niger Delta region of Nigeria. *Journal of Insights in Mining Science & Technology*, *2*(3), 555–586.

[14] Raimi, M. O., Ayinla, L. O., & Ogah, A. (2021). First to respond, last to leave: The role of para-military agencies in disaster management: Evidence from Nigeria. *Sumerianz Journal of Medical and Healthcare*, *4*(2), 96–100. https://ssrn.com/abstract=3851121

[15] Digha, O. N., Ambah, B., & Jacob, E. N. (2017). The effects of crude oil spillage on farmland in Gokana Local Government Area of Rivers State. *European Journal of Basic and Applied Sciences*, *4*(1), 76–96.

[16] Oyebamiji, M. A., & Mba, C. I. (2013). Effects of spillage on community development in the Niger Delta region: Implications for the eradication of poverty and hunger (Millennium Development Goal One) in Nigeria. *World Journal of Social Sciences*, *1*(1), 27–36. https://doi.org/10.5430/wjss.v1n1p27

[17] Nwachukwu, A. N., & Osuagwu, J. C. (2014). Effects of oil spillage on groundwater quality in Nigeria. *American Journal of Engineering Research*, *3*(6), 271–274.

[18] Takon, N. (2014). Environmental damage arising from oil operations in the Niger Delta of Nigeria: How not to continually live with their specific impact in population and ecology. *Environmental Journal of Development and Sustainability*, *3*(9), 1878–1895.

[19] Saleh, M. A., Ashiru, M. A., Sanni, J. E., Ahmed, T. A., & Muhammad, S. (2017). Risk and environmental implications of oil spillage in Nigeria (Niger Delta region). *IIARD International Journal of Geography and Environmental Management*, *3*(2), 44–53.

[20] Wizor, C. H., & Wali, E. (2020). Crude oil theft in the Niger Delta: The oil companies and host communities conundrum. *International Journal of Research and Scientific Innovation*, *7*(1), 22–32.

[21] Akpan, N. P., & Ugwuowo, F. I. (2012). An application of survival analysis and Cox regression model to oil spill data. *Journal of Basic and Applied Scientific Research*, *2*(3), 2071–2080.

[22] Udoumoh, E. F. (2022). Stochastic modelling of oil spill incidences as renewal process. *FUAM Journal of Pure and Applied Science*, *2*(2), 28–33.

[23] Mba, I. C., Mba, E. I., Arazu, W. O., Urama, C. E., Machebe, C. H., & Eze, C. (2019). Application of the stochastic Markov model in predicting the volume of oil spills in Nigeria: A case of the Niger Delta region. *International Journal of Energy Economics and Policy*, *9*(4), 110–114. https://doi.org/10.32479/ijeep.7744

[24] Ugwuowo, F. I., & Udoumoh, E. F. (2011). Multi-stage semi-Markov models for recovery process. *Communications in Statistics - Theory and Methods*, *40*(16), 3011–3019. https://doi.org/10.1080/03610926.2011.563022

[25] Nyitar, J. A., Udoumoh, E. F., & Ekpo, A. (2025). A multi-state assessment of the recovery process of HIV patients under antiretroviral therapy. *Journal of the Royal Statistical Society, Nigeria Group*, *2*(1), 186–202.

[26] Udoumoh, E. F., Ikughur, J. A., & Gemga, A. A. (2025). On the hedging characteristics of gold against exchange rate (NGN-USD) using Markov approach. *Journal of the Royal Statistical Society, Nigeria Group*, *2*(1), 453–473.

[27] Adah, V., Ikughur, J. A., Nwaosu, S. C., & Udoumoh, E. F. (2025). Discretization of climatic characteristics and Markov chain modelling of crop growth. *Journal of the Royal Statistical Society, Nigeria Group*, *2*(1), 99–117.

[28] Ross, S. M. (1983). *Stochastic processes*. John Wiley & Sons.

[29] Howard, R. A. (1977). *Dynamic probabilistic systems: Volume II: Semi-Markov and decision processes*. John Wiley & Sons.

[30] Romeu, J. L., & Grethlein, C. (2000). *A practical guide to statistical analysis of material property data*. Advanced Materials and Processes Technology Information Analysis Center.