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Application of Optimal Control Strategies on Incidence of Medical Complications in Diabetic Patients' Population

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Abstract

Diabetes mellitus is a chronic condition characterized by elevated blood glucose levels, which can lead to severe health complications if not properly managed. The increasing prevalence of diabetes worldwide has made it a major public health concern. This study formulates and analyzes an optimal control model for diabetes management, focusing on minimizing complications and treatment costs. The model is structured around a population of diabetic patients, incorporating dynamic interactions between healthy, susceptible, diabetic, complication, and treatment populations. An objective functional is defined, integrating costs associated with complications and treatment efforts, and is subjected to optimization through control strategies aimed at enhancing patient education, regular monitoring, and comprehensive care. The application of the Pontryagin Maximum Principle provides a solid theoretical foundation for identifying optimal control strategies. Utilizing a fourth-order Runge-Kutta method, the model is simulated under varying control conditions to assess the impact of interventions. The results demonstrate that increasing control measures significantly reduces the incidence of complications while improving treatment rates. The findings highlight the importance of strategic health management interventions in mitigating the burden of diabetes-related complications and emphasize the model's applicability in real-world healthcare settings. This research provides a robust framework for policymakers and healthcare providers to devise effective strategies that enhance the quality of care for diabetic patients.

1. Introduction

Diabetes, a chronic metabolic disorder characterized by either insufficient insulin production or ineffective insulin utilization, poses a significant global health challenge [1]. Diabetes can result from a combination of genetic and environmental factors. A family history of diabetes increases susceptibility [2], while Type 1 diabetes is linked to autoimmune destruction of insulin-producing beta cells [3]. Type 2 diabetes is commonly caused by insulin resistance, often triggered by obesity, physical inactivity, and unhealthy diets [4]. Age, ethnicity, and hormonal changes, such as pregnancy or conditions like PCOS, further raise the risk [5]. Other contributing factors include high blood pressure, abnormal cholesterol levels, chronic stress, and certain

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medications [6]. Metabolic syndrome and a history of gestational diabetes also increase the likelihood of developing type 2 diabetes [7]. Living with diabetes means facing a gradual onset of long-term complications. The longer you navigate this journey—and the more erratic your blood sugar levels—the greater the risk becomes for complications that can profoundly impact your life. From heart and blood vessel issues to nerve damage and kidney dysfunction, diabetes complications can be severe, potentially leading to disability or even death. The pervasive nature of diabetes means it can affect virtually every part of the body, including the feet, eyes, and skin. Indeed, for some individuals, these complications serve as the initial indication of diabetes. Foot issues, in particular, can escalate and lead to severe complications like neuropathy, skin alterations, calluses, foot ulcers, and impaired circulation [8].

The prevalence of diabetes has surged over recent decades, with approximately 422 million adults worldwide living with the condition in 2014, compared to 108 million in 1980. According to [9], an estimated 463 million adults aged 20-79 were living with diabetes in 2019, and projections indicate that this number will rise to 700 million by 2045 if current trends persist [10]. The rise in diabetes prevalence is attributed to various factors, including sedentary lifestyles, unhealthy dietary habits, obesity, aging populations, and genetic predisposition. This rise, mirrored by an alarming increase in associated risk factors such as obesity, underscores the urgent need for effective management strategies.

i. In the dynamic landscape of African societies, a profound shift is underway, characterized by both urbanization and the increasing influence of Western lifestyles. Urban areas are witnessing a surge in materialistic behaviors, with a marked adoption of cosmopolitan norms and the consumption of convenience foods, rich in fat, sugar, and salt. Meanwhile, rural regions grapple with nutritional deficiencies exacerbated by factors such as drought, poverty, and socio-economic disparity, rather than cultural or religious influences.

ii. This dichotomy in dietary habits is further exacerbated by rapid urban population growth, which has strained the production of traditional staples like sorghum, millet, maize, yam, and plantain. Consequently, many African nations are facing challenges in ensuring an adequate daily dietary energy supply, leading to divergent food patterns between urban and rural dwellers and contributing to the rise in diabetes mellitus. South Africa, in particular, grapples with a substantial diabetes burden, with estimates suggesting a considerable number of undiagnosed cases alongside known diabetics. The prevalence of diabetes varies across different ethnic communities, with higher rates observed in certain populations. This underscores the importance of tailored approaches to education and healthcare delivery.

iii. In response to these challenges, there is a pressing need for enhanced nutrition education, particularly among the cosmopolitan diabetic population. However, this demand outpaces the capacity of healthcare providers, who often face resource constraints. Thus, there is an urgent call for the development of innovative and culturally relevant control strategies to meet the evolving needs of diabetic patients in Africa.

Several mathematical models have been developed to for the management of diabetes mellitus. [11] highlighted diabetes as a chronic disease imposing a substantial and escalating socio-economic burden on individuals, families, and society at large. The researchers propose an optimal control framework for modeling the progression from pre-diabetes to diabetes, with and without complications. Their work demonstrates the existence of an optimal control strategy and employs a numerical implicit finite-difference method to track population sizes across various compartments. Results from the model indicate that implementing optimal control measures can lead to a significant reduction in the number of individuals affected by diabetes, both with and without complications, over a 10-year period. [12] proposed a deterministic model that employs distinct control strategies for two separate compartments: one for managing patients before complications arise and another for those already experiencing complications. By treating these groups separately, the study aims to provide proactive and comprehensive care to both prevent and mitigate complications. To achieve this, a fixed-time optimal control problem formulation is utilized to identify the optimal combination of interventions that minimize implementation costs and the incidence of complications within the population. Using Pontryagin's maximum principle, the study derives an optimality system and presents numerical solutions from simulations. These simulations offer valuable insights into the efficacy of the proposed approach in managing diabetes and reducing its burden on individuals and society. [13] addressed diabetes as a chronic ailment imposing a significant burden on both individuals and society as a whole. Their study focuses on constructing an optimal control mathematical model aimed at managing the development of the diabetic population. The model incorporates dynamics related to disabled individuals resulting from diabetes. Additionally, the researchers propose an optimal control approach to mitigate the burden of pre-diabetes. The control strategy aims to prevent the progression of pre-diabetes to diabetes, both with and without complications. The study discusses the existence and characterization of optimal control, with the Pontryagin minimum principle serving as a key analytical tool. Results suggest the presence of optimal control in the mathematical optimization problem concerning the diabetic population model. Furthermore, the efficacy of the optimal control variable (prevention) is shown to be significantly influenced by the number of healthy individuals. [14] introduced an optimal control approach to model the progression from pre-diabetes to diabetes, considering the presence or absence of complications and the influence of living environment. The research demonstrates the existence of an optimal control strategy and employs a numerical implicit finite-difference method to track population sizes within each compartment. Furthermore, [15] in their study proposed an optimal control strategy aimed at raising awareness among individuals with diabetes about the severity of complications associated with the condition, as well as the detrimental effects of an imbalanced lifestyle and environmental factors. Their strategy includes provisions for treatment and psychological support. The researchers employ Pontryagin's maximum principle to delineate the optimal controls and utilize an iterative method to solve the resulting optimality system. Numerical simulations conducted using MATLAB serve to validate the theoretical analysis. [16] delved into the complexities of type 1 diabetes, a severe condition impacting numerous children and adolescents. The disease disrupts the pancreas's ability to produce insulin, the hormone crucial for regulating blood sugar levels. Their study focuses on a mathematical model encompassing the entire blood glucose-insulin system, derived from Bergman's minimal model and tailored to interpret intravenous glucose tolerance tests (IVGTT). The researchers aim to devise a therapeutic regiment tailored to the specific needs of diabetic patients through this mathematical model. Using MATLAB/Simulink TM, they illustrate the results obtained from various examples. [17] developed and analyzed a mathematical model for the dynamics of diabetes mellitus and its complications, incorporating control measures. In this study, the model integrates positive lifestyle choices, such as abstinence from alcohol, smoking, and overeating, along with effective management of diabetes as control strategies. The analytical solution of the model equations is derived using the Homotopy Perturbation Method, while numerical simulations are performed using Maple 18 Mathematical software. The study varies the parameters and presents their effects on the model dynamics graphically. The results demonstrated that the two control measures can effectively reduce the incidence and evolution of complications of diabetes, thereby lowering the morbidity and mortality rates associated with diabetes complications.

In this study, we improved on the work of [17] and applied an optimal control strategy on the formulated model and analyze the impact of the control strategies on the incidence of medical complications. The paper is organized as follows: Section 1 is the introduction, Section 2, the compartmental model for the dynamics of Diabetes and its complication, formulation of optimal control problem and solution to the problem is obtained, discussion of results was done in Section 3 while in Section 4, conclusion was made.

2. The Mathematical Model

The existing model without control measures developed by [17], shown in Figure 2.1, describes the dynamics of diabetes and its complications within a population. This compartmental model categorizes the population into five distinct groups: healthy individuals (H), individuals at risk of developing diabetes (S), diabetics without complications (D), diabetics with complications (C), and diabetics undergoing treatment for complications (T). The model tracks the transitions between these groups over time, driven by various rates and parameters that reflect the progression of diabetes and its complications.

The original system of differential equations governing these dynamics is as follows:

$$\frac{dH(t)}{dt} = \beta\theta - \tau H(t) + \sigma S(t) - \mu H(t) \tag{2.1}$$

$$\frac{dS(t)}{dt} = \beta(1-\theta) - \mu S(t) - \alpha S(t) - \sigma S(t) + \tau H(t) \tag{2.2}$$

$$\frac{dD(t)}{dt} = \alpha S(t) - \mu D(t) - \lambda D(t) + \omega T(t) \tag{2.3}$$

$$\frac{dC(t)}{dt} = \lambda D(t) - \gamma C(t) - \delta C(t) - \mu C(t) \tag{2.4}$$

$$\frac{dT(t)}{dt} = \gamma C(t) - \omega T(t) - \mu T(t) \tag{2.5}$$

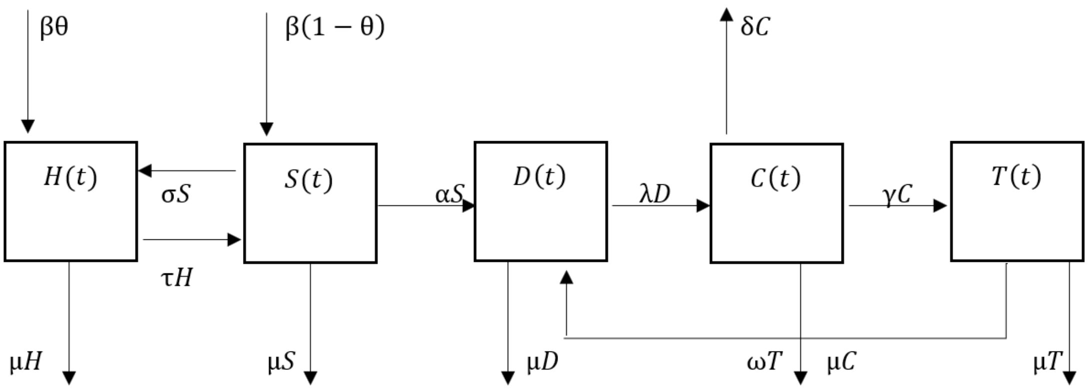

Figure 2.1: Diagram of the model without control.

Table 2.1: Description of variables of the uncontrolled model.

S/N	**Variables**	**Description**
1	$H(t)$	Healthy class
2	$S(t)$	Susceptible class
3	$D(t)$	Diabetics without complications class
4	$C(t)$	Diabetics with complications class
5	$T(t)$	Diabetics with complications undergoing treatment class

Table 2.2: Description of parameters of the uncontrolled model.

S/N	Variables	Parameters
1	α	Probability rate of incidence of diabetes
2	β	Birth rate
3	μ	Natural mortality rate
4	τ	Rate at which healthy individual become susceptible
5	σ	Rate at which susceptible individual become healthy
6	λ	Rate at which D(t) develop a complication
7	γ	Rate at which C(t) are treated
8	ω	Rate at which C(t) after treatment return to D(t)
9	δ	Mortality rate due to complications
10	θ	Proportion of children born into the healthy class
11	$1-\theta$	Proportion of children born into the susceptible class

2.1. Formulating the optimal control problem

In this section, we formulate the optimal control problem aimed at minimizing the incidence of medical complications in a population of diabetic patients. The process involves two key steps which are: defining the control variables and modifying the model equations to include these control variables.

2.2. Definition of the control variables

To effectively manage and reduce the incidence of medical complications among diabetic patients, we introduce control variables into the model. These control variables represent practical interventions that can influence the rates of disease progression and treatment within the population. Specifically, we consider two control measures:

$u_1(t)$: This control variable represents a measure aimed at reducing the incidence rate of complications.

$u_2(t)$: This control variable represents a measure aimed at increasing the treatment rate of complications.

2.3. Modification the model equations

The system of differential equations was modified to incorporate the control variables into the existing model. The control variables $u_1(t)$ *and* $u_2(t)$ were introduced to represent interventions aimed at reducing the rate of complications and increasing the rate of treatment, respectively. By including these controls, the model was designed to directly influence the progression and management of complications within the population. The modified system of equations is outlined as follows:

$$\frac{dH(t)}{dt} = \beta\theta - \tau H(t) + \sigma S(t) - \mu H(t) \tag{2.6}$$

$$\frac{dS(t)}{dt} = \beta(1-\theta) - \mu S(t) - \alpha S(t) - \sigma S(t) + \tau H(t) \tag{2.7}$$

$$\frac{dD(t)}{dt} = \alpha S(t) - \mu D(t) - (\lambda - u_1(t))D(t) + \omega T(t) \tag{2.8}$$

$$\frac{dC(t)}{dt} = (\lambda - u_1(t))D(t) - (\gamma + u_2(t))C(t) - \delta C(t) - \mu C(t) \tag{2.9}$$

$$\frac{dT(t)}{dt} = (\gamma + u_2(t))C(t) - \omega T(t) - \mu T(t) \tag{2.10}$$

where:

$(\lambda - u_1(t))$ reflects the reduced rate at which diabetics without complications $D(t)$ develop complications due to the intervention $u_1(t)$,

$(\gamma + u_2(t))$ reflects the increased rate at which diabetics with complications $C(t)$ receive treatment due to the intervention $u_2(t)$.

2.4. Objective functional

The objective of the optimal control problem is to minimize the total number of diabetics with complications over a fixed time horizon $[0,T]$ while considering the costs associated with implementing control measures. The objective function is formulated as:

$$J = \int_0^T \left[A_1 C(t) + A_2 u_1^2(t) + A_3 u_2^2(t)\right] dt, \tag{2.11}$$

where:

A_1 = weight factor for $C(t)$ which represents the number of diabetics with complications.

A_2 = weight factor for the control effort $u_1(t)$, which reduces the incidence rate of complications.

A_3 = weight factor for the control effort $u_2(t)$, which increases the treatment rate of complications.

To ensure that the control functions are feasible in a practical scenario, we place the following constraints on the controls $u_1(t)$ and $u_2(t)$

$$0 \le u_1(t) \le 1 \text{ and } 0 \le u_2(t) \le 1.$$

These constraints imply that the control efforts are bounded and can range between no control $u_1(t) = 0, u_2(t) = 0$ and full control $u_1(t) = 1, u_2(t) = 1$.

The state variables $H(t), S(t), D(t), C(t), T(t)$ have initial conditions that represent the population sizes at the start of the control period:

$$H(0) = H_0,\ S(0) = S_0,\ D(0) = D_0,\ C(0) = C_0,\ T(0) = T_0. \tag{2.12}$$

2.5. Hamiltonian system

The Hamiltonian function H includes the objective function we want to minimize and the dynamics of the state variables. It is given by:

$$\mathcal{H}(x,u,\lambda,t) = L(x,u,t) + \lambda_T f(x,u,t). \tag{2.13}$$

After applying this to the model, we get:

$$\mathcal{H} = A_1C(t) + A_2u_1^2(t) + A_3u_2^2(t) + \lambda_H\frac{dH(t)}{dt} + \lambda_S\frac{dS(t)}{dt} + \lambda_D\frac{dD(t)}{dt} + \lambda_C\frac{dC(t)}{dt} + \lambda_T\frac{dT(t)}{dt} \tag{2.14}$$

where:

λH, λS, λD, λC, and λT are the adjoint variables.

The Hamiltonian $\mathcal{H}$ is constructed as follows, including the state dynamics, controls, and adjoint variables:

$$\mathcal{H} = A_1C(t) + A_2u_1^2(t) + A_3u_2^2(t) + \lambda_H(\beta\theta - \tau H(t) + \sigma S(t) - \mu H(t)) + \lambda_S (\beta(1-\theta) - \mu S(t) - \alpha S(t) - \sigma S(t) + \tau H(t)) + \lambda_D(\alpha S(t) - \mu D(t) - (\lambda - u_1(t))D(t) + \omega T(t)) + \lambda_C ((\lambda - u_1(t))D(t) - (\gamma + u_2(t))C(t) - \delta C(t) - \mu C(t)) + \lambda_T((\gamma + u_2(t))C(t) - \omega T(t) - \mu T(t)). \tag{2.15}$$

2.6. Solving the adjoint variables

The role of adjoint variables in optimal control theory is to provide necessary conditions for optimality. They help in deriving the optimal control laws for a given system [18]. Specifically, adjoint variables are part of Pontryagin's Maximum Principle, which is a fundamental method used to find the best possible control for a dynamic system over a given period of time [19].

The adjoint variables $\lambda(t)$ evolve according to:

$$\frac{d\lambda(t)}{dt} = -\frac{\partial\mathcal{H}}{\partial x}. \tag{2.16}$$

The adjoint equations are derived from the partial derivatives of the Hamiltonian with respect to the state variables. The adjoint variables for each state variables are:

$$\frac{d\lambda_H}{dt} = -\frac{\partial\mathcal{H}}{\partial H} \tag{2.17}$$

$$\frac{d\lambda_S}{dt} = -\frac{\partial\mathcal{H}}{\partial S} \tag{2.18}$$

$$\frac{d\lambda_D}{dt} = -\frac{\partial\mathcal{H}}{\partial D} \tag{2.19}$$

$$\frac{d\lambda_C}{dt} = -\frac{\partial\mathcal{H}}{\partial C} \tag{2.20}$$

$$\frac{d\lambda_T}{dt} = -\frac{\partial\mathcal{H}}{\partial T} \tag{2.21}$$

Solving the adjoint variables results to

$$\frac{d\lambda_H}{dt} = \lambda_H \cdot \tau + \lambda_H \cdot \mu - \lambda_S \cdot \tau \tag{2.22}$$

$$\frac{d\lambda_S}{dt} = \lambda_S \cdot \mu + \lambda_S \cdot \alpha + \lambda_S \cdot \sigma - \lambda_H \cdot \sigma - \lambda_D \cdot \alpha \tag{2.23}$$

$$\frac{d\lambda_D}{dt} = \lambda_D \cdot \mu + \lambda_D \cdot \lambda - \lambda_D \cdot u_1 - \lambda_C \cdot \lambda + \lambda_C \cdot u_1 \tag{2.24}$$

$$\frac{d\lambda_C}{dt} = \lambda_C \cdot \gamma + \lambda_C \cdot u_2 + \lambda_C \cdot \delta + \lambda_C \cdot \mu - \lambda_T \cdot \gamma - \lambda_T \cdot u_2 - A_1 \tag{2.25}$$

$$\frac{d\lambda_T}{dt} = \lambda_T \cdot \omega + \lambda_T \cdot \mu - \lambda_D \cdot \omega \tag{2.26}$$

2.7. Optimality conditions

To find the optimal control laws $u_1(t)$ and $u_2(t)$, the derivative of the Hamiltonian is taken with respect to each control and set to zero.

$$\frac{\partial H}{\partial u_1(t)} = 0, \quad \frac{\partial H}{\partial u_2(t)} = 0. \tag{2.27}$$

This will give the optimal control laws for $u_1(t)$ and $u_2(t)$:

$$u_1 = \frac{(\lambda_D \cdot \lambda \cdot D) - (\lambda_C \cdot \lambda \cdot D)}{2 \cdot A_2}, \tag{2.28}$$

$$u_2 = \frac{(\lambda_C \cdot \gamma \cdot C) \cdot (\lambda_T \cdot \gamma \cdot C)}{2 \cdot A_3}. \tag{2.29}$$

3. Results

The parameters used in the model were carefully selected from reliable sources to reflect real-world dynamics. Specifically, problem-specific values for rates such as the development of complications, disease onset, natural mortality, treatment success, and others were obtained from the International Diabetes Federation (IDF) database, which provides detailed information on diabetes trends and statistics globally. These values helped to ensure that the model reflects realistic scenarios faced by healthcare systems dealing with diabetes management.

Table 3.1: Description of parameters of the uncontrolled model with specific values.

S/N	Variables	Parameters	Specific Values
1	α	Probability rate of incidence of diabetes	0.1
2	β	Birth rate	0.5
3	μ	Natural mortality rate	0.01
4	τ	Rate at which healthy individual become susceptible	0.2
5	σ	Rate at which susceptible individual become healthy	0.05
6	λ	Rate at which $D(t)$ develop a complication	0.2
7	γ	Rate at which $C(t)$ are treated	0.3
8	ω	Rate at which $C(t)$ after treatment return to $D(t)$	0.4
9	δ	Mortality rate due to complications	0.05
10	θ	Proportion of children born into the healthy class	0.8
11	1–θ	Proportion of children born into the susceptible class	0.2

3.1. Graphical representation of results of model without control

The initial phase of the analysis involved simulating the dynamics of the diabetic population without the implementation of any control measures. The following graphs illustrate the population changes over a period of 10 years for five key subpopulations.

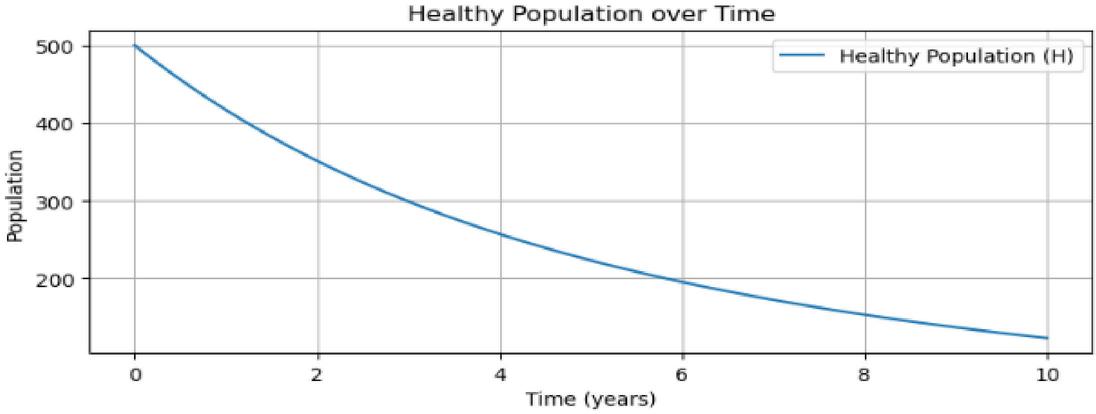

Figure 3.1: Graph of the healthy population without control.

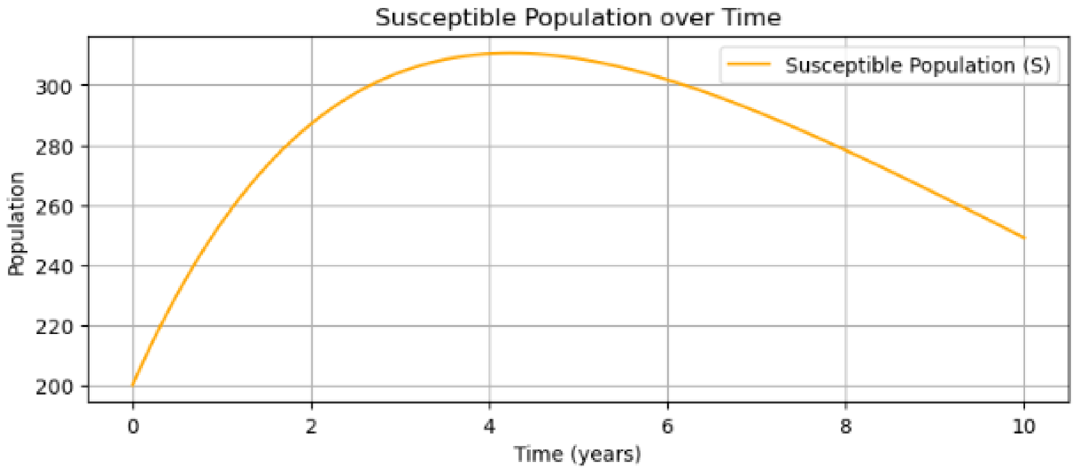

Figure 3.2: Graph of the susceptible population without control.

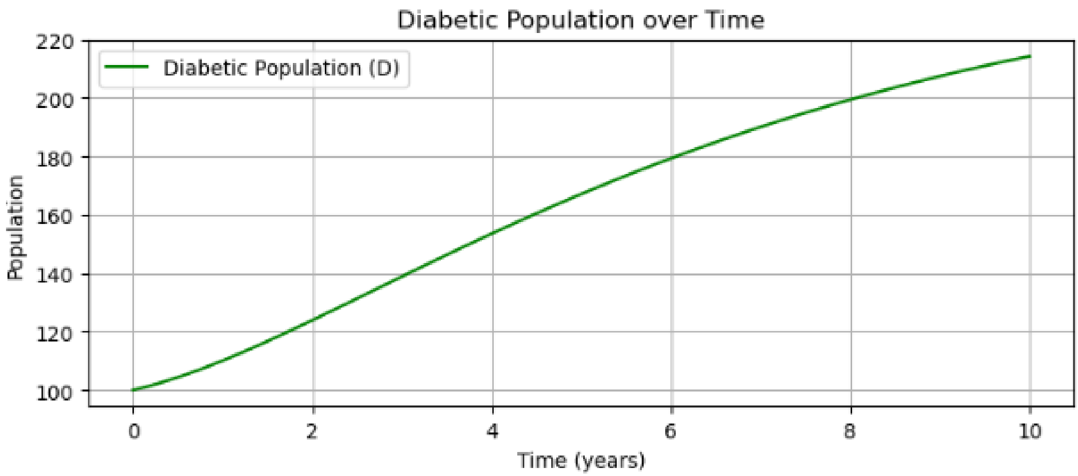

Figure 3.3: Graph of the diabetic population without control.

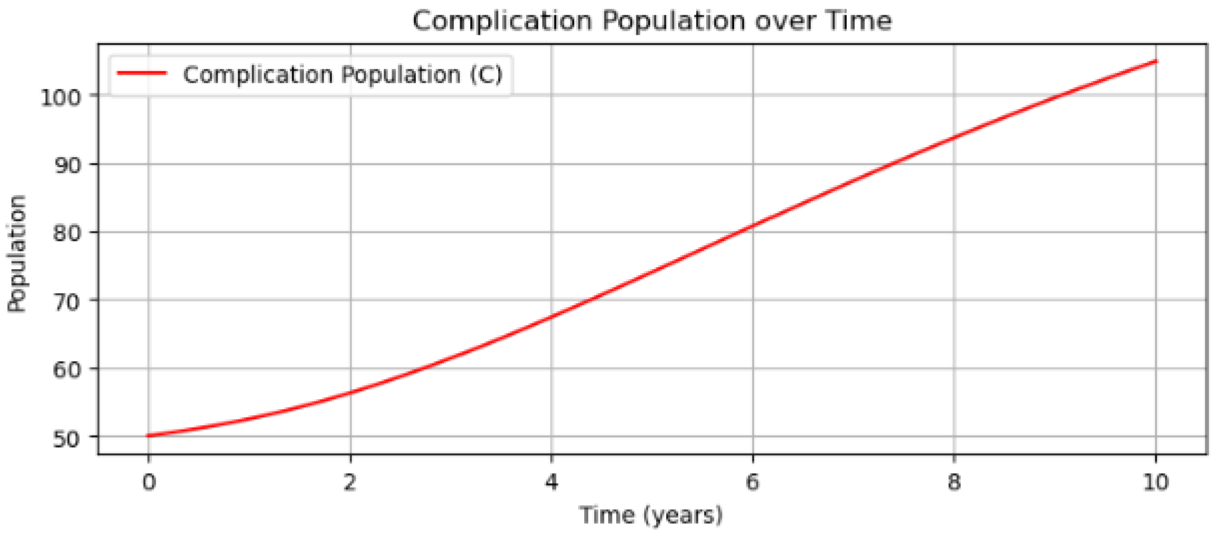

Figure 3.4: Graph of the diabetic with complication population without control**.**

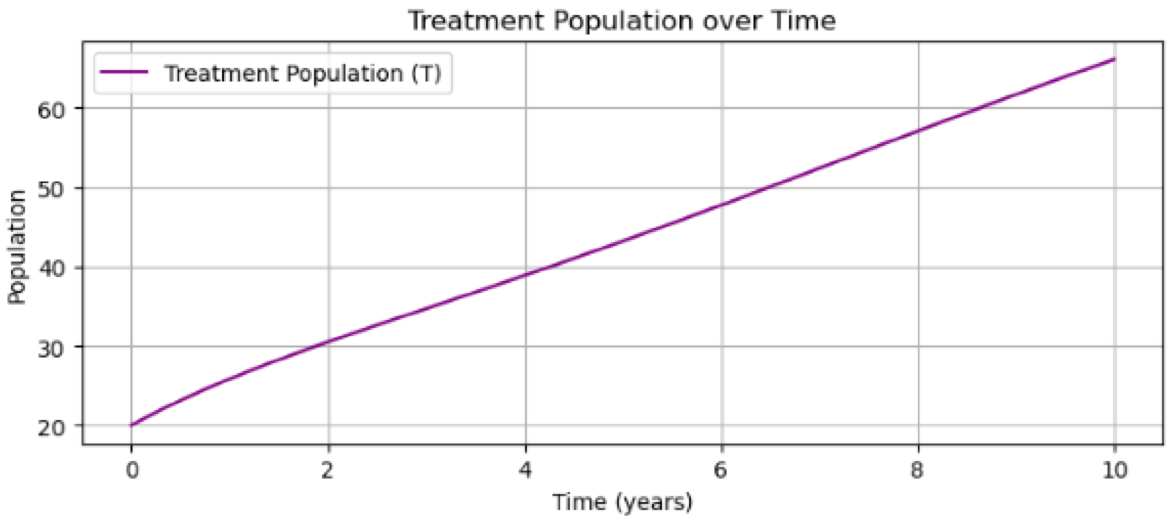

Figure 3.5: Graph of the treatment of diabetic with complication population without control.

3.2. Graphical representation of results of model with control

In this section, the modified model incorporating optimal control strategies aimed at managing the incidence and treatment of medical complications in a diabetic patient population was presented. Unlike the initial model without control, where the dynamics of the population were allowed to progress naturally, this version introduces two control measures denoted as $u_1(t)$ and $u_2(t)$ designed to mitigate complications and improve treatment outcomes over a period of 10 years.

The goal of implementing controls was to optimize the population dynamics by:

i. Reducing the number of patients who develop complications due to diabetes.
ii. Increasing the rate at which patients with complications receive treatment.

Specifically, the introduction of controls aims to maintain a lower population in the complication class while simultaneously improving the effectiveness of treatment measures, leading to better overall health outcomes.

3.3. Description of control variables

The control variables, $u_1(t)$ and $u_2(t)$ reflect real-life strategies that can be applied to achieve the above objectives:

i. $u_1(t)$ - (Patient education, regular monitoring, and preventive treatment):

This control measures efforts such as awareness campaigns, preventive care, and frequent health monitoring to reduce the transition of individuals from the diabetic class to the complication classes. In this model, a value of $u_1(t) = 0.19$ indicates moderate but effective application of these preventive strategies. The primary influence of $u_1(t)$ is on reducing the rate at which patients develop complications by controlling the parameter λ which governs complication progression.

ii. $u_2(t)$ - (Comprehensive care for complications, accessibility of healthcare, rehabilitation, and follow-up care):

This control represents medical interventions aimed at treating complications and managing the long-term care of diabetic patients. A value of $u_2(t) = 0.2$ reflects the quality and accessibility of healthcare services, rehabilitation programs, and follow-up care. In the model, this influences the parameter γ which governs the recovery rate from complications, improving patient outcomes by increasing the rate of recovery from complications.

In the preceding sections, the results of the model without control were presented, where complications grew significantly, and treatment efforts struggled to keep pace. Here, the results from the model with control to those of the uncontrolled model were being compared. The controls are expected to yield more desirable population outcomes, including lower complication rates and improved recovery rates.

For this simulation, the Runge-Kutta 4th order (RK4) method to numerically solve the system of differential equations representing the population dynamics was implemented. The parameters for the modified model were chosen based on available data and adjusted to reflect the influence of the control variables.

The following graph illustrated the dynamics of the healthy, susceptible, diabetic, complication, and treatment populations with the applied controls. The time horizon for this simulation still remained 10 years, and the effects of $u_1(t)$ and $u_2(t)$ on the respective populations are shown, providing a direct comparison to the previously presented uncontrolled model.

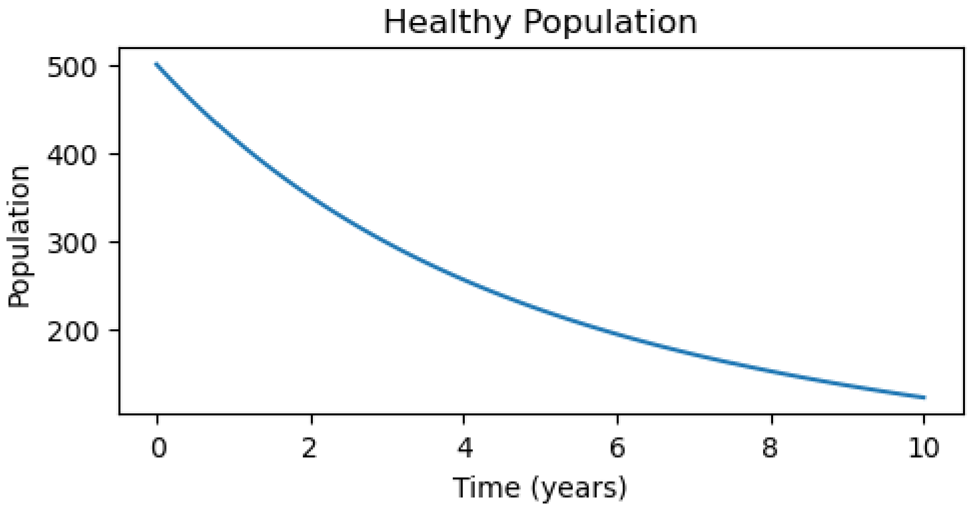

Figure 3.6: Graph of the healthy population with control.

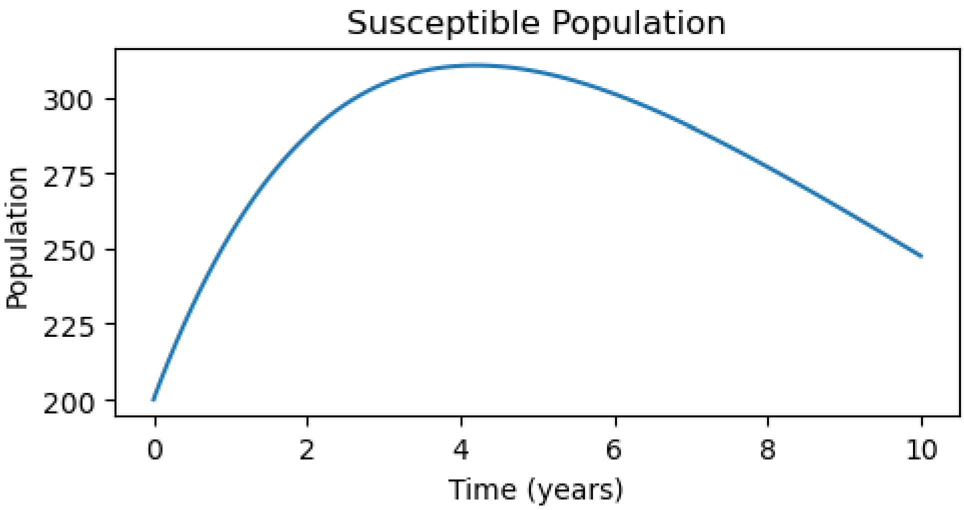

Figure 3.7: Graph of the susceptible population with control.

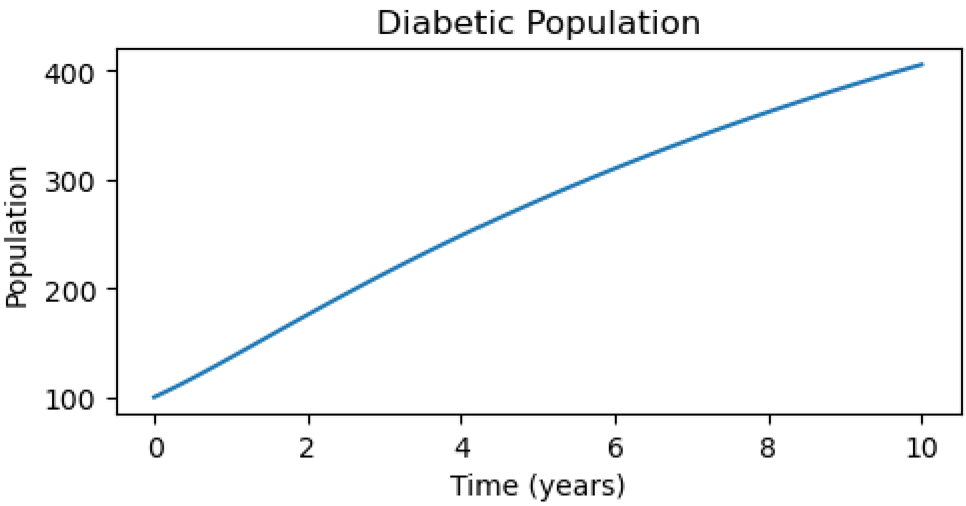

Figure 3.8: Graph of the diabetic population with control.

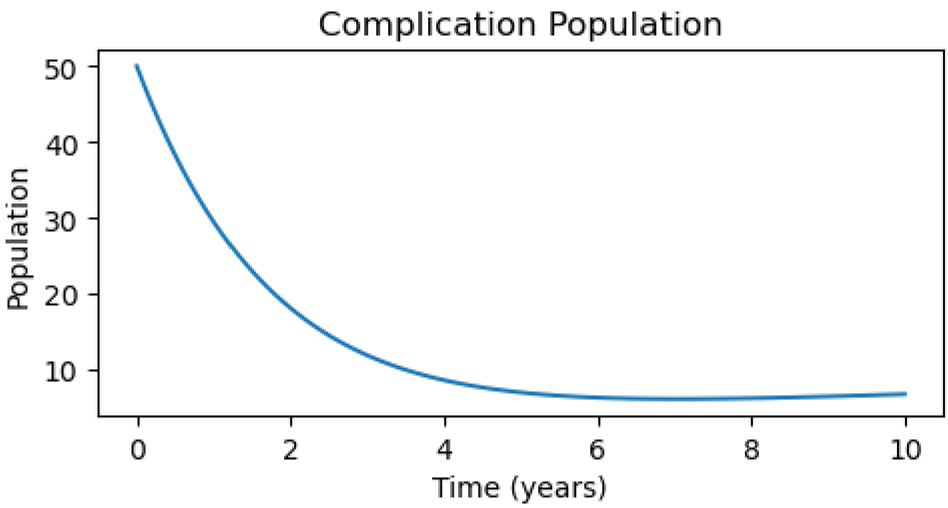

Figure 3.9: Graph of the diabetic with complication population with control.

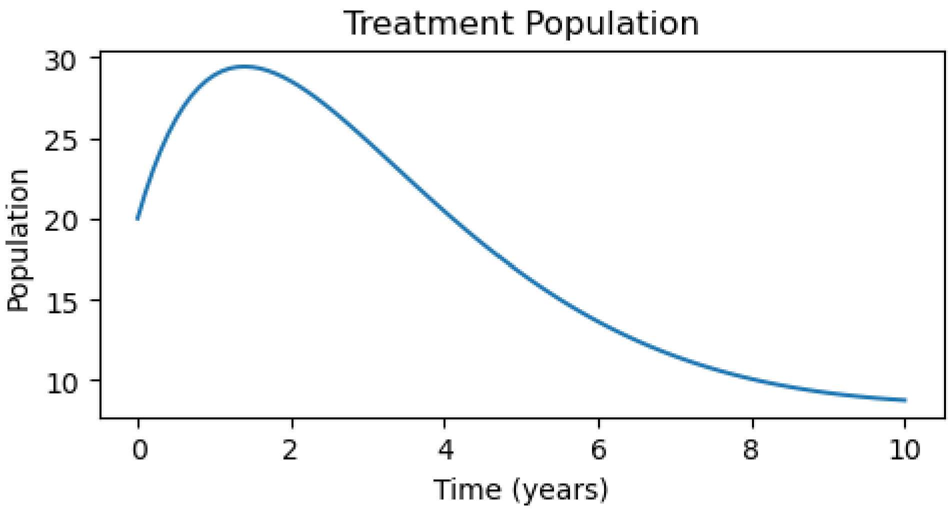

Figure 3.10: Graph of the treatment of diabetic with complication population with control.

The observed increase in the treatment population during the early years can be attributed to the immediate effects of implementing control measures. Initially, enhanced patient education, regular monitoring, and preventive treatments (represented by u_1) lead to greater awareness of complications among diabetic patients. This heightened awareness results in an increase in the number of individuals seeking treatment.

However, as time progresses and these control measures take effect, a noticeable decrease in the complication population $C(t)$ occurs. This reduction implies that fewer patients require treatment, leading to a decline in the treatment population $T(t)$. Therefore, the initial rise in the treatment class is countered by the subsequent decrease in complications, demonstrating the importance of comprehensive care for complications, accessibility to healthcare, rehabilitation, and follow-up care (represented by u_2).

Interestingly, the healthy and susceptible populations remain relatively unchanged despite the implementation of control measures. This phenomenon can be attributed to the stability of the transition rates between these classes, as the parameters governing these transitions (such as σ, τ) and some other parameters have not been altered. As a result, the dynamics of healthy individuals transitioning to susceptible status and vice versa do not exhibit significant shifts, highlighting that while control measures impact treatment and complications, they may not directly affect the underlying population distributions of health and susceptibility.

Table 3.2: Description of parameters of the controlled model with specific values

S/N	**Variables**	**Parameters**	**Specific Values after Control**
1	α	Probability rate of incidence of diabetes	0.1
2	β	Birth rate	0.5
3	μ	Natural mortality rate	0.01
4	τ	Rate at which healthy individual become susceptible	0.2
5	σ	Rate at which susceptible individual become healthy	0.05

6	λ	Rate at which $D(t)$ develop a complication	0.01
7	γ	Rate at which $C(t)$ are treated	0.5
8	ω	Rate at which $C(t)$ after treatment return to $D(t)$	0.4
9	δ	Mortality rate due to complications	0.05
10	θ	Proportion of children born into the healthy class	0.8
11	$1-\theta$	Proportion of children born into the susceptible class	0.2

4. Conclusion

In this study, the dynamics of diabetic populations with a focus on the impact of control measures on health outcomes was investigated. Through mathematical modeling, the interactions between healthy, susceptible, diabetic, complication, and treatment populations, revealing significant insights into the effects of control strategies were explored. The results indicated that implementing control measures, such as patient education, regular monitoring, and comprehensive care, led to an initial increase in the treatment population as patients became more aware of complications and sought for medical assistance. However, as those measures took effect, there was a consequential reduction in the complication population, resulting in fewer individuals requiring treatment over time. This underscored the effectiveness of preventive strategies in managing diabetic complications. Moreover, the analysis highlighted that while control measures significantly influenced the treatment and complication dynamics, the healthy and susceptible populations remained relatively stable. This stability suggested that transition rates between these classes are not markedly affected by the control interventions, emphasizing the need for ongoing public health initiatives aimed at promoting healthy lifestyle and preventing the onset of diabetes.

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