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Concept of Anti Multigroups and its Properties

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Abstract

The concept of multigroups is an application of multiset to group theory. Multigroup is an algebraic structure of a multiset whose underlying set is a group. The objective of this paper is to introduce the concept of anti multigroups and deduce some related results. We establish that a multiset defined over a group is a multigroup if and only if its complement is an anti multigroup. Finally, some results that connect cuts of multigroups to anti multigroups are considered.

1. Introduction

The term multisets as buttressed by Knuth [22], was first suggested by N. G. de Bruijn (cf. [6]) in a private communication to D. E. Knuth, as an important generalization of set theory, by relaxing the idea of distinct collection of elements in a set. Multiset theory has been explored in literature [9, 21, 25, 27]. The notion of multisets is a boost to the concept of multigroups via multisets, which generalizes group theory. Nazmul et al. [23] proposed the concept of multigroups in multisets framework and presented a number of results. The notion is parallel to fuzzy groups [24]. A comprehensive account on the concept of multigroups was carried out in [18], and it was established that multigroup via multiset is a generalization of group theory.

The concept of multigroups via multisets has been researched upon since inception.

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A number of algebraic properties of order of an element in a multigroup were considered in [3] and some results on multigroups which cut across some homomorphic properties were explored in [4, 10]. The notions of upper and lower cuts of multigroups were proposed and discussed in details with some number of results in [7], and the notions were extended to homomorphic sense and a number of results were explored [14]. Some group's analogous concepts like normal subgroups, characteristic subgroups, direct product, cosets, factor groups and group actions, etc. have been established in multigroup context [1, 2, 8, 11-13, 15-17, 19, 20, 26].

The motivation of this paper is to extend the notion of anti fuzzy groups [5] to multigroups context. In this paper, we propose the notion of anti multigroups and obtain some of its properties. The paper is organized as follows: In Section 2, preliminaries on multisets and multigroups are reviewed. Section 3 introduces anti multigroups with some number of results. Meanwhile, Section 4 draws conclusion to the paper and suggests areas of future works.

2. Preliminaries

In this section, we review some existing definitions and results for the sake of completeness and reference.

Definition 2.1. [27] Let X be a set. A multiset A over X is just a pair $\langle X, C_A \rangle$, where

$$C_A : X \to \mathcal{N} = \{0, 1, 2, ...\}$$

is a function, such that for $x \in X$ implies $A(x)$ is a cardinal and $A(x) = C_A(x) > 0$, where $C_A(x)$ denoted the number of times an object x occur in A. Whenever $C_A(x) = 0$, implies $x \notin X$.

Any ordinary set B is actually a multiset $\langle B, \chi_B \rangle$, where χ_B is its characteristic function. The set X is called the ground or generic set of the class of all multisets containing objects from X.

Take X to be the set from which multisets are constructed. The multiset X^n is the set of all multisets of X such that no element occurs more than n times. Likewise, the multiset X^∞ is the set of all multisets of X such that there is no limit on the number of occurrences of an element. We denote the set of all multisets over X by $MS(X)$. Our interest is on $MS(X)$ that is contained in X^n.

For example, a multiset $A = [a, a, b, b, c, c, c]$ of $X = \{a, b, c\}$ can be represented as $A = [a^2, b^2, c^3]$. Other forms of multiset representations can be found in literature.

Definition 2.2. [21] Let X be a nonempty set and X^n be the multiset space defined over X. Then, for any $A \in MS(X) \subseteq X^n$, the *complement of* A in X^n denoted by A^c is a multiset such that $\forall x \in X$,

$$C_{A^c}(x) = n - C_A(x).$$

Henceforth, whenever we write $MS(X)$ implies the set of all multisets over X drawn from the multiset space X^n.

Definition 2.3. [27] Let $A, B \in MS(X)$. Then, A is called a *submultiset* of B written as $A \subseteq B$ if $C_A(x) \le C_B(x)\ \forall x \in X$. Also, if $A \subseteq B$ and $A \ne B$, then A is called a *proper submultiset* of B and denoted as $A \subset B$. A multiset is called the *parent* in relation to its submultiset.

Definition 2.4. [25] Let $A, B \in MS(X)$. Then, the *intersection*, *union* and *sum* of A and B, denoted by $A \cap B$, $A \cup B$ and $A + B$, respectively, are defined by the rules that for any object $x \in X$,

(i) $C_{A\cap B}(x) = C_A(x) \wedge C_B(x)$,

(ii) $C_{A\cup B}(x) = C_A(x) \vee C_B(x)$,

(iii) $C_{A+B}(x) = C_A(x) + C_B(x)$,

where $\wedge$ and $\vee$ denote minimum and maximum, respectively.

Definition 2.5. [25] Let $A, B \in MS(X)$. Then, A and B are *comparable* to each other if and only if $A \subseteq B$ or $B \subseteq A$, and $A = B$ if and only if $C_A(x) = C_B(x)$ $\forall x \in X$.

Definition 2.6. [15] Let X be a group. A multiset A over X is called a *multigroupoid* of X if for all $x, y \in X$,

$$C_A(xy) \geq C_A(x) \wedge C_A(y),$$

where C_A denotes count function of A from X into a natural number $\mathbb{N}$.

Definition 2.7. [15, 23] Let X be a group. A multiset A of X is said to be a *multigroup* of X if it satisfies the following two conditions:

(i) A is a multigroupoid of X,

(ii) $C_A(x^{-1}) = C_A(x)\ \forall x \in X$.

The set of all multigroups of X is denoted by $MG(X)$.

It can be easily verified that if A is a multigroup of X, then

$$C_A(e) = \bigvee_{x \in X} C_A(x)\ \forall x \in X,$$

that is, $C_A(e)$ is the tip of A, where e is the identity element of X.

Remark 2.1. [23] Let X be a group and A be a multiset over X. If

$$C_A(xy^{-1}) \geq C_A(x) \wedge C_A(y),$$

for all $x, y \in X$, then A is called a *multigroup* of X.

Definition 2.8. [15] Let $A \in MG(X)$. A submultiset B of A is called a *submultigroup* of A denoted by $B \sqsubseteq A$ if B is a multigroup. A submultigroup B of A is a *proper submultigroup* denoted by $B \sqsubset A$, if $B \sqsubseteq A$ and $A \neq B$.

Definition 2.9. [7] Let $A \in MG(X)$. Then, the sets $A_{[n]}$ and $A_{(n)}$ defined by

$$A_{[n]} = \{x \in X \mid C_A(x) \geq n,\ n \in \mathbb{N}\}$$

and

$$A_{(n)} = \{x \in X \mid C_A(x) > n,\ n \in \mathbb{N}\}$$

are called the *strong* and *weak upper cuts* of A. Cleary, $A_{(n)} \subseteq A_{[n]}$.

Theorem 2.1. [7] *Let* $A \in MG(X)$. *Then* $A_{[n]}, n \in \mathbb{N}$ *is a subgroup of X for* $n \leq C_A(e)$.

Definition 2.10. [23] The *inverse* of an element $x \in X$ in a multigroup A of X is defined by

$$C_A(x^{-1}) = C_{A^{-1}}(x) \ \forall x \in X.$$

It is deducible that, $C_{A^{-1}}(x) = C_A(x) = C_{(A^{-1})^{-1}}(x)$.

3. Anti Multigroups and Some Properties

This section presents anti multigroup as a multigroup in reverse order. We denote a group by *X* unless otherwise stated.

3.1. Concept of anti multigroups

Here, we define anti multigroup and discuss some of its properties.

Definition 3.1. Suppose *X* is a groupoid. Then, a multiset *A* of *X* is called an *anti multigroupoid* of *X* if

$$C_A(xy) \leq C_A(x) \vee C_A(y) \ \forall x, y \in X.$$

Definition 3.2. A multiset *A* of *X* is called an *anti multigroup* of *X* if the following conditions hold:

(i) $C_A(xy) \leq C_A(x) \vee C_A(y) \ \forall x, y \in X$.

(ii) $C_A(x^{-1}) \leq C_A(x) \ \forall x \in X$.

We denote the set of all anti multigroups of *X* by $AMG(X)$.

Example 3.1. Let $X = \{e, a, b, c\}$ be a group such that

$$ab = c, ac = b, bc = a, a^2 = b^2 = c^2 = e.$$

Then, the multiset $A = \{e^2, a^5, b^4, c^5\}$ is an anti multigroup of *X*.

Proposition 3.1. *If A is an anti multigroup of X*, *then the following hold*:

(i) $C_A(x^{-1}) = C_A(x)\ \forall x \in X$.

(ii) $C_A(e) \leq C_A(x)\ \forall x \in X$, *where e is the identity element of X.*

(iii) $C_A(x^n) \leq C_A(x)\ \forall x \in X,\ n \in \mathbb{N}$.

Proof. We present the verifications of (i) to (iii) as below.

(i) By Definition 3.2, $C_A(x^{-1}) \leq C_A(x)\ \forall x \in X$. Also,

$$C_A(x) = C_A((x^{-1})^{-1}) \leq C_A(x^{-1}).$$

This completes the proof of (i).

(ii) Suppose $x \in X$. Certainly, $xx^{-1} = e$. Thus,

$$\begin{aligned} C_A(e) = C_A(xx^{-1}) &\leq C_A(x) \vee C_A(x) \\ &= C_A(x). \end{aligned}$$

Hence $C_A(e) \leq C_A(x)\ \forall x \in X$.

(iii) For $n \in \mathbb{N}$, we have

$$\begin{aligned} C_A(x^n) &\leq C_A(x^{n-1}) \vee C_A(x) \\ &\leq C_A(x^{n-2}) \vee C_A(x) \vee C_A(x) \\ &\leq C_A(x) \vee C_A(x) \vee \ldots \vee C_A(x) \\ &= C_A(x)\ \forall x \in X. \end{aligned}$$

Proposition 3.2. *If A and B are anti multigroups of X, then $A \cap B$ is an anti multigroup of X.*

Proof. Let $x, y \in X$. We have

$$\begin{aligned} C_{A\cap B}(xy^{-1}) &= C_A(xy^{-1}) \wedge C_B(xy^{-1}) \\ &\leq [C_A(x) \vee C_A(y)] \wedge [C_B(x) \vee C_B(y)] \\ &= [C_A(x) \wedge C_B(x)] \vee [C_A(y) \wedge C_B(y)] \end{aligned}$$

$$= C_{A \cap B}(x) \vee C_{A \cap B}(y).$$

Hence the result.

Corollary 3.1. *If* $\{A_i\}_{i \in I}$ *is a family of anti multigroups of* X, *then* $\bigcap_{i \in I} A_i \in AMG(X)$.

Proof. Straightforward from Proposition 3.2.

Remark 3.1. Let $A, B \in AMG(X)$. Then, $A \bigcup B$ is not an anti multigroup of X except either $A \subseteq B$ or $B \subseteq A$.

Definition 3.3. The family of anti multigroups $\{A_i\}_{i \in I}$ of X is said to *have inf/sup assuming chain* if either $A_1 \subseteq A_2 \subseteq \ldots \subseteq A_n$ or $A_1 \supseteq A_2 \supseteq \ldots \supseteq A_n$, respectively.

Theorem 3.1. *Let* $\{A_i\}_{i \in I}$ *be a family of anti multigroups of X. If* $\{A_i\}_{i \in I}$ *have sup/inf assuming chain, then* $\bigcup_{i \in I} A_i \in AMG(X)$.

Proof. Let $A = \bigcup_{i \in I} A_i$, then $C_A(x) = \bigvee_{i \in I} C_{A_i}(x)$. We show that

$$C_A(xy^{-1}) \leq C_A(x) \vee C_A(y) \; \forall x, \; y \in X.$$

Let $C_A(x) > 0, C_A(y) > 0$, then we have $\bigvee_{i \in I} C_{A_i}(x) > 0, \bigvee_{i \in I} C_{A_i}(y) > 0$. From the fact that $\{A_i\}_{i \in I}$ possesses sup/inf assuming chain, $\exists i_0 \in I$ such that $C_{A_{i_0}}(x) = \bigvee_{i \in I} C_{A_i}(x)$, and also $\exists j_0 \in I$ such that $C_{A_{j_0}}(x) = \bigvee_{i \in I} C_{A_i}(x)$. Then, we have

Case I: $A_{i_0} \subseteq A_{j_0}$ or

Case II: $A_{j_0} \subseteq A_{i_0}$.

By Case I, we get $C_{A_{i_0}}(x) \leq C_{A_{j_0}}(x)$. And so

$$C_A(xy^{-1}) = C_{A_{j_0}}(xy^{-1})$$

$$\leq C_{A_{j_0}}(x) \vee C_{A_{j_0}}(y)$$

$$\leq C_{A_{i_0}}(x) \vee C_{A_{i_0}}(y)$$

$$= \bigvee_{i \in I} C_{A_i}(x) \vee \bigvee_{i \in I} C_{A_i}(y)$$

$$= C_A(x) \vee C_A(y).$$

By Case II, it implies that $C_{A_{j0}}(x) \le C_{A_{i0}}(x)$. Thus

$$\begin{aligned} C_A(xy^{-1}) &= C_{A_{i0}}(xy^{-1}) \\ &\le C_{A_{i0}}(x) \vee C_{A_{i0}}(y) \\ &\le C_{A_{j0}}(x) \vee C_{A_{j0}}(y) \\ &= \bigvee_{i \in I} C_{A_i}(x) \vee \bigvee_{i \in I} C_{A_i}(y) \\ &= C_A(x) \vee C_A(y). \end{aligned}$$

The proof is completed.

Theorem 3.2. *If A and B are anti multigroups of X, then the sum of A and B is an anti multigroup of X.*

Proof. Let $x, y \in X$. We have

$$\begin{aligned} C_{A \oplus B}(xy^{-1}) &= C_A(xy^{-1}) + C_B(xy^{-1}) \\ &\le [C_A(x) \vee C_A(y)] + [C_B(x) \vee C_B(y)] \\ &= [C_A(x) + C_B(x)] \vee [C_A(y) + C_B(y)] \\ &= C_{A \oplus B}(x) \vee C_{A \oplus B}(y). \end{aligned}$$

Hence $A \oplus B \in AMG(X)$.

Remark 3.2. Let $\{A_i\}_{i \in I} \in AMG(X)$. Then $\sum_{i \in I} A_i \in AMG(X)$.

Proposition 3.3. *A multiset A is an anti multigroup of X if and only if* $C_A(xy^{-1}) \le C_A(x) \vee C_A(y)\ \forall x, y \in X$.

Proof. Assume that *A* is an anti multigroup of *X*. Then the following conditions hold;

$$C_A(xy) \le C_A(x) \vee C_A(y)\ \forall x, y \in X \text{ and } C_A(x^{-1}) \le C_A(x)\ \forall x \in X.$$

By combining the conditions, we get

$$C_A(xy^{-1}) \leq C_A(x) \vee C_A(y) \; \forall x, \; y \in X.$$

Conversely, suppose the given condition is satisfied. Combining the following facts:

$$C_A(e) \leq C_A(x), \; C_A(x^{-1}) = C_A(x) \; \forall x \in X$$

and

$$C_A(xy) \leq C_A(x(y^{-1})^{-1}) \leq C_A(x) \vee C_A(y^{-1})$$

$$= C_A(x) \vee C_A(y) \; \forall x, \; y \in X,$$

we conclude that *A* is an anti multigroup of *X*.

Theorem 3.3. *If A is an anti multigroupoid of a finite group X, then A is an anti multigroup.*

Proof. Let $x \in X$, $x \neq e$. Since X is finite, x has a finite order. Thus $x^n = e \Rightarrow x^{-1} = x^{n-1}$. Now using the definition of an anti multigroupoid repeatedly, it follows that

$$C_A(x^{-1}) = C_A(x^{n-1}) = C_A(x^{n-2}x)$$

$$\leq C_A(x^{n-2}) \vee C_A(x)$$

$$\leq C_A(x) \vee \ldots \vee C_A(x)$$

$$= C_A(x).$$

Hence the result.

Theorem 3.4. *Let A be a multiset of X. Then* $A \in MG(X)$ *if and only if* $A^c \in AMG(X)$.

Proof. Suppose $A \in MG(X)$. It implies that, $\forall x, \; y \in X$, we have

$$C_A(xy^{-1}) \geq C_A(x) \wedge C_A(y)$$

$$\Rightarrow C_{(A^c)^c}(xy^{-1}) \geq C_{(A^c)^c}(x) \wedge C_{(A^c)^c}(y)$$

$$\Rightarrow 1 - C_{A^c}(xy^{-1}) \geq 1 - C_{A^c}(x) \wedge 1 - C_{A^c}(y)$$

$$\Rightarrow -C_{A^c}(xy^{-1}) \geq -1 + [1 - C_{A^c}(x) \wedge 1 - C_{A^c}(y)]$$

$$\Rightarrow C_{A^c}(xy^{-1}) \leq 1 - [1 - C_{A^c}(x) \wedge 1 - C_{A^c}(y)]$$

$$\Rightarrow C_{A^c}(xy^{-1}) \leq C_{A^c}(x) \vee C_{A^c}(y).$$

Hence $A^c \in AMG(X)$.

Conversely, suppose A^c is an anti multigroup of *X*. Then for all $x, y \in Y$, we have

$$C_{A^c}(xy^{-1}) \leq C_{A^c}(x) \vee C_{A^c}(y)$$

$$\Rightarrow 1 - C_A(xy^{-1}) \leq 1 - C_A(x) \vee 1 - C_A(y)$$

$$\Rightarrow -C_A(xy^{-1}) \leq -1 + [1 - C_A(x) \vee 1 - C_A(y)]$$

$$\Rightarrow C_A(xy^{-1}) \geq 1 - [1 - C_A(x) \vee 1 - C_A(y)]$$

$$\Rightarrow C_A(xy^{-1}) \geq C_A(x) \wedge C_A(y).$$

Hence $A \in MG(X)$.

Proposition 3.4. *Let* $A \in AMG(X)$. *If* $C_A(x) > C_A(y)$ *for some* $x, y \in X$. *Then* $C_A(xy) = C_A(x) = C_A(yx)$.

Proof. Suppose $C_A(x) > C_A(y)$ for some $x, y \in X$. Now,

$$C_A(xy) \leq C_A(x) \vee C_A(y) = C_A(x).$$

Similarly,

$$C_A(x) = C_A(xyy^{-1}) \leq C_A(xy) \vee C_A(y) = C_A(xy).$$

Thus, $C_A(xy) = C_A(x)$. In the same vein, $C_A(yx) = C_A(x)$. The result follows.

Proposition 3.5. *Let* $A \in AMG(X)$. *Then* $C_A(xy^{-1}) = C_A(e)$ *if and only if* $C_A(x) = C_A(y)$.

Proof. Assume that $C_A(xy^{-1}) = C_A(e)\ \forall x,\ y \in X$, where e is the identity of X. Then

$$C_A(x) = C_A(x(y^{-1}y)) = C_A((xy^{-1})y)$$

$$\leq C_A(xy^{-1}) \vee C_A(y)$$

$$= C_A(y).$$

Similarly,

$$C_A(y) = C_A((x^{-1}x)y^{-1}) = C_A(x^{-1}(xy^{-1}))$$

$$\leq C_A(x) \vee C_A(xy^{-1})$$

$$\leq C_A(x).$$

Hence $C_A(x) = C_A(y)$.

Conversely, assume $C_A(x) = C_A(y)\ \forall x,\ y \in X$. Thus, we have

$$C_A(xy^{-1}) = C_A(yy^{-1}) \Rightarrow C_A(xy^{-1}) = C_A(e).$$

Proposition 3.6. *Let* $A \in AMG(X)$. *Then* $C_A(xy) = C_A(y)\ \forall x,\ y \in X$ *if and only if* $C_A(x) = C_A(e)$.

Proof. Suppose $C_A(xy) = C_A(y)\ \forall y \in X$. Then by letting $y = e$, we have $C_A(x) = C_A(e)\ \forall x \in X$.

Conversely, suppose that $C_A(x) = C_A(e)$. Then $C_A(y) \geq C_A(x)$ and so

$$C_A(xy) \leq C_A(x) \vee C_A(y) = C_A(y).$$

Also,

$$C_A(y) = C_A(x^{-1}xy) \leq C_A(x) \vee C_A(xy)$$

$$= C_A(xy).$$

Hence $C_A(xy) = C_A(y)\ \forall y \in X$.

Theorem 3.5. *Let* $A \in AMG(X)$ *and if* $x, y \in X$ *with* $C_A(x) \neq C_A(y)$, *then* $C_A(xy) = C_A(yx) = C_A(x) \vee C_A(y)$.

Proof. Let $x, y \in X$. Since $C_A(x) \neq C_A(y)$, it implies that $C_A(x) < C_A(y)$ or $C_A(y) < C_A(x)$. Suppose $C_A(x) < C_A(y)$. Then $C_A(xy) \leq C_A(y)$ and

$$\begin{aligned} C_A(y) = C_A(x^{-1}xy) &\leq C_A(x^{-1}) \vee C_A(xy) \\ &= C_A(x) \vee C_A(xy) \\ &= C_A(xy). \end{aligned}$$

It follows that

$$\begin{aligned} C_A(y) \leq C_A(xy) &\leq C_A(x) \vee C_A(y) \\ &= C_A(y). \end{aligned}$$

From here, we see that $C_A(xy) \leq C_A(x) \vee C_A(y)$ and $C_A(x) \vee C_A(y) \leq C_A(xy)$ implying that $C_A(xy) = C_A(x) \vee C_A(y)$.

Similarly, suppose $C_A(y) < C_A(x)$. We have $C_A(yx) \leq C_A(x)$ and

$$\begin{aligned} C_A(x) = C_A(y^{-1}yx) &\leq C_A(y^{-1}) \vee C_A(yx) \\ &= C_A(y) \vee C_A(yx) \\ &= C_A(yx). \end{aligned}$$

Thus, we get

$$\begin{aligned} C_A(x) \leq C_A(yx) &\leq C_A(y) \vee C_A(x) \\ &= C_A(x). \end{aligned}$$

Clearly, $C_A(yx) = C_A(y) \vee C_A(x)$. Hence the result follows.

Corollary 3.2. *If A is an anti multigroup of X, then* $C_A(xy) = C_A(x) \vee C_A(y)$ $\forall x, y \in X$ *with* $C_A(x) \neq C_A(y)$.

Proof. Let $x, y \in X$. Assume that $C_A(x) < C_A(y)$, then

$$C_A(xy) \leq C_A(x) \vee C_A(y) = C_A(y) \ \forall x,\ y \in X$$

and

$$\begin{aligned} C_A(x) \vee C_A(y) = C_A(x^{-1}xy) &\leq C_A(x^{-1}) \vee C_A(xy) \\ &= C_A(x) \vee C_A(xy) \\ &= C_A(xy). \end{aligned}$$

Thus $C_A(xy) = C_A(x) \vee C_A(y)$.

3.2. Cuts of anti multigroups

In this subsection, we propose the idea of cuts of anti multigroups and outline some results.

Definition 3.4. Let $A \in AMG(X)$. Then, the set $\mathbf{A}_{[n]}$ for $n \in \mathbb{N}$ defined by

$$\mathbf{A}_{[n]} = \{x \in X \mid C_A(x) \leq n\}$$

is called a *cut* of A.

Clearly, $\mathbf{A}_{[n]} \cup A_{[n]} = X$ for $n \in \mathbb{N}$.

Proposition 3.7. *Let A be an anti multigroup of X. Then for* $n \in \mathbb{N}$ *such that* $n \geq C_A(e)$, $\mathbf{A}_{[n]}$ *is a subgroup of X.*

Proof. For all $x,\ y \in \mathbf{A}_{[n]}$, it follows that

$$C_A(xy^{-1}) \leq [C_A(x) \vee C_A(y)] \leq n,$$

which concludes the proof.

Proposition 3.8. *Let A be a multiset of X such that* $\mathbf{A}_{[n]}$ *is a subgroup of* $X \ \forall n \in \mathbb{N}$ *with* $n \geq C_A(e)$. *Then A is an anti multigroup of X.*

Proof. Let $x,\ y \in X$ and $C_A(x) = n_1,\ C_A(y) = n_2$. Suppose $n_2 \geq n_1$. Then $x,\ y \in \mathbf{A}_{[n]}$ so that $xy^{-1} \in \mathbf{A}_{[n]}$. Hence

$$C_A(xy^{-1}) \leq n_2 = n_1 \vee n_2 = C_A(x) \vee C_A(y).$$

4. Conclusion

We have proposed the concept of anti multigroups and deduced some properties of anti multigroups. It was established that a multiset of a group is a multigroup if and only if the complement of the multiset is an anti multigroup. For future research, some analogous results in multigroups could be investigated in anti multigroup setting.

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