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# Fourier Transform of Set-Valued Functions with an Application in Signal Processing

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**Abstract**

In this paper, we have presented an application with respect to the Fourier transform of an interval-valued function. Thanks to this application we have obtained some information about the Fourier transform of the signals with inexact data. For this purpose we have identified the interval signal that most closely resembles this non-deterministic signal with a very small error. This continuous-time interval signal is called as *model interval signal*. Therefore, we have obtained approximate results for the Fourier transform of non-deterministic signals by calculating the Fourier transform of this model interval signal.

## 1 Introduction

The solutions of many engineering problems are based on the analysis of set-valued functions. For example, they are frequently used in processing of continuous or discrete-time signals with inexact data. We can see examples of such applications in [14], [4] and [1]. Let us consider the following problem: Is it possible to guess the Fourier transforms of a non-deterministic signal with incomplete information on a certain set? In order to predict the transformations of the signal, we must first determine which interval or set we need to know that the non-deterministic signal stays in. Further, sometimes only we need to know the non-deterministic signal stays between two deterministic signals. Signals in a situation like this are called signals with inexact data. To be able to process such signals, we have introduced the notion of set-valued signals. The most useful type of these signals are interval-valued signals. These are the types of signals that we do not come across naturally, but that we come up with in order to process a signals with inexact data. We should start by associating these signals with interval-valued or a set-valued function. Once this is done, the properties of the family of the interval-valued functions, for example, in the problem, must be studied. In general, we know that families of the interval or set-valued functions has an algebraic structure called as a quasilinear space. The most important quasilinear space to be used in our work will be the space $L^2(\mathbb{R}, \Omega(\mathbb{C}))$ of all set-valued functions whose norm is square integrable.

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The motivation and innovation of this article is that if certain characteristics of a non-deterministic signal are known, can some approximate results be obtained concerning the Fourier transform of this signal? In the article some new techniques are developed to obtain some approximate estimations of this type signals. The techniques used in this article provide an implementation of the Fourier transform and its inverse implemented in the function that is based on research of Marcio Gameioro. Among others, this function forms the basis for research such as [5], [6] and [12].

Classical Fourier transform converts a set of time domain data vectors into a set of frequency domain vectors. The Fourier transform $\mathcal{T}$ of a function $f : \mathbb{R} \to \mathbb{C}$ is given by $\mathcal{T}(f) = \hat{f}$ where

$$\hat{f}(w) = \int_{\mathbb{R}} f(t) e^{-2\pi i w t} dt \quad , \quad w \in \mathbb{R},$$

[7]. For example, consider the box function

$$f(t) = \begin{cases} 1/\sqrt{t_0} & , \text{ for } |t| \leq \frac{1}{2} t_0; \\ 0 & , \text{ otherwise.} \end{cases}$$

for some positive real number $t_0$. The Fourier transform of $f$ is

$$\hat{f}(w) = \frac{t_0}{\sqrt{t_0}} sinc(\pi t_0 w)$$

(see, [7]) where

$$sinc(t) = \begin{cases} \frac{\sin t}{t} & , \; t \neq 0 \\ 1 & , \; t = 0 \end{cases}.$$

Let us consider interval-valued function

$$F(t) = \begin{cases} [-1/\sqrt{t_0}, 1/\sqrt{t_0}] & , \text{ for } |t| \leq \frac{1}{2} t_0; \\ \{0\} & , \text{ otherwise.} \end{cases}$$

Is there a definition giving the Fourier transform of $F$? In [15], we have studied on the Fourier transform of such functions. While we are creating required mathematical background of this process, we used the notion of *quasilinear space* which is introduced by Aseev in [11]. He presented an approach for the analysis of the function spaces of set-valued mappings in [11].

The most popular quasilinear spaces are $\Omega(E)$ and $\Omega_C(E)$ which are defined as the sets of all non-empty closed bounded and non-empty convex closed bounded subsets of any normed linear space $E$, respectively. Especially, $\Omega_C(\mathbb{R})$ is defined as the set of all non-empty compact convex subsets of real numbers and it is a subset of $\Omega(\mathbb{R})$. An element of $\Omega_C(\mathbb{R})$ is called as a real interval and the interval $x$ denoted by $x = [\underline{x}, \overline{x}]$ where $\underline{x}$ and $\overline{x}$ are the left and right endpoints of $x$, respectively. We say that $x$ is a degenerate interval if $\underline{x} = \overline{x}$ and it is denoted by $\{x\}$ or $[x, x]$ and it contains a single real number $x$. For $x, y \in \Omega_C(\mathbb{R})$ and $\lambda \in \mathbb{R}$, the Minkowski sum and scalar multiplication operations are defined by

$$x + y = [\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$$

and

$$\lambda x = \begin{cases} [\lambda \underline{x}, \lambda \overline{x}] & , \quad \lambda \geq 0 \\ [\lambda \overline{x}, \lambda \underline{x}] & , \quad \lambda < 0, \end{cases}$$

respectively.

The investigation of $\Omega(\mathbb{C})$ contributes to interval and convex analysis and this space is an excellent tool for mathematical formulation of many real-life situations, for example signal processing. In this paper we are interested in the analysis of the space of $\Omega(\mathbb{C})$-valued functions.

## 2 Basic Concepts and Motivation

Let us start with some basic definitions and theorems.

Suppose that $X$ is a quasilinear space and $Y \subseteq X$. Then $Y$ is called a *subspace of* $X$ whenever $Y$ is a quasilinear space with the same partial order on $X$, [2].

$Y$ is subspace of quasilinear space $X$ if and only if for every $x, y \in Y$ and $\alpha, \beta \in \mathbb{K}$, $\alpha.x + \beta.y \in Y$, [2].

An element $x$ in a quasilinear space $X$ is said to be *symmetric* if $-x = x$ and $X_{sym}$ denotes the set of all symmetric elements. Also, $X_r$ stands for the set of all regular elements of $X$ while $X_s$ stands for the sets of all singular elements and zero in $X$. Further, it can be easily shown that $X_r$, $X_{sym}$ and $X_s$ are subspaces of $X$. They are called *regular, symmetric* and *singular subspaces* of $X$, respectively, [13].

Let $X$ be a real or complex quasilinear space. The real-valued function on $X$ is called a *norm* if the following conditions hold:

$$\|x\| > 0 \text{ if } x \neq 0, \tag{1}$$

$$\|x + y\| \leq \|x\| + \|y\|, \tag{2}$$

$$\|\alpha x\| = |\alpha| \, \|x\|, \tag{3}$$

$$\text{if } x \preceq y, \text{ then } \|x\| \leq \|y\|, \tag{4}$$

$$\begin{gathered}\text{if for any } \varepsilon > 0 \text{ there exists an element } x_\varepsilon \in X \text{ such that} \\ x \preceq y + x_\varepsilon \text{ and } \|x_\varepsilon\| \leq \varepsilon \text{ then } x \preceq y,\end{gathered} \tag{5}$$

here $x, y, x_\varepsilon$ are arbitrary element in $X$ and $\alpha$ is any scalar, [11]. A quasilinear space $X$ with a norm defined on it, is called *normed quasilinear space*. Hausdorff metric or norm metric on $X$ is defined by the equality

$$h_X(x, y) = \inf \left\{ r \geq 0 : x \preceq y + a_1^{(r)},\, y \preceq x + a_2^{(r)} \text{ and } \left\| a_i^{(r)} \right\| \leq r, i = 1, 2 \right\}.$$

A norm on $\Omega(E)$ is defined by

$$\|A\|_\Omega = \sup_{a \in A} \|a\|_E \ .$$

Hence $\Omega_C(E)$ and $\Omega(E)$ are normed quasilinear spaces, [11].

Now let us give the lemma which enables to in some applications.

**Lemma 2.1.** *[9] If an interval-valued $F$ is both continuos and inclusion isotonic then the interval integral of $F$ is equivalent to*

$$\int_{[a,b]} F(t)dt = [\int_{[a,b]} \underline{F}(t)dt, \int_{[a,b]} \overline{F}(t)dt]$$

*where $\underline{F}$ and $\overline{F}$ are two continuous real-valued function such that, for real $t$,*

$$F(t) = [\underline{F}(t), \overline{F}(t)].$$

Let us now introduce the spaces $L^p(\mathbb{R}, \Omega(\mathbb{C}))$ $(1 \leq p < \infty)$ which plays an important role for the Fourier transform of the set-valued functions.

For $1 \leq p < \infty$, the space $L^p(\mathbb{R}, \Omega(\mathbb{C}))$ consists of all set-valued measurable functions $F : \mathbb{R} \to \Omega(\mathbb{C})$ such that the Lebesgue integral

$$\int_{\mathbb{R}} \|F(x)\|_{\Omega}^p \, dx$$

is exist.

The operations of algebraic sum, multiplication by a complex scalar and the partial order relation are defined as follows:

$$(F_1 + F_2)(x) = F_1(x) + F_2(x),$$

$$(\lambda F)(x) = \lambda F(x)$$

and

$$F_1 \preceq F_2 \Leftrightarrow F_1(x) \subseteq F_1(x) \text{ for almost everywhere (a.e.) } x \in \mathbb{R}.$$

$L^p(\mathbb{R}, \Omega(\mathbb{C}))$ is a quasilinear space over the field $\mathbb{C}$ with the above algebraic operations and the partial order relation, [3].

For $1 \leq p < \infty$, the expression

$$\|F\|_p = \left\| \overline{\{\int_{\mathbb{R}} |f(x)|^p \, dx : f \in S^p(F)\}} \right\|_{\Omega}^{1/p} \tag{6}$$

defines a norm on $L^p(\mathbb{R}, \Omega(\mathbb{C}))$, where $S^p(F) = \{f : \mathbb{R} \to \mathbb{C} : \int_{\mathbb{R}} |f(x)|^p \, dx < \infty$ and $f(x) \in F(x)$ for any $x \in \mathbb{R}\}$. [3].

Let us first give the set-valued Fourier transform on $L^2(\mathbb{R}, \Omega(\mathbb{C}))$ as follows:

**Definition 2.1.** *[15] The set-valued Fourier transform of $F \in L^2(\mathbb{R}, \Omega(\mathbb{C}))$ is the set-valued function $\hat{F}$ defined by*

$$\hat{F}(w) = \overset{(A)}{\int_{\mathbb{R}}} F(x)e^{-2\pi iwx}dx = \{\int_{\mathbb{R}} f(x)e^{-2\pi iwx}dx : f \in S^2(F)\} \tag{7}$$

*for any $w \in \mathbb{R}$ where $S^2(F) = \{f : \mathbb{R} \to \mathbb{C} : \int_{\mathbb{R}} |f(x)|^2 dx < \infty$ and $f(x) \in F(x)$ for any $x \in \mathbb{R}\}$.*

The integral $\overset{(A)}{\int_{\mathbb{R}}} F(x)e^{-2\pi iwx}dx$ is the Aumann integral and it is the set of integrals $\int_{\mathbb{R}} f(x)e^{-2\pi iwx}dx$ for any integrable selections $f$ of $F$, (for Aumann integral, see [10]).

The set-valued Fourier transform of $F$ is also denoted by

$$(\mathcal{F}F)(w) = \hat{F}(w) \quad ,w \in \mathbb{R}.$$

Note that the set-valued Fourier transform of $F \in L^2(\mathbb{R}, \Omega(\mathbb{C}))$ is the set of Fourier transforms of integrable selections of $F$, i.e.,

$$\hat{F}(w) = \{\hat{f}(w) : \mathcal{T}(f) = \hat{f},\ f \in S^2(F)\}.$$

*We will employ the following notation throughout the text:*

*$\mathcal{F}(F) = \hat{F}$ means $\hat{F}$ is the set-valued Fourier transform of the function $F : \mathbb{R} \to \Omega(\mathbb{C})$. Further, $\hat{f} = \mathcal{T}(f)$ means $\hat{f}$ is the Fourier transform of the function $f : \mathbb{R} \to \mathbb{C}$ in the meaning of usual.*

## 3 An Application on Set-Valued Fourier Transform

Mathematically, a signal is defined as a function from a subset $A$ of $\mathbb{R}$ into $\mathbb{C}$. If $A = \mathbb{R}$ then the signal is called as continuous-time signal, [7]. In real life, sometimes a signal value in a time $t$ may not be known exactly. The notion of *interval signal* provides a suitable way for processing of such non-deterministic signals. The closed bounded subset of real numbers is said an *interval, i.e., the interval* $x$ denoted by $[\underline{x}, \overline{x}]$ is the set given by $[\underline{x}, \overline{x}] = \{a \in \mathbb{R} : \underline{x} \leq a \leq \overline{x}\}$ where $\underline{x}$ and $\overline{x}$ are the left and right endpoints of the interval $x$, respectively. It is called that $x$ is a *degenerate interval* if $\underline{x} = \overline{x}$ and it is shown as $\{x\}$ or $[x, x]$, [9]. In this article, we represent the set of all intervals as $\mathbb{I}_{\mathbb{R}}$. Further, this space is a normed quasilinear space.

In our previous work, we defined that an interval signal is a function from a subset of $\mathbb{R}$ into $\mathbb{I}_{\mathbb{C}}$. If the domain of the interval signal is the set $\mathbb{R}$ then this signal is called as *continuous-time interval signal,* [4]. An element $u$ of $\mathbb{I}_{\mathbb{C}}$ is called as complex interval and it is the set given by

$$u = \left[\underline{u_r}, \overline{u_r}\right] + i\left[\underline{u_s}, \overline{u_s}\right]$$

where $\left[\underline{u_r}, \overline{u_r}\right]$ and $\left[\underline{u_s}, \overline{u_s}\right]$ are nonempty closed (real) intervals of $\mathbb{R}$ and $i = \sqrt{-1}$, the complex unit, [14]. Further, $\mathbb{I}_{\mathbb{R}}$ is a subset of $\mathbb{I}_{\mathbb{C}}$.

The problem is that if certain characteristics of a non-deterministic signal are known, can some approximate results be obtained concerning the Fourier transform? In the previous sections of this article, we developed the mathematical techniques necessary to answer this question. Firstly, we have identify an interval signal that most closely resembles this non-deterministic signal with a very small error. This continuous-time interval signal is called as *model interval signal.* Since the notion of Fourier transform for set-valued functions given in the previous section is also valid for the interval signals, we calculate the Fourier transform of this model interval signal. Consequently, we obtain that the Fourier transform of the non-deterministic signal remains in the Fourier transform of its model interval signal.

Now we will see how the Fourier transform of the interval signal is calculated if the model interval signal is a function given by 8.

**Example 3.1.** *Let us consider the interval-valued function $F$ given by*

$$F(t) = \begin{cases} [-1/\sqrt{2}, 1/\sqrt{2}] & , \text{ for } |t| \leq 1; \\ \{0\} & , \text{ otherwise.} \end{cases} \tag{8}$$

*($F$ is illustrated in Figure-1). We say $F \in L^1(\mathbb{R}, \Omega(\mathbb{C}))$ since*

$$\int_{\mathbb{R}} \|F(x)\|_{\Omega} \, dx = \int_{-1}^{1} \left\|[-1/\sqrt{2}, 1/\sqrt{2}]\right\|_{\Omega} dx = \sqrt{2} < \infty.$$

*By the equality (7) and by the properties of Aumann integral, the Fourier transform of $F$ is as follows:*

$$\begin{aligned} \hat{F}(w) &= \int_{[-1,1]}^{(A)} F(x) e^{-2\pi i w x} dx \\ &= \int_{[-1,1]}^{(A)} [-1/\sqrt{2}, 1/\sqrt{2}] \cos(2\pi w x) dx - i \int_{[-1,1]}^{(A)} [-1/\sqrt{2}, 1/\sqrt{2}] \sin(2\pi w x) dx. \end{aligned}$$

*Further, functions (integrands) above are continuous from $[-1, 1]$ to $\mathbb{I}_{\mathbb{R}}$ and they are clearly inclusion isotonic. Let us only prove the integrand $V(x) = [-1/\sqrt{2}, 1/\sqrt{2}] \cos(2\pi w x)$ is continuous: First of all we say that the metric on $\mathbb{I}_{\mathbb{R}}$ is just defined as*

$$d(X, Y) = \max\left\{\left|\underline{X} - \underline{Y}\right|, \left|\overline{X} - \overline{Y}\right|\right\}$$

*where $X = [\underline{X}, \overline{X}]$ and $Y = [\underline{Y}, \overline{Y}]$. Now let us assume $x_n \to x$ in $[-1, 1]$ with the usual metric on real numbers. Then of course $|x_n - x| \to 0$ for $n \to \infty$. We should prove that $d(V(x_n), V(x)) \to 0$, $n \to \infty$. Since $V(x_n), V(x) \in \mathbb{I}_{\mathbb{R}}$ then $V(x_n) = [\underline{V(x_n)}, \overline{V(x_n)}]$ and $V(x) = [\underline{V(x)}, \overline{V(x)}]$ where $\underline{V(x_n)} =$*

$-1/\sqrt{2}\cos(2\pi w x_n)$, $\overline{V(x_n)} = 1/\sqrt{2}\cos(2\pi w x_n)$ *and* $\underline{V(x)} = -1/\sqrt{2}\cos(2\pi w x)$,$\overline{V(x)} = 1/\sqrt{2}\cos(2\pi w x)$. *Thus,*

$$\begin{aligned} d(V(x_n), V(x)) &= d([\underline{V(x_n)}, \overline{V(x_n)}], [\underline{V(x)}, \overline{V(x)}]) \\ &= \max\{\left|\underline{V(x_n)} - \underline{V(x)}\right|, \left|\overline{V(x_n)} - \overline{V(x)}\right|\} \\ &= \frac{1}{\sqrt{2}}\left|\cos(2\pi w x_n) - \cos(2\pi w x)\right| \to 0 \ \textit{as } n \to \infty \end{aligned}$$

*since the classical* $\cos(2\pi w x)$ *is continuous. This proves the above assertion.*

*Now by using the Lemma (2.1) we can write*

$$\begin{aligned} \hat{F}(w) &= [\int_{-1}^{1} -1/\sqrt{2}\cos(2\pi w x)dx, \int_{-1}^{1} 1/\sqrt{2}\cos(2\pi w x)dx] \\ &- i[\int_{-1}^{1} -1/\sqrt{2}\sin(2\pi w x)dx, \int_{-1}^{1} 1/\sqrt{2}\sin(2\pi w x)dx] \\ &= [\frac{-1}{\sqrt{2}\pi w}\sin(2\pi w), \frac{1}{\sqrt{2}\pi w}\sin(2\pi w)] - i\{0\} \\ &= [-\sqrt{2}sinc(2\pi w), \sqrt{2}sinc(2\pi w)]. \end{aligned}$$

*In classical analysis, we know that the Fourier transform of the box function*

$$f(t) = \begin{cases} 1/\sqrt{2} & , \ \textit{for } |t| \le 1; \\ 0 & , \ \textit{otherwise} \end{cases}$$

*is just the function*

$$\hat{f}(w) = \sqrt{2}sinc(2\pi w).$$

*This example shows that the interval-valued function* $F$ *given by*

$$F(t) = \begin{cases} [-1/\sqrt{2}, 1/\sqrt{2}] & , \ \textit{for } |t| \le 1; \\ \{0\} & , \ \textit{otherwise} \end{cases}$$

*is the interval-valued function* $\hat{F}$ *given by*

$$\begin{aligned} \hat{F}(w) &= [-\sqrt{2}sinc(2\pi w), \sqrt{2}sinc(2\pi w)] \\ &= sinc(2\pi w)[-\sqrt{2}, \sqrt{2}]. \end{aligned}$$

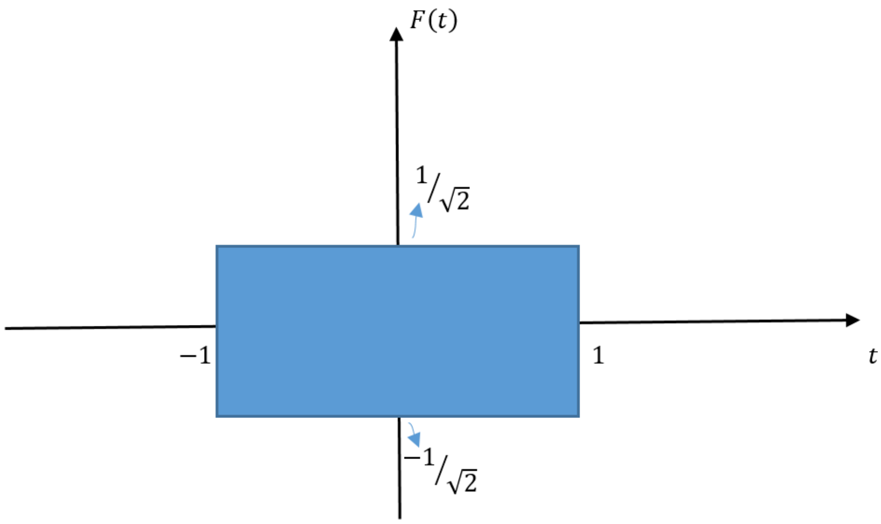


Figure 1: The graph of the interval-valued function $F$

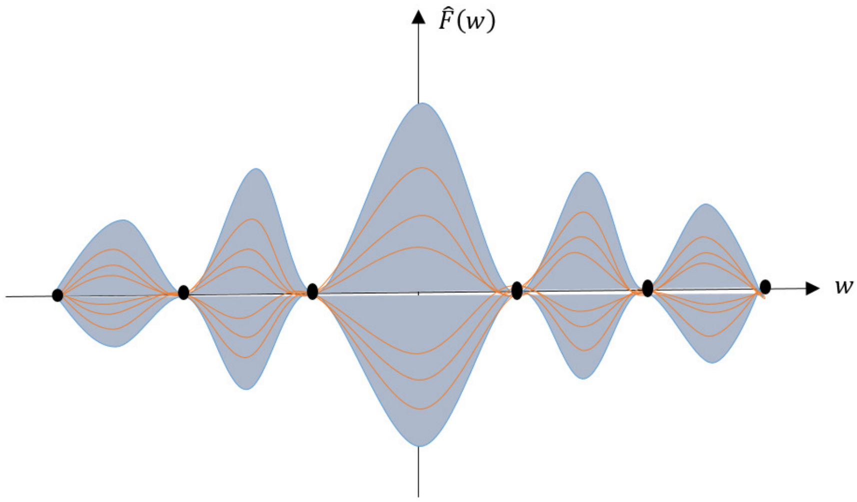


Figure 2: The estimated graph of Fourier transform of the interval-valued function $F$

**Remark 3.1.** *Here, we should note that the lower and upper bounds of the interval $\hat{F}(w)$ is not affected by the changes of $w$ frequency values since $[-\sqrt{2}, \sqrt{2}]$ is a symmetric interval. $\hat{F}$ is illustrated in Figure-2. For any selection $g$ of $F$, its Fourier transform $\hat{g}$ must be a selection of $\hat{F}$. Further if $g$ is a linear selection of $F$, that is $g$ is a linear real-valued function, then its Fourier transform $\hat{g}(w)$ must be similar to the function $sinc(2\pi w)$, and it must obey to the restrictions*

$$-sinc(2\pi w) \leq \hat{g}(w) \leq sinc(2\pi w)$$

*or*

$$sinc(2\pi w) \leq \hat{g}(w) \leq -sinc(2\pi w)$$

*since it is a selection of $\hat{F}$. This is also illustrated in Figure 2.*

**Conclusion 3.1.** *If certain characteristics of a non-deterministic signal are known, can some approximate results be obtained concerning the frequency, Fourier transform, z-transform, Laplace transform or other characteristics of the signal?.By using quasilinear functional analysis and interval-valued functions, we can have important information about the frequency content of some non-deterministic signals obeying certain restrictions.*

*The set-valued Fourier transform helps us obtain such information. In some cases, even a little information about the frequency content of the signals may be sufficient to solve some important problems. In the future, the z-transform and Laplace transform of signals with inexact data can be defined using quasilinear functional analysis techniques and interval analysis with the help of similar methods in this study.*

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