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A Spatial Analysis of Multimodal Transport in Kisii Town Using the LWR Model

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Abstract

Traffic flow in most urban areas is augmenting due to the growth in transport and continual demand for it. It is multimodal and includes use of different types of vehicles, motorcycles and even walking. The assessment of uninterrupted traffic flow is traditionally based on empirical methods. This study was based on the macroscopic model which is a mathematical model that formulates the relationships among traffic flow characteristics like density, flow, mean and speed of a traffic stream. The study considered traffic models first developed by Lighthill and Whitham [14] and later Richards [20] shortly called LWR traffic flow model. Simulation by use of this method enables control strategies of congestion dissipation and has suggested some recommended measures to rationalize the design of roads and implementation of regulations of road users considering some regulations and infrastructural gaps in Kisii town. This paper focuses on two finite difference schemes, that is, first order Explicit Upwind Difference Scheme-$EUDS$ (forward time, backward space) and second order Lax-Wendroff Difference Scheme-$LWDS$ (forward time, centred space) for solving first order PDE as well as the traffic density $\rho(t,x)$ was computed by solving LWR macroscopic conservation form of traffic flow model using both schemes. The conditions of stability were numerically verified and it is shown that $LWDS$ is superior to $EUDS$ in terms of time step selection. The results obtained were compared with average key data, which provided the initial and boundary conditions used for numerical simulation.

1 Introduction

This chapter focuses on traffic flow, specifically the use of models to control traffic congestion. Traffic flow can be defined as the study of how motorized transport moves between origin and destination, and how

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individual drivers interact with others. Since driver behaviour cannot be predicted with absolute certainty, mathematical models have been built [2]. The analysis of traffic flow and modelling of vehicular congestion has mainly relied on fundamental laws inspired by physics, using analogies from fluid mechanics and many-particle systems. One main difference between physical systems and vehicular traffic is that humans make choices in terms of routes, destinations, and driving behaviour, which creates additional complexity in the system [10]. While most traffic science theories make a clear distinction between free-flow and congested traffic states, empirical analysis of spatio-temporal congestion patterns has revealed additional complexity, as noted by Munoz and Daganzo [18]. These patterns have serious spill-over effects in urban areas. According to Kumar et al. [12], most towns are undergoing multifaceted problems because of rapid urbanization. Traffic congestion is intolerable in most towns, mainly due to the upsurge of motorcycle transport and rural-urban migration. The increase in motorized transport poses a severe challenge to traffic flow control for all large and growing towns [19].

2 Background of the Study

Interest in modelling traffic flow has a long history. In the 1930s, the scientific study of traffic flow began with Greenshields et al. [6], who developed a two-variable model focused on volume and speed. During this period, photography was also used for monitoring traffic flow. Later, probability theory was introduced in the description of traffic by Adams [1]. Traffic performance at street intersections was also investigated by Greenshields et al. [6]. The first papers to discuss traffic congestion problems were written by Lighthill and Whitham [14] and Richards [20]. Their work was initially motivated by flood movement in long rivers but was later applied to the theory of traffic flow on long crowded roads, culminating in the well-known LWR (Lighthill-Whitham-Richards) model by Morgan [17]. More recently, in order to understand and optimize traffic flow dynamics, mathematical modelling and simulation have provided deeper insights into developing more effective transportation systems and well-informed policy decisions.

2.1 Kisii Town

Kisii town has a strategic location, and its elevation to county status has contributed to some infrastructural development, especially of roads serving the town. Some roads that were previously single-lane carriageways have now been upgraded to dual carriageways. These roads serve the Central Business District (CBD). Unfortunately, little has been done to match the infrastructural development outside the town centre with what is happening within it and in some parts of its environs. No road within the CBD is a dual carriageway; they are all single-lane carriageway roads. The streets are poorly connected, with competing modes of transport. The known fundamental diagram, initially observed for a stretch of road to describe traffic behaviour, does not sufficiently capture the additional complexity in traffic systems, as noted by Helbing et al. [8]. It also contains experimental errors in congested highway stretches [11] or in city streets, as reported by Daganzo and Geroliminis [5]. The town has paved roads radiating from its centre, with several intersections and two traffic circles. Alongside many of these roads are market-like scenarios encroaching onto the road.

2.2 Modelling

The analysis of the town's network and traffic flow was conducted using a model whose primary purpose was to study the consistent behaviour of traffic streams through relationships such as flow (q), density (ρ), and velocity (v) [3]. The continuum traffic flow model, first developed by Lighthill and Whitham [14] and later Richards [20], is macroscopic in nature and is based on the assumption of mass density conservation; that is, the number of vehicles between any two points is conserved if there are no entrances or exits. The LWR model is a first-order model in the sense that it is formulated as a scalar hyperbolic conservation law and is often solved by finite difference methods [16]. The non-linear first-order partial differential equation is posed as an initial and boundary value problem (IBVP). The numerical solution of this equation was obtained using the Explicit Upwind Difference Scheme ($EUDS$) and the Lax-Wendroff Difference Scheme ($LWDS$), both subject to appropriate initial and boundary conditions. The traffic density $\rho(x, t)$ was computed using both schemes, recalling the continuity equation in mathematical modelling.

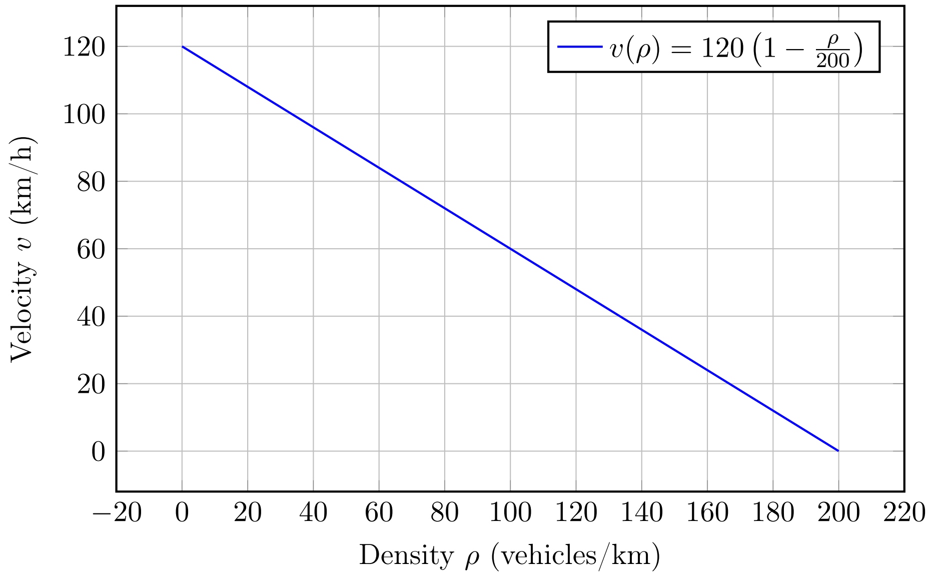
$$\rho t + (\rho v)_x = 0. \tag{2.1}$$

With ρ denoting the density of cars on the road and v their velocity, it is difficult to track individual vehicles; instead, an average is taken over a given road segment. Here, $\rho = 0$ corresponds to an empty road, while $\rho = 1$ represents bumper-to-bumper traffic. To capture the variation in driver speeds depending on density, the velocity is modeled as a linearly decreasing function of density, as shown below.

To illustrate the linear decrease of velocity with increasing traffic density, we use the fundamental relationship:

$$v(\rho) = v_{\max}\left(1 - \frac{\rho}{\rho_{\max}}\right),$$

where $v_{\max} = 120\,\text{km/h}$ is the maximum free-flow speed and $\rho_{\max} = 200\,\text{vehicles/km}$ is the jam density. The corresponding graph depicts how velocity declines as density approaches $\rho_{\max}$.



It is noticeable that v approaches zero as ρ approaches $\rho_{\max}$, corresponding to the normalized density interval $[0, 1]$.

Making velocity a linearly decreasing function of density:

$$v(\rho) = 1 - \rho. \tag{2.2}$$

Combining equations 2.1 and 2.2, then the conservation law is;

$$\rho t + [\rho(1-\rho)]_x = 0. \tag{2.3}$$

The function $\rho(1-\rho)$ is the flux or rate of flow of cars. We can have a non-linear flux resulting from advection equation. It can be made look linear using chain rule.

$$f(\rho)_x = f^1(\rho)\rho_x = (1-2\rho)\rho_x. \tag{2.4}$$

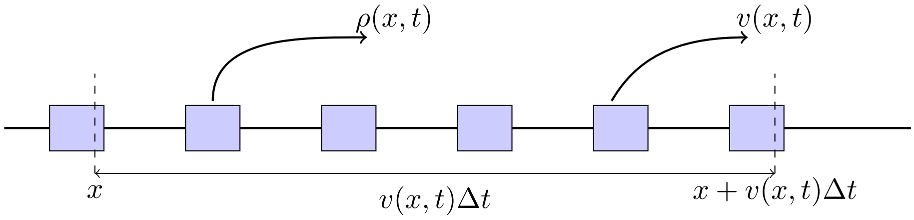
Then,

$$\rho_t + (1-2\rho)\rho_x = 0. \tag{2.5}$$

Equation 2.5 the advection equation but with a velocity $(1-2\rho)$ that depends on the density of the cars. The value $f^1(\rho) = 1 - 2\rho$ is referred to as the characteristic speed; which is not the speed at which the cars move, rather the speed at which information is transmitted along the road.

2.3 Relationship of Traffic Flow Variables

Let $\rho(x, t)$ and $v(x, t)$ be continuous functions of space x and time t. Consider a very small time interval Δt, during which the values of $\rho(x, t)$ and $v(x, t)$ can be approximated as constants. Hence, over the interval $(x,t) \to (x,t+\Delta t)$, the number of vehicles present in a given road segment can be determined, as illustrated in the figure below [9].



Therefore, the number of vehicles passing an observer can be expressed as $v(x,t)\,\Delta t\,\rho(x,t)$. Hence, by definition,

$$Q(x,t) = \rho(x,t)\,v(x,t), \tag{2.6}$$

where $Q(x,t)$ denotes the traffic flow rate.

Speed (v) in traffic flow theory is defined as the travel distance per unit time. In practice, the precise speed of every vehicle is difficult to measure. Instead, the average speed of sampled vehicles is often calculated. This is referred to as the *time mean speed*, defined as the average speed of a traffic stream passing a fixed point along a roadway over a given period of time [9].

$$v_t = \frac{1}{m}\sum_{i=1}^{m} v_i, \tag{2.7}$$

where m is the number of vehicles passing the fixed point and v_i is the speed of the i-th vehicle [22]. Traffic congestion, as reflected in reduced speeds, has caused considerable inconvenience to people's daily activities, underscoring the importance of addressing this problem.

3 Literature Review

3.1 Introduction

This chapter reviews relevant literature related to the spatial analysis of multimodal transport in Kisii town. It highlights various techniques for evaluating differential equation–based traffic flow models that explain the dynamics of traffic. Traffic flow encompasses the movement of vehicles, people, and other transport-related entities. Spatial analysis of traffic flow helps in understanding and managing traffic patterns and congestion. It is useful in identifying bottlenecks, optimizing routes, and ensuring efficient traffic management. Moreover, it assists urban planners and traffic engineers in making informed decisions about traffic decongestion (Helbing et al. [8]). It also informs the design and implementation of public transportation systems and other infrastructural improvements ([5]).

López [15] noted that individual efforts to avert congestion often seem ineffective and suggested collective action, which includes the use of modern traffic models. Traffic control models provide insights into various traffic metrics used to evaluate the impact of different interventions, such as travel time, speed, flow, and density. A key research concern is how these individual models can be integrated to form a traffic control system that can collectively address disordered traffic.

There is a need for a constant and reliable means of evaluating traffic performance in a network under various traffic and geometric configurations. The development of such performance models extends traffic flow theory to the network level and provides traffic engineers with tools to evaluate system-wide control strategies, particularly in urban areas.

Traffic systems consist of the network topology (e.g., street width and configuration), the number of trips between origin and destination points, and varying demand levels. The proposed models can be categorized as follows:

3.1.1 Microscopic Model

This model describes the dynamics between vehicles' positions and velocities. The purpose of this model is to determine how cars follow one another. The basic assumption is that vehicles maintain a minimum time and distance gap between each other. If the lead vehicle changes its speed, the following vehicle will also adjust its speed accordingly. The speed of vehicle n is denoted as

$$\frac{dx_n(t)}{dt} = \dot{x}_n(t), \tag{3.1}$$

and the acceleration of vehicle n is given by

$$\frac{d^2x_n(t)}{dt^2} = \ddot{x}_n(t). \tag{3.2}$$

Chandler et al. [4] first developed the linear car-following model, which can be expressed as

$$\ddot{x}_{n+1}(t+T) = \alpha\big[\dot{x}_n(t) - \dot{x}_{n+1}(t)\big], \tag{3.3}$$

where α is the sensitivity coefficient, $\ddot{x}_{n+1}(t+T)$ is the acceleration of the $(n+1)^{\text{th}}$ car at time $t+T$, $\dot{x}_n(t)$ is the speed of the n^{th} car at time t, and $\dot{x}_{n+1}(t)$ is the speed of the $(n+1)^{\text{th}}$ car at time t.

3.1.2 Macroscopic Model

This model describes traffic flow by using fluid dynamics differential equations:

$$n(x)\frac{\partial c(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = 0, \tag{3.4}$$

where $c(x,t)$ is the traffic density in vehicles per lane per kilometre at location x and time t, $n(x)$ is the number of lanes at position x, and $q(x,t)$ is the traffic flow (traffic intensity) in vehicles per hour at location x and time t.

The aggregated variables $c(x,t)$ and $q(x,t)$ are continuous functions of space and time. The equation expresses the physical principle of traffic flow, which can also be expressed in terms of density and speed as

$$\begin{aligned}&\frac{\partial g(x,v,v_{\text{des}},t)}{\partial t}+v\frac{\partial g(x,v,v_{\text{des}},t)}{\partial x}\\&+\frac{\partial}{\partial v}\left[\frac{(v_{\text{des}}-v)}{C}\,g(x,v,v_{\text{des}},t)\right]\\&=\int_0^\infty(1-\rho_{\text{pass}})(v-v^1)f(x,v,t)\,dv,\end{aligned}\tag{3.5}$$

where

$$f(x,v,t)=\int_0^\infty v(x,v,v_{\text{des}},t)\,dv_{\text{des}},\tag{3.6}$$

ρ_{pass} is the probability of vehicle passing, and $v(x,v,v_{\text{des}},t)$ is the velocity distribution function.

Borrowing from the three models, other scholars proposed traffic control models.

3.1.3 Kinematic Wave Model

The kinematic wave model is the simplest dynamic traffic flow model that reproduces the propagation of traffic waves. It is based on the following elements:

i) Conservation law,

ii) The fundamental diagram,

iii) Initial and boundary conditions.

The governing equations are derived from the continuity equation and the momentum equation. Vehicles on a highway can be considered as a compressible fluid. The density varies from 0 on an empty highway to 1 in bumper-to-bumper traffic [13].

Consider a section of a highway with two counting stations, p_1 and p_2. Let c_1 be the number of cars passing p_1 in time t with corresponding flow q_1, and let c_2 be the number of cars passing p_2 in time t with flow q_2.

Assuming $c_1 > c_2$, we have

$$\Delta c=-\Delta v\,\Delta t.\tag{a}$$

Similarly, for $\rho_1 > \rho_2$,

$$\Delta c = \Delta\rho\,\Delta x. \tag{b}$$

Combining (a) and (b) gives the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{3.7}$$

Flow is taken as a function of density, that is,

$$q = \rho v,$$

where ρ is the traffic density and v is the average speed of vehicles.

3.1.4 Payne–Whitham Model

The Payne–Whitham (PW) model independently studied macroscopic traffic behavior. The continuity equation is the same as in the LWR model, while the second equation characterizes vehicle acceleration [17]. The PW model is given by:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0, \tag{3.8}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x} = \frac{v_c(\rho) - v}{\tau}. \tag{3.9}$$

The term $\frac{c_0^2}{\rho}\frac{\partial \rho}{\partial x}$ represents the driver's spatial adjustment to forward traffic conditions. Traffic alignment occurs during the relaxation time τ. During this process, traffic speed tends towards the equilibrium velocity $v_c(\rho)$, which depends on the density distribution, and is characterized by the relaxation term $\frac{v_c(\rho) - v}{\tau}$.

The constant c_0 is the driver spatial density adjustment parameter. It is a non-negative constant, usually ranging between 2–4 and up to about 57 m/s. However, it cannot fully represent variations in driver behavior and may produce unrealistic results. The anticipation term in the PW model can lead to large changes in acceleration and deceleration when there are abrupt changes in density [5].

3.1.5 Wardrop's Third Principle (Urban Traffic Congestion and Traffic Policy)

Early works by Wardrop (1952) and Smeed (1968) focused on the development of macroscopic models for arterial roads, which were later extended to general network models. Smeed introduced the concept of the number of vehicles that can *usefully* enter the control area of a city, denoted by N, as well as the number of vehicles per unit that can enter the city center.

In general, N depends on several factors including the design of the road network, road width, type of intersection control, distribution of destinations, and vehicle mix. For towns of similar network structure, shape, and control type, the principal variables are: the area of the control zone (a), the total area of the town (A), the fraction of area devoted to roads (f), and the capacity (c) expressed in vehicles per unit time per unit road width. These factors are related through the expression:

$$N = \alpha f c \sqrt{A}, \tag{3.10}$$

where α is a proportionality constant.

4 Materials and Methods

4.1 Introduction

This paper is based on both primary and secondary data. The primary data were collected directly through vehicle counts conducted during peak hours, while the secondary data were obtained from journals and other relevant literature. The study focuses on macroscopic traffic flow using the Lighthill-Whitham-Richards (LWR) model.

4.2 Governing Equation of the LWR Traffic Flow Model

The general mathematical form of the Lighthill-Whitham-Richards (LWR) traffic flow model with initial conditions can be written as an Initial Value Problem [7]:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left[v_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right)\right] = 0, \tag{4.1}$$

with the initial condition

$$\rho(t_0, x) = \rho_0(x),$$

where $\rho(x,t)$ denotes the traffic density, $v_{\max}$ is the maximum speed, and $\rho_{\max}$ is the maximum (jam) density.

4.3 Exact Solution of the Non-Linear PDE of the LWR Traffic Flow Model

The non-linear Initial Value Problem (IVP) in Equation (4.1) can be solved by the method of characteristics. Its exact solution is given by

$$\rho(t,x) = \rho_0\left[x - v_{\max}\left(\frac{1-2\rho}{\rho_{\max}}\right)t\right], \tag{4.2}$$

where $\rho_0(x)$ denotes the initial density distribution.

In practice, however, it is difficult to accurately approximate the initial density function $\rho_0(x)$ from measured traffic data. This limitation necessitates the use of efficient numerical methods for solving the IVP in Equation (4.1). Numerical schemes are thus essential for approximating realistic traffic density evolution in situations where the exact solution cannot be obtained directly from data.

4.4 Finite Difference Method for the LWR Traffic Flow Model

We consider the non-linear partial differential equation (PDE) of the LWR traffic flow model as an Initial Boundary Value Problem (IBVP):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\big[q(\rho)\big] = 0, \qquad t_0 \le t \le T,\ a \le x \le b,$$

with initial condition

$$\rho(t_0, x) = \rho_0(x), \qquad a \le x \le b,$$

and boundary condition

$$\rho(t, a) = \rho_0(t), \qquad t_0 \le t \le T.$$

Here $q(\rho)$ denotes the traffic flux function, which is related to the velocity–density relation by

$$q(\rho) = \rho\, v(\rho),$$

where

$$v(\rho) = v_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right). \tag{4.3}$$

Thus, the governing PDE of the LWR model can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\big[q(\rho)\big] = 0. \tag{4.4}$$

4.5 Explicit Upwind Difference Scheme using FTBS Technique

A finite difference scheme for the non-linear LWR traffic flow model is constructed by discretizing both space and time. The governing IBVP in Equation (4.3) is considered on a uniform grid, where

$$t^{n+1} = t^n + \Delta t, \qquad x_{i+1} = x_i + \Delta x,$$

for $n = 0, 1, \ldots, N-1$ and $i = 1, 2, \ldots, M$.

The time derivative $\frac{\partial \rho}{\partial t}$ is approximated using the forward difference:

$$\frac{\partial \rho}{\partial t} \approx \frac{\rho_i^{n+1} - \rho_i^n}{\Delta t}. \tag{4.5}$$

The spatial derivative of the flux term $\frac{\partial q}{\partial x}$ is approximated using the backward difference:

$$\frac{\partial q}{\partial x} \approx \frac{q(\rho_i^n) - q(\rho_{i-1}^n)}{\Delta x}. \tag{4.6}$$

Substituting (4.5) and (4.6) into Equation (4.2), and denoting $\rho_i^n \approx \rho(t^n, x_i)$, the first-order explicit upwind (FTBS) scheme takes the form

$$\frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} + \frac{q(\rho_i^n) - q(\rho_{i-1}^n)}{\Delta x} = 0. \tag{4.7}$$

Rearranging, we obtain

$$\rho_i^{n+1} = \rho_i^n - \frac{\Delta t}{\Delta x}\Big[q(\rho_i^n) - q(\rho_{i-1}^n)\Big].$$

Here, the flux function $q(\rho)$ is defined as

$$q(\rho) = \rho\, v(\rho) = v_{\max}\left(\rho - \frac{\rho^2}{\rho_{\max}}\right). \tag{4.8}$$

Alternatively, the scheme can be expressed in terms of the characteristic speed $q'(\rho)$ as

$$\rho_i^{n+1} = \rho_i^n - q'(\rho_i^n)\frac{\Delta t}{\Delta x}\left(\rho_i^n - \rho_{i-1}^n\right),$$

where

$$\lambda = q'(\rho_i^n)\,\frac{\Delta t}{\Delta x}. \tag{4.9}$$

Thus, the explicit FTBS upwind scheme is obtained as

$$\rho_i^{n+1} = (1-\lambda)\,\rho_i^n + \lambda\,\rho_{i-1}^n,$$

subject to the CFL condition $\lambda \leq 1$ for numerical stability [7].

4.6 Lax–Wendroff Difference Scheme by FTCS

The Lax–Wendroff difference scheme can be derived by discretizing the time derivative as

$$\frac{\partial \rho}{\partial t} \approx \frac{\rho_i^{n+1} - \rho_i^n}{\Delta t}.$$

The discretization of the spatial derivative $\frac{\partial \rho}{\partial x}$ is obtained by a second-order central difference formula, derived from the Taylor series expansion:

$$\rho(x, t + \Delta t) = \rho(x, t) + \Delta t \frac{\partial \rho}{\partial t} + \frac{(\Delta t)^2}{2!} \frac{\partial^2 \rho}{\partial t^2} + \mathcal{O}((\Delta t)^3). \tag{4.10}$$

Furthermore, the flux derivative is approximated by

$$\frac{\partial}{\partial x}\left[q(\rho)(t^n, x_i)\right] \approx \frac{q(\rho_{i+1}^n) - q(\rho_{i-1}^n)}{2\Delta x}. \tag{4.11}$$

From the conservation law

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0, \tag{4.12}$$

the Cauchy–Kowalewski procedure is applied to replace the time derivatives in (4.10) with spatial derivatives.

From (4.12) we have

$$\frac{\partial \rho}{\partial t} = -\frac{\partial q(\rho)}{\partial x}.$$

Differentiating again with respect to t gives

$$\frac{\partial^2 \rho}{\partial t^2} = -\frac{\partial}{\partial t}\left(\frac{\partial q(\rho)}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{\partial q(\rho)}{\partial t}\right).$$

Using the chain rule and noting that $\frac{\partial q(\rho)}{\partial t} = q'(\rho)\frac{\partial \rho}{\partial t}$, we obtain

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial}{\partial x}\left[q'(\rho)\frac{\partial q(\rho)}{\partial x}\right].$$

Substituting these expressions into (4.10) yields

$$\rho(x, t + \Delta t) = \rho(x, t) - \Delta t \frac{\partial q(\rho)}{\partial x} + \frac{(\Delta t)^2}{2} \frac{\partial}{\partial x}\left[q'(\rho)\frac{\partial q(\rho)}{\partial x}\right] + \mathcal{O}((\Delta t)^3). \tag{4.13}$$

The implementation of the explicit upwind difference scheme (EUDS) and the Lax–Wendroff difference scheme (LWDS) requires care. Since vehicles move in a single direction, the characteristic speed

$$\frac{\partial q}{\partial \rho}$$

must remain positive to ensure numerical stability and physical consistency.

5 Results and Discussion

5.1 Computational Results and Discussion

To test the model, a section of the highway was selected. The section considered was the dual carriageway from Gesonso to the Kisii–Migori Junction, connecting with the double-lane single carriageway through Darajambili to Kisii town. We implemented two numerical finite difference schemes: the first-order Explicit Upwind Difference Scheme (EUDS) and the second-order Lax–Wendroff Difference Scheme (LWDS). Numerical simulations and computer programming were carried out to compare these schemes.

We present the numerical simulation results based on the EUDS and LWDS. The density profiles of the exact solution at different time steps were examined when $V_{\max} = 80\,\text{km/hr}$. Comparisons were made among the exact solution, EUDS, and LWDS at the 60th, 120th, and 180th time steps.

The density profiles obtained using LWDS were found to be much closer to the exact solution compared to those obtained using EUDS. While EUDS results were reasonably close to LWDS, they deviated more from the exact solution. In the graphical representation, the solid line corresponds to the exact solution, the dotted line represents EUDS, and the red line represents LWDS. It was observed that the LWDS density profile showed oscillations (zigzag patterns) near the right boundary at higher time steps.

When discretization parameters were chosen as $\Delta t = 0.1$ and $\Delta x = 0.2$, the right boundary displayed notable oscillations. Reducing the time step to $\Delta t = 0.05$ or $\Delta t = 0.04$ significantly reduced these oscillations. Finally, with $\Delta t = 0.01$, the density profile exhibited almost no oscillations, thereby improving accuracy.

These results demonstrate that both density and velocity of traffic evolve consistently with the initial approximations, and that smaller discretization parameters enhance stability and accuracy of the LWDS.

6 Conclusion and Recommendation

6.1 Conclusion

This study highlighted the usefulness of traffic flow modeling as an important tool for traffic flow control and management. We derived both the exact solution and numerical solutions of the LWR traffic flow model using EUDS and LWDS. The findings showed that the LWDS density profile is much closer to the exact solution, with significantly lower error compared to EUDS. This can be attributed to the higher-order accuracy of LWDS, which is a second-order scheme, whereas EUDS is only first-order accurate.

6.2 Recommendation and Justification

Traffic control aims to ensure accessibility for individuals traveling to different destinations. Road networks should serve as many trips as possible in order to maximize accessibility for a given distribution of travelers. However, developing new realistic traffic flow models remains a challenging task.

This study employed the LWR model, first proposed by Lighthill, Whitham, and Richards, and applied it to the case of urban transport in Kisii town within Kisii County. The findings provide insights into mitigating traffic congestion in Kisii County, where transport demand continues to grow. A major challenge identified is that while peripheral roads have been expanded, little or no development has been done within the Central Business District (CBD). Therefore, significant road expansion within the CBD is recommended.

Finally, this study suggests that ramp modeling could be a logical next step for future research, as it would further enhance the understanding and control of traffic dynamics in growing urban centers.

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