Earthline Journal of Mathematical Sciences
E-ISSN: 2581-8147
Volume 16, Number 2, March 2026, Pages 307-317
https://doi.org/10.34198/ejms.16226.24.307317

Some New Integral Inequalities Involving a Monotonic Function and an Adjustable Parameter

Christophe Chesneau

Department of Mathematics, LMNO, University of Caen-Normandie, 14032 Caen, France
e-mail: christophe.chesneau@gmail.com

Abstract

This article presents and proves two new theorems concerning integral inequalities involving monotonic functions. Each theorem incorporates an adjustable parameter, allowing for greater flexibility and generality. Several related propositions are also derived from these theorems, thereby extending their scope. To demonstrate the applicability and effectiveness of the new inequalities, the theoretical findings are presented alongside selected numerical examples.

1 Introduction

Integral inequalities involving monotonic functions, such as the well-known Chebyshev integral inequality, play a key role in the theory of analysis and its numerous applications. These inequalities are powerful tools for estimating solutions to differential and integral equations, analyzing the stability of dynamic systems, and studying the convergence properties of various functional sequences. Foundational results and classical approaches in this area can be found in [1, 2, 8, 10, 19, 20].

Over the past decade, there has been a significant increase in research focused on refining, extending and generalizing integral inequalities associated with monotonic functions. Recent contributions, including those presented in [3–7, 9, 11–18], have introduced innovative techniques and new classes of inequalities. The ongoing study of integral inequalities involving monotonic functions continues to strengthen the theoretical foundations of mathematical analysis. Formulating new, more general forms of such inequalities, in particular, deepens our understanding of the interplay between monotonicity, integrability, and functional relationships.

This article makes a contribution to this active area of research by presenting two new theorems that depend on an adjustable parameter. The first theorem takes the following form:

$$\int_0^1 x^{\alpha} f^{\alpha+1}(x)dx \leq \frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1},$$

Received: January 17, 2026; Accepted: February 26, 2026; Published online: March 6, 2026
2020 Mathematics Subject Classification: 26D15.
Keywords and phrases: integral inequalities, Chebyshev integral inequality.

where $\alpha \geq 0$ is an adjustable parameter and $f : [0,1] \to [0,+\infty)$ is a non-increasing function. The proof relies solely on elementary integration techniques and the fundamental properties of non-increasing functions. This makes it both accessible and elegant. This result can also be regarded as a kind of reverse Hölder-type integral inequality involving a power weight function x^α. This inequality establishes a useful relationship between the weighted integral of a power of a monotonic function and the power of its unweighted integral, revealing a structural duality between the two forms. The two theorems enable us to derive four new propositions that extend the scope of integral inequalities involving monotonic functions even further. Several numerical examples illustrate the theory and demonstrate the sharpness of the inequalities. Overall, our results strengthen the theoretical framework of the subject by providing new insights and tools for analyzing inequalities related to monotonicity and integrability.

The remainder of the article is organized as follows: Section 2 investigates integral inequalities involving a non-increasing function, referred to as the non-increasing case. Section 3 presents the analogous results for the non-decreasing case. Finally, Section 4 provides concluding remarks and discusses possible directions for future research.

2 The Non-increasing Case

2.1 Main result

The first theorem, concerning the non-increasing case, is stated below, followed immediately by its proof.

Theorem 2.1. *Let $\alpha \geq 0$ and $f : [0,1] \to [0,+\infty)$ be a non-increasing function. Then we have the following inequality:*

$$\int_0^1 x^\alpha f^{\alpha+1}(x)dx \leq \frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1}.$$

Proof. The proof relies on a clever intermediate integral inequality. Since f is non-increasing, for any $t \in (0,x)$, we have $f(x) \leq f(t)$. Therefore, the following inequality holds:

$$x^\alpha f^\alpha(x) = \left(f(x)\int_0^x dt\right)^\alpha \leq \left(\int_0^x f(t)dt\right)^\alpha.$$

Using this and standard power primitive developments, including $\left(\int_0^x f(t)dt\right)' = f(x)$ and $\int_0^0 f(t)dt = 0$, we get

$$\begin{aligned}\int_0^1 x^\alpha f^{\alpha+1}(x)dx &= \int_0^1 f(x)x^\alpha f^\alpha(x)dx \\ &\leq \int_0^1 f(x)\left(\int_0^x f(t)dt\right)^\alpha dx \\ &= \left[\frac{1}{\alpha+1}\left(\int_0^x f(t)dt\right)^{\alpha+1}\right]_0^1 \\ &= \frac{1}{\alpha+1}\left(\int_0^1 f(t)dt\right)^{\alpha+1}.\end{aligned}$$

This completes the proof of the theorem. □

Despite the relative simplicity of the proof, to the best of our knowledge, the integral inequality stated in Theorem 2.1 is a new addition to the existing literature. As mentioned in the introduction, it can be viewed as a kind of reverse Hölder-type integral inequality involving a power weight function.

To complete this theorem, note that, by applying the Hölder integral inequality, we have

$$\left(\int_0^1 f(x)dx\right)^{\alpha+1} \leq \int_0^1 f^{\alpha+1}(x)dx.$$

This and Theorem 2.1 yield the elegant inequalities:

$$\int_0^1 x^\alpha f^{\alpha+1}(x)dx \leq \frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1} \leq \frac{1}{\alpha+1}\int_0^1 f^{\alpha+1}(x)dx.$$

In particular, they are considerably sharper than the following trivial integral inequality, which relies only on the fact that $x^\alpha \in [0,1]$ for any $x \in [0,1]$, without taking into account the monotonic nature of the function f:

$$\int_0^1 x^\alpha f^{\alpha+1}(x)dx \leq \int_0^1 f^{\alpha+1}(x)dx.$$

Let us illustrate Theorem 2.1 with a numerical example. Setting $f(x) = 1/(2+x)$, $x \in [0,1]$, which is obviously non-increasing, and $\alpha = 1$, we get

$$\int_0^1 x^\alpha f^{\alpha+1}(x)dx = \int_0^1 x\left(\frac{1}{2+x}\right)^2 dx \approx 0.0721$$

and

$$\frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1} = \frac{1}{2}\left(\int_0^1 \frac{1}{2+x}dx\right)^2 \approx 0.0822.$$

Of course, we have $0.0721 < 0.0822$. This illustrates the desired inequality.

The remainder of this section is devoted to the presentation of several additional integral inequalities that are derived as consequences of Theorem 2.1.

2.2 Special cases $\alpha = n$

A non-trivial integral inequality, derived as a consequence of Theorem 2.1, is presented in the proposition below.

Proposition 2.2. *Let $f : [0,1] \to [0,1)$ be a non-increasing function such that $\int_0^1 f(x)dx < 1$. Then we have the following inequality:*

$$\int_0^1 \frac{f(x)}{1 - xf(x)} dx \leq -\log\left(1 - \int_0^1 f(x)dx\right).$$

Proof. Applying Theorem 2.1 to $\alpha = n$ for $n \in \mathbb{N}$, we have

$$\int_0^1 x^n f^{n+1}(x)dx \leq \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1}.$$

Summing both sides with respect to n, we get

$$\sum_{n=0}^{+\infty}\left(\int_0^1 x^n f^{n+1}(x)dx\right) \leq \sum_{n=0}^{+\infty} \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1}. \tag{1}$$

Using the fact that $xf(x) \in [0,1)$ for any $x \in [0,1)$, together with the uniform convergence of the geometric series expansion, we obtain

$$\begin{aligned} \sum_{n=0}^{+\infty}\left(\int_0^1 x^n f^{n+1}(x)dx\right) &= \int_0^1 f(x)\sum_{n=0}^{+\infty}(xf(x))^n dx \\ &= \int_0^1 \frac{f(x)}{1 - xf(x)}dx. \end{aligned} \tag{2}$$

Applying the logarithmic series expansion directly, taking into account that $\int_0^1 f(x)dx \in [0,1)$, we have

$$\sum_{n=0}^{+\infty} \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1} = -\log\left(1 - \int_0^1 f(x)dx\right). \tag{3}$$

Combining Equations (1), (2) and (3), we get

$$\int_0^1 \frac{f(x)}{1 - xf(x)} dx \leq -\log\left(1 - \int_0^1 f(x)dx\right).$$

This ends the proof of the proposition. □

Let us illustrate Proposition 2.2 with a numerical example. Setting $f(x) = 1/(2+x)$, $x \in [0,1]$, we get

$$\int_0^1 \frac{f(x)}{1 - xf(x)} dx = \int_0^1 \frac{1/(2+x)}{1 - x/(2+x)} dx = 0.5$$

and

$$-\log\left(1-\int_0^1 f(x)dx\right) = -\log\left(1-\int_0^1 \frac{1}{2+x}dx\right) \approx 0.5199.$$

Clearly, $0.5 < 0.5199$, which illustrates the desired inequality.

Another result derived as a consequence of Theorem 2.1 is presented in the proposition below. The proof employs series expansion techniques similar to those developed in the proof of Proposition 2.2.

Proposition 2.3. *Let $f : [0,1] \to [0,1)$ be a non-increasing function such that $\int_0^1 f(x)dx < 1$. Then we have the following inequality:*

$$\int_0^1 \frac{f(x)}{(1-xf(x))^2}dx \leq \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}.$$

Proof. Applying Theorem 2.1 to $\alpha = n$ for $n \in \mathbb{N}$, we have

$$\int_0^1 x^n f^{n+1}(x)dx \leq \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1},$$

so that

$$\int_0^1 (n+1)x^n f^{n+1}(x)dx \leq \left(\int_0^1 f(x)dx\right)^{n+1}.$$

Summing both sides with respect to n, we get

$$\sum_{n=0}^{+\infty}\left(\int_0^1 (n+1)x^n f^{n+1}(x)dx\right) \leq \sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^{n+1}. \tag{4}$$

Using the fact that $xf(x) \in [0,1)$ for any $x \in [0,1)$, together with the uniform convergence of the geometric series expansion, we obtain

$$\begin{aligned}\sum_{n=0}^{+\infty}\left(\int_0^1 (n+1)x^n f^{n+1}(x)dx\right) &= \int_0^1 f(x)\sum_{n=0}^{+\infty}(n+1)(xf(x))^n dx \\ &= \int_0^1 \frac{f(x)}{(1-xf(x))^2}dx. \end{aligned} \tag{5}$$

Using the geometric series expansion taking into account that $\int_0^1 f(x)dx \in [0,1)$, we have

$$\sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^{n+1} = \left(\int_0^1 f(x)dx\right)\sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^n = \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}. \tag{6}$$

Combining Equations (4), (5) and (6), we get

$$\int_0^1 \frac{f(x)}{(1-xf(x))^2}dx \leq \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}.$$

This concluses the proof of the proposition. □

Let us illustrate Proposition 2.3 with a numerical example. Setting $f(x) = 1/(2+x)$, $x \in [0,1]$, we get

$$\int_0^1 \frac{f(x)}{(1-xf(x))^2}dx = \int_0^1 \frac{1/(2+x)}{(1-x/(2+x))^2}dx = 0.625$$

and

$$\frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx} = \frac{\int_0^1 1/(2+x)dx}{1-\int_0^1 1/(2+x)dx} \approx 0.6820.$$

Obviously, we have $0.625 < 0.6820$. This illustrates the desired inequality.

The remainder of the article is devoted to the non-decreasing case.

3 The Non-decreasing Case

3.1 Main result

The second theorem, concerning the non-decreasing case, is stated below, followed immediately by its proof.

Theorem 3.1. *Let $\alpha \geq 0$ and $f : [0,1] \to [0,+\infty)$ be a non-decreasing function. Then we have the following inequality:*

$$\int_0^1 (1-x)^\alpha f^{\alpha+1}(x)dx \leq \frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1}.$$

Proof. The proof relies on a first clever inequality. Since f is non-decreasing, for any $t \in (x,1)$, we have $f(x) \leq f(t)$. Therefore, the following inequality holds:

$$(1-x)^\alpha f^\alpha(x) = \left(f(x)\int_x^1 dt\right)^\alpha \leq \left(\int_x^1 f(t)dt\right)^\alpha.$$

Using this and standard power primitive developments, including $\left(\int_x^1 f(t)dt\right)' = -f(x)$ and $\int_1^1 f(t)dt = 0$, we get

$$\begin{aligned}\int_0^1 (1-x)^\alpha f^{\alpha+1}(x)dx &= \int_0^1 f(x)(1-x)^\alpha f^\alpha(x)dx \\ &\leq \int_0^1 f(x)\left(\int_x^1 f(t)dt\right)^\alpha dx \\ &= \left[-\frac{1}{\alpha+1}\left(\int_x^1 f(t)dt\right)^{\alpha+1}\right]_0^1 \\ &= \frac{1}{\alpha+1}\left(\int_0^1 f(t)dt\right)^{\alpha+1}.\end{aligned}$$

This completes the proof of the theorem. □

To the best of our knowledge, the integral inequality stated in Theorem 3.1 is a new addition to the existing literature. Compared with Theorem 2.1, we can see that the change in monotonicity has resulted in a change to the weight function, which is now $(1-x)^{\alpha}$ rather than x^{α}. However, the upper bound remains unchanged. This inequality can be viewed as a kind of reverse Hölder-type integral inequality involving a power weight function, i.e., $(1-x)^{\alpha}$.

To complete it, note that, by applying the Hölder integral inequality, we have

$$\left(\int_0^1 f(x)dx\right)^{\alpha+1} \leq \int_0^1 f^{\alpha+1}(x)dx.$$

This and Theorem 3.1 yield the elegant inequalities:

$$\int_0^1 (1-x)^{\alpha} f^{\alpha+1}(x)dx \leq \frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1} \leq \frac{1}{\alpha+1}\int_0^1 f^{\alpha+1}(x)dx.$$

Let us illustrate Theorem 3.1 with a numerical example. Setting $f(x) = x/2$, $x \in [0,1]$, which is obviously non-decreasing, and $\alpha = 1$, we get

$$\int_0^1 (1-x)^{\alpha} f^{\alpha+1}(x)dx = \int_0^1 (1-x)\left(\frac{x}{2}\right)^2 dx \approx 0.0208$$

and

$$\frac{1}{\alpha+1}\left(\int_0^1 f(x)dx\right)^{\alpha+1} = \frac{1}{2}\left(\int_0^1 \frac{x}{2}dx\right)^2 \approx 0.0312.$$

Clearly, $0.0208 < 0.0312$, which illustrates the desired inequality.

3.2 Special cases $\alpha = n$

A non-trivial integral inequality, derived as a consequence of Theorem 3.1, is presented in the proposition below.

Proposition 3.2. *Let $f : [0,1] \to [0,1)$ be a non-decreasing function such that $\int_0^1 f(x)dx < 1$. Then we have the following inequality:*

$$\int_0^1 \frac{f(x)}{1-(1-x)f(x)}dx \leq -\log\left(1 - \int_0^1 f(x)dx\right).$$

Proof. Applying Theorem 3.1 to $\alpha = n$ for $n \in \mathbb{N}$, we have

$$\int_0^1 (1-x)^n f^{n+1}(x)dx \leq \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1}.$$

Summing both sides with respect to n, we get

$$\sum_{n=0}^{+\infty}\left(\int_0^1 (1-x)^n f^{n+1}(x)dx\right) \leq \sum_{n=0}^{+\infty}\frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1}. \tag{7}$$

Using the fact that $(1-x)f(x) \in [0,1)$ for any $x \in [0,1)$, together with the uniform convergence of the geometric series expansion, we obtain

$$\begin{aligned}\sum_{n=0}^{+\infty}\left(\int_0^1 (1-x)^n f^{n+1}(x)dx\right) &= \int_0^1 f(x)\sum_{n=0}^{+\infty}((1-x)f(x))^n dx\\ &= \int_0^1 \frac{f(x)}{1-(1-x)f(x)}dx. \end{aligned} \tag{8}$$

Applying the logarithmic series expansion directly, taking into account that $\int_0^1 f(x)dx \in [0,1)$, we have

$$\sum_{n=0}^{+\infty}\frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1} = -\log\left(1-\int_0^1 f(x)dx\right). \tag{9}$$

Combining Equations (7), (8) and (9), we get

$$\int_0^1 \frac{f(x)}{1-(1-x)f(x)}dx \leq -\log\left(1-\int_0^1 f(x)dx\right).$$

This concludes the proof of the proposition. □

Let us illustrate Proposition 3.2 with a numerical example. Setting $f(x) = x/2$, $x \in [0,1]$, we get

$$\int_0^1 \frac{f(x)}{1-(1-x)f(x)}dx = \int_0^1 \frac{x/2}{1-(1-x)x/2}dx \approx 0.2731$$

and

$$-\log\left(1-\int_0^1 f(x)dx\right) = -\log\left(1-\int_0^1 \frac{x}{2}dx\right) \approx 0.2876.$$

Obviously, we have $0.2731 < 0.2876$. This illustrates the desired inequality.

Another result derived as a consequence of Theorem 3.1 is presented in the proposition below. The proof employs series expansion techniques similar to those developed in the proof of Proposition 3.2.

Proposition 3.3. *Let $f : [0,1] \to [0,1)$ be a non-decreasing function such that $\int_0^1 f(x)dx < 1$. Then we have the following inequality:*

$$\int_0^1 \frac{f(x)}{(1-(1-x)f(x))^2}dx \leq \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}.$$

Proof. Applying Theorem 3.1 to $\alpha = n$ for $n \in \mathbb{N}$, we have

$$\int_0^1 (1-x)^n f^{n+1}(x)dx \leq \frac{1}{n+1}\left(\int_0^1 f(x)dx\right)^{n+1},$$

so that

$$\int_0^1 (n+1)(1-x)^n f^{n+1}(x)dx \leq \left(\int_0^1 f(x)dx\right)^{n+1}.$$

Summing both sides with respect to n, we get

$$\sum_{n=0}^{+\infty}\left(\int_0^1 (n+1)(1-x)^n f^{n+1}(x)dx\right) \leq \sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^{n+1}. \tag{10}$$

Using the fact that $(1-x)f(x) \in [0,1)$ for any $x \in [0,1)$, together with the uniform convergence of the geometric series expansion, we obtain

$$\begin{aligned} \sum_{n=0}^{+\infty}\left(\int_0^1 (n+1)(1-x)^n f^{n+1}(x)dx\right) &= \int_0^1 f(x)\sum_{n=0}^{+\infty}(n+1)((1-x)f(x))^n dx \\ &= \int_0^1 \frac{f(x)}{(1-(1-x)f(x))^2}dx. \end{aligned} \tag{11}$$

Using the geometric series expansion, taking into account that $\int_0^1 f(x)dx \in [0,1)$, we have

$$\sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^{n+1} = \left(\int_0^1 f(x)dx\right)\sum_{n=0}^{+\infty}\left(\int_0^1 f(x)dx\right)^{n} = \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}. \tag{12}$$

Combining Equations (10), (11) and (12), we get

$$\int_0^1 \frac{f(x)}{(1-(1-x)f(x))^2}dx \leq \frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx}.$$

This ends the proof of the proposition. □

Let us illustrate Proposition 3.3 with a numerical example. Setting $f(x) = x/2$, $x \in [0,1]$, we get

$$\int_0^1 \frac{f(x)}{(1-(1-x)f(x))^2}dx = \int_0^1 \frac{x/2}{(1-(1-x)x/2)^2}dx \approx 0.2989$$

and

$$\frac{\int_0^1 f(x)dx}{1-\int_0^1 f(x)dx} = \frac{\int_0^1 (x/2)dx}{1-\int_0^1 (x/2)dx} \approx 0.3333.$$

Clearly, $0.2989 < 0.3333$, which illustrates the desired inequality.

4 Conclusion

In this article, we introduced two new integral inequalities involving monotonic functions, each with an adjustable parameter, and derived several related propositions. We presented numerical examples to demonstrate the effectiveness and applicability of the results. Future research could explore extending these inequalities to multidimensional settings and to those involving more general weight functions. They could also be applied to the study of integral operators, differential equations and approximation theory.

References

[1] Bainov, D., & Simeonov, P. (1992). *Integral inequalities and applications* (Mathematics and Its Applications, Vol. 57). Kluwer Academic Publishers. https://doi.org/10.1007/978-94-011-2446-1

[2] Beckenbach, E. F., & Bellman, R. (1961). *Inequalities*. Springer-Verlag. https://doi.org/10.1007/978-3-642-64971-4

[3] Benaissa, B., & Budak, H. (2021). On Hardy-type integral inequalities with negative parameter. *Turkish Journal of Inequalities, 5*(1), 42–47. https://doi.org/10.53453/tjiaas.935106

[4] Benaissa, B., & Senouci, A. (2022). New integral inequalities relating to a general integral operator through monotone functions. *Sahand Communications in Mathematical Analysis, 19*(1), 41–56. https://doi.org/10.22130/scma.2021.528430.932

[5] Chesneau, C. (2024). A novel multivariate integral ratio operator: Theory and applications including inequalities. *Asian Journal of Mathematics and Applications, 2024*, 1–37.

[6] Chesneau, C. (2024). A generalization of the Du integral inequality. *Transactions on Journal of Mathematical Analysis and Applications, 12*, 45–52.

[7] Chesneau, C. (2025). Integral inequalities under diverse parametric primitive exponential-weighted integral inequality assumptions. *Annals of Communications in Mathematics, 8*, 43–56.

[8] Cvetkovski, Z. (2012). *Inequalities: Theorems, techniques and selected problems*. Springer. https://doi.org/10.1007/978-3-642-23792-8

[9] Du, W.-S. (2023). New integral inequalities and generalizations of Huang-Du's integral inequality. *Applied Mathematics Sciences, 17*(6), 265–272. https://doi.org/10.12988/ams.2023.91717

[10] Hardy, G. H., Littlewood, J. E., & Pólya, G. (1934). *Inequalities*. Cambridge University Press. https://doi.org/10.1017/CBO9780511661846

[11] Huang, H., & Du, W.-S. (2022). On a new integral inequality: Generalizations and applications. *Axioms, 11*(11), 601. https://doi.org/10.3390/axioms11110601

[12] Móri, T. F. (2009). A general inequality of Ngo-Thang-Dat-Tuan type. *Journal of Inequalities in Pure and Applied Mathematics, 10*(3), Art. 84.

[13] Ngo, Q. A., Thang, D. D., Dat, T. T., & Tuan, D. A. (2006). Notes on an integral inequality. *Journal of Inequalities in Pure and Applied Mathematics, 7*(4), Art. 120.

[14] Senouci, A., Benaissa, B., & Sofrani, M. (2023). Some new integral inequalities for negative summation parameters. *Survey in Mathematics and its Applications, 18*, 123–133.

[15] Sulaiman, W. T. (2008). Notes on integral inequalities. *Demonstratio Mathematica, 41*(4), 887–894. https://doi.org/10.1515/dema-2013-0111

[16] Sulaiman, W. T. (2011). New several integral inequalities. *Tamkang Journal of Mathematics, 42*(4), 505–510. https://doi.org/10.5556/j.tkjm.42.2011.890

[17] Sulaiman, W. T. (2011). Several ideas on some integral inequalities. *Advances in Pure Mathematics, 1*(3), 63–66. https://doi.org/10.4236/apm.2011.13012

[18] Sulaiman, W. T. (2012). A study on several new integral inequalities. *South Asian Journal of Mathematics, 2*(4), 333–339.

[19] Walter, W. (1970). *Differential and integral inequalities*. Springer-Verlag. https://doi.org/10.1007/978-3-642-86259-5

[20] Yang, B. C. (2009). *Hilbert-type integral inequalities*. Bentham Science Publishers. https://doi.org/10.2174/97816080504061090101